Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.3-a+b-x^2-^p-c+d-x^2-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [349]. This is test number [20].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (349)	% 0. (0)
Mathematica	% 100. (349)	% 0. (0)
Maple	% 70.49 (246)	% 29.51 (103)
Maxima	% 6.88 (24)	% 93.12 (325)
Fricas	% 38.97 (136)	% 61.03 (213)
Sympy	% 30.37 (106)	% 69.63 (243)
Giac	% 31.81 (111)	% 68.19 (238)

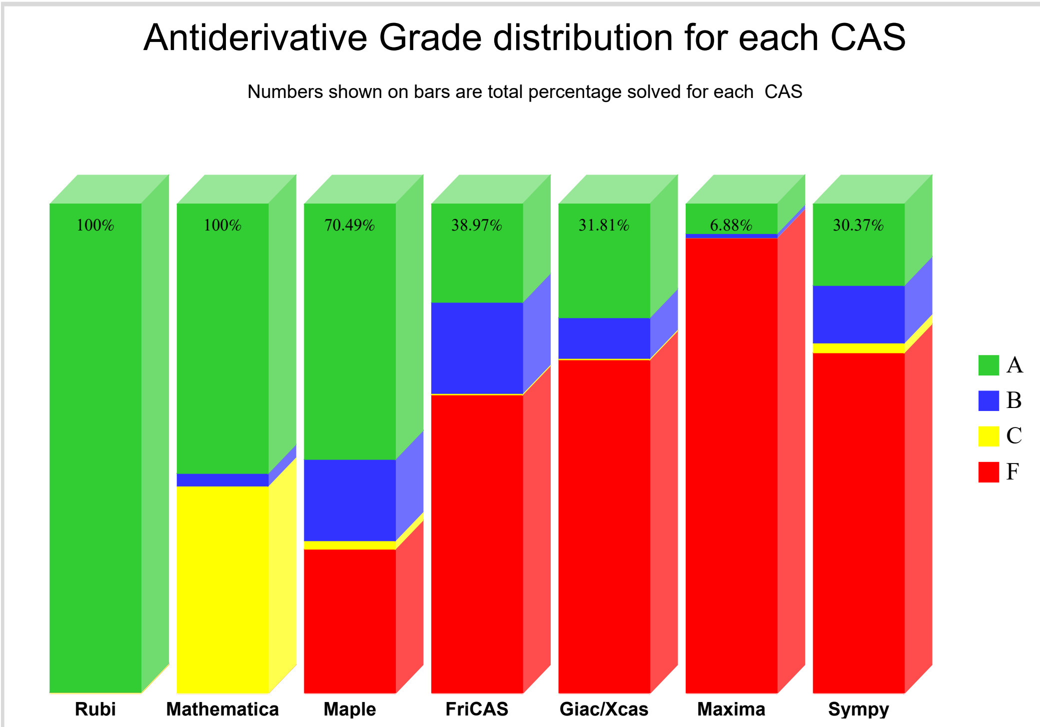
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

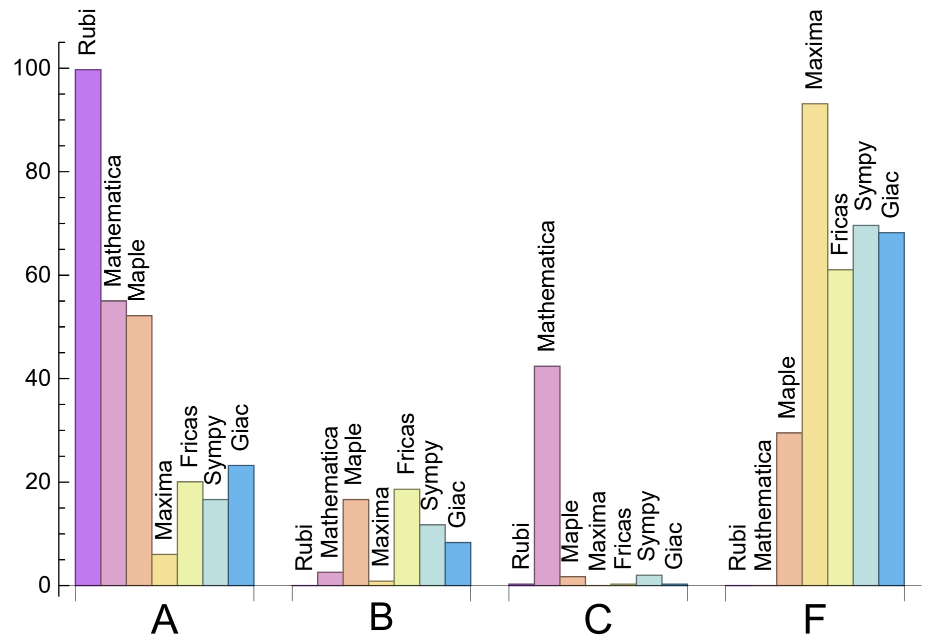
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.71	0.	0.29	0.
Mathematica	55.01	2.58	42.41	0.
Maple	52.15	16.62	1.72	29.51
Maxima	6.02	0.86	0.	93.12
Fricas	20.06	18.62	0.29	61.03
Sympy	16.62	11.75	2.01	69.63
Giac	23.21	8.31	0.29	68.19

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	178.99	1.	116.	1.
Mathematica	0.59	132.27	1.04	102.	1.
Maple	0.02	859.28	5.19	110.	1.21
Maxima	1.1	122.79	1.67	93.5	1.36
Fricas	9.48	1179.68	9.29	725.	7.8
Sympy	8.16	227.78	2.38	96.	1.75
Giac	2.34	352.34	2.44	176.	1.55

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {301}

Mathematica {46, 50, 54, 59, 63, 69, 75, 83, 86, 87, 88, 91, 94, 101, 102, 110, 112, 114, 115, 117, 119, 120, 121, 124, 126, 127, 128, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

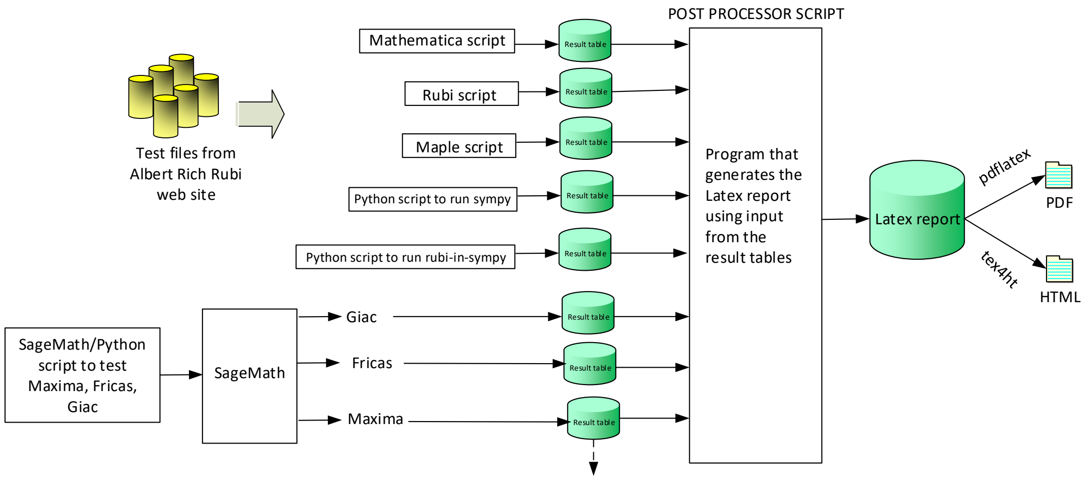
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

B grade: { }

C grade: { 301 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 77, 78, 79, 80, 81, 82, 84, 85, 89, 90, 92, 93, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 137, 163, 164, 165, 168, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 198, 202, 203, 204, 211, 214, 215, 216, 217, 218, 220, 221, 223, 225, 226, 227, 231, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 302, 303, 326, 342, 343, 344, 345, 349 }

B grade: { 50, 72, 224, 243, 293, 341, 346, 347, 348 }

C grade: { 46, 54, 63, 75, 83, 86, 87, 88, 91, 94, 101, 102, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 212, 213, 219, 222, 228, 229, 230, 232, 233, 241, 248, 249, 250, 252, 253, 254, 292, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 33, 34, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 108, 137, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 349 }

B grade: { 18, 19, 26, 27, 28, 29, 35, 36, 37, 40, 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 170, 171, 176, 177, 179, 205, 206, 213, 259, 260, 263, 264, 275, 276, 278, 281, 302, 303 }

C grade: { 180, 197, 232, 241, 252, 294 }

F grade: { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 297, 298, 299, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 14, 15, 16, 43, 44, 85, 92, 93, 98, 99, 100, 108, 137, 163

B grade: { 72, 73, 97 }

C grade: { }

F grade: { 5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 39, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 137, 163, 164, 244, 349 }

B grade: { 12, 13, 18, 19, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 41, 42, 50, 51, 52, 59, 60, 61, 68, 69, 70, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 108, 146, 147, 148, 149, 160, 162, 165, 180, 224, 231, 251, 304, 305, 306, 307, 308, 309, 312, 314, 315, 316, 317, 320 }

C grade: { 313 }

F grade: { 72, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 39, 43, 44, 47, 48, 56, 65, 74, 75, 76, 77, 84, 85, 109, 110, 111, 116, 117, 118, 122, 123, 124, 125, 131, 138, 180, 184, 185, 186, 188, 189, 190, 218, 220, 222, 224, 225, 226, 227, 231, 235, 236, 237, 246, 254, 256, 291 }

B grade: { 5, 6, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 45, 46, 53, 54, 55, 62, 63, 64, 92, 93, 99, 100, 108, 187, 221, 234 }

C grade: { 241, 245, 247, 342, 343, 344, 345 }

F grade: { 26, 34, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 223, 228, 229, 230, 232, 233, 238, 239, 240, 242, 243, 244, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348, 349 }

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A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 105, 107, 108, 180 }

B grade: { 24, 33, 42, 50, 51, 52, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 79, 80, 87, 88, 94, 95, 96, 102, 103, 104, 106, 224 }

C grade: { 231 }

F grade: { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	231	107	132
normalized size	1	1.	1.	1.03	1.38	2.46	1.14	1.4
time (sec)	N/A	0.064	0.019	0.002	1.14	1.588	0.075	1.828

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	169	76	99
normalized size	1	1.	1.	1.04	1.36	2.41	1.09	1.41
time (sec)	N/A	0.042	0.014	0.002	0.962	1.556	0.069	1.143

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	120	53	68
normalized size	1	1.	1.	0.98	1.3	2.4	1.06	1.36
time (sec)	N/A	0.027	0.011	0.001	1.02	1.561	0.065	1.076

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.013	0.005	0.001	1.466	1.517	0.054	1.084

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	45	0	223	82	46
normalized size	1	1.	1.	1.12	0.	5.58	2.05	1.15
time (sec)	N/A	0.02	0.023	0.004	0.	1.775	0.387	1.069

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	381	112	77
normalized size	1	1.	1.	1.08	0.	6.05	1.78	1.22
time (sec)	N/A	0.02	0.044	0.007	0.	1.624	0.517	1.262

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	90	0	621	150	105
normalized size	1	1.	0.89	0.98	0.	6.75	1.63	1.14
time (sec)	N/A	0.031	0.058	0.007	0.	1.771	0.689	1.3

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	296	136	177
normalized size	1	1.	1.	1.02	1.37	2.43	1.11	1.45
time (sec)	N/A	0.074	0.022	0.001	0.978	1.482	0.079	1.289

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	209	97	123
normalized size	1	1.	1.	1.06	1.35	2.55	1.18	1.5
time (sec)	N/A	0.046	0.016	0.001	0.976	1.612	0.072	1.183

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	120	53	68
normalized size	1	1.	1.	0.98	1.3	2.4	1.06	1.36
time (sec)	N/A	0.028	0.007	0.001	0.987	1.473	0.063	1.241

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	390	172	97
normalized size	1	1.	0.94	1.51	0.	6.19	2.73	1.54
time (sec)	N/A	0.043	0.048	0.003	0.	1.841	0.526	1.092

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	129	0	612	236	128
normalized size	1	1.	1.09	1.57	0.	7.46	2.88	1.56
time (sec)	N/A	0.099	0.059	0.007	0.	1.858	0.842	1.065

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	121	147	0	921	223	170
normalized size	1	1.	1.04	1.27	0.	7.94	1.92	1.47
time (sec)	N/A	0.072	0.093	0.009	0.	1.757	1.161	1.083

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	161	177	225	414	189	252
normalized size	1	1.	1.05	1.15	1.46	2.69	1.23	1.64
time (sec)	N/A	0.103	0.028	0.001	1.081	1.509	0.086	1.095

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	296	136	177
normalized size	1	1.	1.	1.02	1.37	2.43	1.11	1.45
time (sec)	N/A	0.07	0.021	0.002	0.983	1.476	0.081	1.081

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	169	76	99
normalized size	1	1.	1.	1.04	1.36	2.41	1.09	1.41
time (sec)	N/A	0.044	0.012	0.001	0.949	1.419	0.068	1.102

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	161	0	613	240	176
normalized size	1	1.	0.95	1.64	0.	6.26	2.45	1.8
time (sec)	N/A	0.064	0.06	0.003	0.	1.751	0.671	1.284

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	205	0	896	313	205
normalized size	1	1.	1.	1.92	0.	8.37	2.93	1.92
time (sec)	N/A	0.096	0.058	0.01	0.	1.791	1.22	1.636

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	141	266	0	1224	422	243
normalized size	1	1.	1.08	2.05	0.	9.42	3.25	1.87
time (sec)	N/A	0.165	0.078	0.01	0.	1.93	2.189	1.109

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	136	246	0	892	323	267
normalized size	1	1.	0.96	1.73	0.	6.28	2.27	1.88
time (sec)	N/A	0.093	0.086	0.004	0.	1.851	0.882	1.114

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	161	0	613	240	174
normalized size	1	1.	0.94	1.64	0.	6.26	2.45	1.78
time (sec)	N/A	0.059	0.066	0.003	0.	1.698	0.699	1.169

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	390	172	97
normalized size	1	1.	0.94	1.51	0.	6.19	2.73	1.54
time (sec)	N/A	0.041	0.049	0.003	0.	2.086	0.538	1.123

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	223	82	45
normalized size	1	1.	1.03	1.15	0.	5.72	2.1	1.15
time (sec)	N/A	0.015	0.025	0.003	0.	2.099	0.4	1.1

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	608	712	257
normalized size	1	1.	0.87	0.79	0.	8.69	10.17	3.67
time (sec)	N/A	0.027	0.044	0.005	0.	2.075	2.215	1.225

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1451	2033	165
normalized size	1	1.	0.87	1.32	0.	13.31	18.65	1.51
time (sec)	N/A	0.084	0.168	0.009	0.	2.893	15.033	1.162

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	158	310	0	3217	0	293
normalized size	1	1.	0.99	1.94	0.	20.11	0.	1.83
time (sec)	N/A	0.192	0.233	0.01	0.	6.807	0.	1.211

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	192	402	0	1675	498	413
normalized size	1	1.	1.	2.09	0.	8.72	2.59	2.15
time (sec)	N/A	0.163	0.098	0.009	0.	1.811	2.288	1.169

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	296	0	1261	398	297
normalized size	1	1.	1.	2.08	0.	8.88	2.8	2.09
time (sec)	N/A	0.12	0.087	0.009	0.	1.783	1.708	1.122

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	205	0	896	313	205
normalized size	1	1.	1.	1.93	0.	8.45	2.95	1.93
time (sec)	N/A	0.093	0.06	0.008	0.	1.818	1.238	1.159

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	129	0	612	236	127
normalized size	1	1.	1.07	1.57	0.	7.46	2.88	1.55
time (sec)	N/A	0.104	0.063	0.008	0.	1.796	0.862	1.154

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	381	112	77
normalized size	1	1.	1.	1.08	0.	6.05	1.78	1.22
time (sec)	N/A	0.021	0.044	0.005	0.	1.847	0.533	1.172

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	109	144	0	1451	2033	163
normalized size	1	1.	1.01	1.33	0.	13.44	18.82	1.51
time (sec)	N/A	0.081	0.145	0.008	0.	2.421	15.387	1.111

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	136	238	0	3294	3662	1928
normalized size	1	1.	0.81	1.43	0.	19.72	21.93	11.54
time (sec)	N/A	0.201	0.314	0.013	0.	6.509	141.098	1.501

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	197	403	0	6472	0	448
normalized size	1	1.	0.86	1.75	0.	28.14	0.	1.95
time (sec)	N/A	0.309	0.414	0.014	0.	22.834	0.	1.123

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	484	0	2195	614	459
normalized size	1	1.	1.	2.47	0.	11.2	3.13	2.34
time (sec)	N/A	0.227	0.122	0.013	0.	1.609	6.524	1.133

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	160	367	0	1689	513	343
normalized size	1	1.	1.	2.29	0.	10.56	3.21	2.14
time (sec)	N/A	0.197	0.095	0.01	0.	1.765	3.717	1.107

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	139	266	0	1224	422	240
normalized size	1	1.	1.07	2.05	0.	9.42	3.25	1.85
time (sec)	N/A	0.166	0.08	0.009	0.	1.865	2.189	1.107

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	124	147	0	918	223	170
normalized size	1	1.	1.07	1.27	0.	7.91	1.92	1.47
time (sec)	N/A	0.077	0.095	0.008	0.	1.877	1.191	1.18

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	89	0	621	150	105
normalized size	1	1.	0.91	0.97	0.	6.75	1.63	1.14
time (sec)	N/A	0.033	0.063	0.007	0.	1.606	0.714	1.195

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	158	309	0	3212	0	294
normalized size	1	1.	0.98	1.92	0.	19.95	0.	1.83
time (sec)	N/A	0.197	0.265	0.01	0.	6.134	0.	1.148

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	197	403	0	6472	0	450
normalized size	1	1.	0.83	1.71	0.	27.42	0.	1.91
time (sec)	N/A	0.311	0.417	0.014	0.	23.112	0.	1.135

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	233	568	0	9997	0	4545
normalized size	1	1.	0.74	1.8	0.	31.74	0.	14.43
time (sec)	N/A	0.451	0.915	0.015	0.	72.782	0.	1.884

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	23	45	73	31	27
normalized size	1	1.	0.71	0.68	1.32	2.15	0.91	0.79
time (sec)	N/A	0.01	0.007	0.005	1.024	1.485	0.127	1.093

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	33	65	159	46	73
normalized size	1	1.	0.87	0.7	1.38	3.38	0.98	1.55
time (sec)	N/A	0.016	0.012	0.007	1.46	1.532	0.157	1.444

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	181	310	0	886	484	271
normalized size	1	1.	0.78	1.34	0.	3.84	2.1	1.17
time (sec)	N/A	0.179	5.108	0.013	0.	2.334	17.983	1.808

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	160	190	0	585	291	174
normalized size	1	1.	1.07	1.28	0.	3.93	1.95	1.17
time (sec)	N/A	0.088	2.476	0.006	0.	1.816	10.179	1.144

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	96	0	371	144	95
normalized size	1	1.	0.98	1.1	0.	4.26	1.66	1.09
time (sec)	N/A	0.028	0.155	0.005	0.	1.651	5.138	1.14

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	232	41	50
normalized size	1	1.	1.07	0.78	0.	5.04	0.89	1.09
time (sec)	N/A	0.01	0.02	0.	0.	1.571	1.799	1.109

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	932	0	1288	0	150
normalized size	1	1.	1.02	11.37	0.	15.71	0.	1.83
time (sec)	N/A	0.054	0.045	0.033	0.	1.918	0.	1.136

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	165	2521	0	765	0	293
normalized size	1	1.	2.01	30.74	0.	9.33	0.	3.57
time (sec)	N/A	0.034	0.231	0.024	0.	2.137	0.	3.031

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	176	5101	0	1439	0	657
normalized size	1	1.	1.18	34.23	0.	9.66	0.	4.41
time (sec)	N/A	0.094	0.528	0.022	0.	2.616	0.	2.521

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	227	7922	0	2485	0	1293
normalized size	1	1.	1.09	38.09	0.	11.95	0.	6.22
time (sec)	N/A	0.213	0.957	0.026	0.	6.191	0.	21.984

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	220	393	0	1102	665	351
normalized size	1	1.	0.81	1.44	0.	4.05	2.44	1.29
time (sec)	N/A	0.218	5.124	0.012	0.	3.016	45.902	1.112

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	157	249	0	776	440	236
normalized size	1	1.	0.8	1.27	0.	3.96	2.24	1.2
time (sec)	N/A	0.116	2.509	0.007	0.	2.065	26.204	1.104

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	109	131	0	483	253	139
normalized size	1	1.	0.92	1.11	0.	4.09	2.14	1.18
time (sec)	N/A	0.04	0.199	0.003	0.	1.729	13.238	1.107

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	0	294	70	66
normalized size	1	1.	1.	0.78	0.	4.52	1.08	1.02
time (sec)	N/A	0.016	0.093	0.	0.	1.632	2.971	1.082

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	110	1845	0	1582	0	205
normalized size	1	1.	0.97	16.33	0.	14.	0.	1.81
time (sec)	N/A	0.108	0.195	0.014	0.	2.982	0.	1.135

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	142	4621	0	1941	0	428
normalized size	1	1.	1.08	35.27	0.	14.82	0.	3.27
time (sec)	N/A	0.09	0.121	0.016	0.	2.781	0.	1.165

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	163	9059	0	1072	0	609
normalized size	1	1.	1.44	80.17	0.	9.49	0.	5.39
time (sec)	N/A	0.057	0.67	0.022	0.	2.515	0.	2.304

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	247	13766	0	1994	0	1241
normalized size	1	1.	1.24	69.18	0.	10.02	0.	6.24
time (sec)	N/A	0.115	0.775	0.03	0.	3.951	0.	20.881

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	362	18791	0	3330	0	2102
normalized size	1	1.	1.21	62.64	0.	11.1	0.	7.01
time (sec)	N/A	0.365	1.33	0.038	0.	15.271	0.	6.296

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	270	476	0	1385	796	433
normalized size	1	1.	0.77	1.36	0.	3.97	2.28	1.24
time (sec)	N/A	0.247	5.173	0.015	0.	4.512	87.831	1.188

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	241	158	308	0	963	537	298
normalized size	1	1.	0.66	1.28	0.	4.	2.23	1.24
time (sec)	N/A	0.148	2.624	0.007	0.	2.996	50.966	1.176

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	130	166	0	605	316	182
normalized size	1	1.	0.87	1.11	0.	4.06	2.12	1.22
time (sec)	N/A	0.052	0.222	0.003	0.	2.243	26.27	1.153

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	76	66	0	347	97	85
normalized size	1	1.	0.9	0.79	0.	4.13	1.15	1.01
time (sec)	N/A	0.023	0.103	0.001	0.	1.635	4.188	1.141

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	140	3053	0	2022	0	290
normalized size	1	1.	0.89	19.45	0.	12.88	0.	1.85
time (sec)	N/A	0.199	0.121	0.014	0.	8.09	0.	1.166

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	144	7345	0	2584	0	547
normalized size	1	1.	0.82	41.97	0.	14.77	0.	3.13
time (sec)	N/A	0.226	0.16	0.02	0.	6.335	0.	1.24

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	184	14133	0	3125	0	890
normalized size	1	1.	0.95	72.85	0.	16.11	0.	4.59
time (sec)	N/A	0.194	0.193	0.026	0.	4.266	0.	1.234

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	201	21220	0	1438	0	1142
normalized size	1	1.	1.4	147.36	0.	9.99	0.	7.93
time (sec)	N/A	0.073	0.778	0.033	0.	3.205	0.	22.01

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	306	28625	0	2612	0	1955
normalized size	1	1.	1.23	114.96	0.	10.49	0.	7.85
time (sec)	N/A	0.137	1.008	0.048	0.	8.07	0.	4.835

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	111	0	128
normalized size	1	1.	1.	1.1	0.	3.7	0.	4.27
time (sec)	N/A	0.014	0.028	0.011	0.	1.52	0.	1.219

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	64	84	80	0	0	95
normalized size	1	1.	2.37	3.11	2.96	0.	0.	3.52
time (sec)	N/A	0.014	0.025	0.008	1.483	0.	0.	1.196

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	187	149	192	0	159
normalized size	1	1.	1.	7.48	5.96	7.68	0.	6.36
time (sec)	N/A	0.013	0.01	0.029	1.511	1.568	0.	1.141

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	140	228	0	675	400	203
normalized size	1	1.	0.83	1.35	0.	3.99	2.37	1.2
time (sec)	N/A	0.145	5.094	0.012	0.	1.859	11.967	1.166

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	160	131	0	440	238	122
normalized size	1	1.	1.48	1.21	0.	4.07	2.2	1.13
time (sec)	N/A	0.056	2.356	0.006	0.	1.667	6.267	1.122

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	62	0	275	126	66
normalized size	1	1.	0.98	1.07	0.	4.74	2.17	1.14
time (sec)	N/A	0.017	0.02	0.004	0.	1.604	2.506	1.121

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	153	17	31
normalized size	1	1.	1.	0.84	0.	6.12	0.68	1.24
time (sec)	N/A	0.006	0.005	0.002	0.	1.579	0.993	1.17

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	300	0	513	0	95
normalized size	1	1.	1.	6.12	0.	10.47	0.	1.94
time (sec)	N/A	0.022	0.018	0.013	0.	1.991	0.	1.143

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	126	809	0	957	0	327
normalized size	1	1.	1.25	8.01	0.	9.48	0.	3.24
time (sec)	N/A	0.049	0.287	0.015	0.	2.768	0.	1.186

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	192	1815	0	1750	0	726
normalized size	1	1.	1.18	11.13	0.	10.74	0.	4.45
time (sec)	N/A	0.119	0.622	0.02	0.	4.639	0.	3.546

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	172	340	0	1272	0	317
normalized size	1	1.	0.67	1.32	0.	4.95	0.	1.23
time (sec)	N/A	0.259	5.196	0.017	0.	2.327	0.	1.155

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	122	219	0	888	0	212
normalized size	1	1.	0.72	1.3	0.	5.25	0.	1.25
time (sec)	N/A	0.197	5.097	0.008	0.	1.81	0.	1.161

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	105	160	123	0	597	0	124
normalized size	1	1.17	1.78	1.37	0.	6.63	0.	1.38
time (sec)	N/A	0.062	2.369	0.006	0.	1.638	0.	1.117

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	70	54	0	367	60	68
normalized size	1	1.	1.3	1.	0.	6.8	1.11	1.26
time (sec)	N/A	0.017	0.058	0.004	0.	1.545	3.704	1.176

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	47	17	19
normalized size	1	1.	1.	0.94	1.19	2.94	1.06	1.19
time (sec)	N/A	0.002	0.003	0.002	0.976	1.491	0.534	1.147

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	309	618	0	911	0	144
normalized size	1	1.	3.91	7.82	0.	11.53	0.	1.82
time (sec)	N/A	0.039	0.682	0.013	0.	2.362	0.	1.141

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	758	1439	0	1706	0	429
normalized size	1	1.	5.3	10.06	0.	11.93	0.	3.
time (sec)	N/A	0.109	2.528	0.017	0.	4.487	0.	5.46

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	1392	2919	0	2967	0	868
normalized size	1	1.	6.19	12.97	0.	13.19	0.	3.86
time (sec)	N/A	0.243	4.771	0.02	0.	9.686	0.	14.873

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	157	351	0	1430	0	320
normalized size	1	1.	0.62	1.38	0.	5.61	0.	1.25
time (sec)	N/A	0.245	5.162	0.02	0.	2.787	0.	1.189

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	125	228	0	1008	0	213
normalized size	1	1.	0.73	1.33	0.	5.86	0.	1.24
time (sec)	N/A	0.157	5.097	0.007	0.	1.964	0.	1.169

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	214	136	0	655	0	139
normalized size	1	1.	2.04	1.3	0.	6.24	0.	1.32
time (sec)	N/A	0.05	4.135	0.006	0.	1.651	0.	1.166

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	92	115	144	54
normalized size	1	1.	0.79	0.72	1.96	2.45	3.06	1.15
time (sec)	N/A	0.01	0.016	0.003	0.96	1.545	10.792	1.119

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	99	95	36
normalized size	1	1.	0.74	0.67	1.08	2.54	2.44	0.92
time (sec)	N/A	0.006	0.006	0.003	0.96	1.541	0.783	1.139

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	775	1070	0	1530	0	432
normalized size	1	1.	6.35	8.77	0.	12.54	0.	3.54
time (sec)	N/A	0.103	2.628	0.014	0.	4.241	0.	1.179

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	170	2371	0	2849	0	837
normalized size	1	1.	0.84	11.74	0.	14.1	0.	4.14
time (sec)	N/A	0.228	5.482	0.02	0.	9.31	0.	7.721

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	221	4495	0	4520	0	1364
normalized size	1	1.	0.71	14.36	0.	14.44	0.	4.36
time (sec)	N/A	0.4	5.653	0.024	0.	35.954	0.	3.709

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	163	190	628	487	0	294
normalized size	1	1.	0.73	0.85	2.8	2.17	0.	1.31
time (sec)	N/A	0.101	0.101	0.007	1.059	3.349	0.	1.172

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	107	115	336	319	0	186
normalized size	1	1.	0.61	0.66	1.93	1.83	0.	1.07
time (sec)	N/A	0.071	0.064	0.006	1.014	1.946	0.	1.255

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	59	57	139	185	566	97
normalized size	1	1.	0.65	0.63	1.53	2.03	6.22	1.07
time (sec)	N/A	0.029	0.022	0.003	0.97	1.559	36.603	1.177

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	99	95	36
normalized size	1	1.	0.74	0.67	1.08	2.54	2.44	0.92
time (sec)	N/A	0.006	0.009	0.002	0.952	1.492	0.792	1.161

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	236	628	0	914	0	144
normalized size	1	1.	2.99	7.95	0.	11.57	0.	1.82
time (sec)	N/A	0.046	2.59	0.033	0.	2.307	0.	1.17

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	405	823	0	954	0	304
normalized size	1	1.	4.05	8.23	0.	9.54	0.	3.04
time (sec)	N/A	0.055	0.757	0.024	0.	2.618	0.	1.186

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	130	5177	0	1439	0	657
normalized size	1	1.	0.87	34.74	0.	9.66	0.	4.41
time (sec)	N/A	0.079	5.17	0.028	0.	2.896	0.	3.775

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	179	13964	0	1994	0	1241
normalized size	1	1.	0.9	70.17	0.	10.02	0.	6.24
time (sec)	N/A	0.113	5.268	0.044	0.	4.26	0.	23.152

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	55	0	24
normalized size	1	1.	1.	0.95	0.	2.75	0.	1.2
time (sec)	N/A	0.005	0.007	0.003	0.	1.502	0.	1.117

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	0	69	0	69
normalized size	1	1.	1.	1.12	0.	2.76	0.	2.76
time (sec)	N/A	0.008	0.005	0.003	0.	1.508	0.	1.146

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	513	0	95
normalized size	1	1.	1.	6.24	0.	10.47	0.	1.94
time (sec)	N/A	0.02	0.014	0.009	0.	1.999	0.	1.107

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	108	31	34
normalized size	1	1.	1.	0.93	1.2	7.2	2.07	2.27
time (sec)	N/A	0.004	0.012	0.006	1.458	1.514	3.453	1.085

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	99	0	0	0	136	0
normalized size	1	1.	0.15	0.	0.	0.	0.21	0.
time (sec)	N/A	0.597	5.047	0.032	0.	0.	4.06	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	617	617	176	0	0	0	99	0
normalized size	1	1.	0.29	0.	0.	0.	0.16	0.
time (sec)	N/A	0.426	3.225	0.026	0.	0.	2.909	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	62	0	0	0	63	0
normalized size	1	1.	0.11	0.	0.	0.	0.11	0.
time (sec)	N/A	0.369	0.072	0.023	0.	0.	2.026	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	740	740	162	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.321	0.15	0.047	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	584	86	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	0.057	0.055	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	818	818	252	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.311	0.049	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	849	849	265	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.651	0.181	0.056	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	110	0	0	0	139	0
normalized size	1	1.	0.16	0.	0.	0.	0.21	0.
time (sec)	N/A	0.571	5.049	0.03	0.	0.	6.248	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	637	637	173	0	0	0	131	0
normalized size	1	1.	0.27	0.	0.	0.	0.21	0.
time (sec)	N/A	0.484	2.775	0.027	0.	0.	4.907	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	608	68	0	0	0	100	0
normalized size	1	1.	0.11	0.	0.	0.	0.16	0.
time (sec)	N/A	0.42	0.062	0.024	0.	0.	3.478	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	765	765	231	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.444	0.133	0.06	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	775	775	235	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	0.14	0.047	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	815	815	252	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.613	0.22	0.048	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	98	0	0	0	165	0
normalized size	1	1.	0.15	0.	0.	0.	0.25	0.
time (sec)	N/A	0.527	5.056	0.03	0.	0.	4.46	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	88	0	0	0	129	0
normalized size	1	1.	0.14	0.	0.	0.	0.21	0.
time (sec)	N/A	0.44	5.047	0.026	0.	0.	3.454	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	597	597	158	0	0	0	94	0
normalized size	1	1.	0.26	0.	0.	0.	0.16	0.
time (sec)	N/A	0.381	4.433	0.025	0.	0.	2.547	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	568	62	0	0	0	60	0
normalized size	1	1.	0.11	0.	0.	0.	0.11	0.
time (sec)	N/A	0.305	0.024	0.023	0.	0.	1.569	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.029	0.039	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	787	787	234	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.439	0.153	0.045	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	818	818	255	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.543	0.162	0.045	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	623	623	76	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	5.05	0.057	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	592	592	62	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	5.044	0.051	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	53	0	0	0	60	0
normalized size	1	1.	0.09	0.	0.	0.	0.11	0.
time (sec)	N/A	0.304	0.017	0.024	0.	0.	3.801	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	776	776	226	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.443	0.129	0.041	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	807	807	236	0	0	0	0	0
normalized size	1	1.	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.534	0.172	0.046	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	849	849	256	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.653	0.23	0.046	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	653	653	96	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.512	5.071	0.064	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	83	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.426	5.067	0.048	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	45	90	0	0
normalized size	1	1.	0.55	0.55	1.02	2.05	0.	0.
time (sec)	N/A	0.017	5.026	0.004	1.14	1.774	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	590	74	0	0	0	60	0
normalized size	1	1.	0.13	0.	0.	0.	0.1	0.
time (sec)	N/A	0.366	0.038	0.023	0.	0.	11.292	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	796	796	248	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.537	0.206	0.042	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	827	827	259	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.634	0.241	0.046	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	163	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.124	0.044	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	166	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.151	0.046	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	153	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.148	0.046	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.03	0.	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	156	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.151	0.043	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	5544	0	0
normalized size	1	1.	1.04	0.	0.	49.06	0.	0.
time (sec)	N/A	0.021	0.04	0.001	0.	10.867	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	4852	0	0
normalized size	1	1.	1.14	0.	0.	44.51	0.	0.
time (sec)	N/A	0.014	0.035	0.	0.	8.789	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	143	0	0	775	0	0
normalized size	1	1.	1.49	0.	0.	8.07	0.	0.
time (sec)	N/A	0.018	0.144	0.035	0.	54.84	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	143	0	0	911	0	0
normalized size	1	1.	1.51	0.	0.	9.59	0.	0.
time (sec)	N/A	0.017	0.098	0.034	0.	55.362	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	169	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.162	0.044	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	167	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.17	0.044	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	168	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.122	0.043	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	172	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.159	0.042	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	148	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.146	0.043	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	148	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.162	0.043	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	136	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.113	0.028	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	136	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.119	0.049	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	136	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.101	0.051	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	136	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.107	0.027	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	124	0	0	4397	0	0
normalized size	1	1.	1.77	0.	0.	62.81	0.	0.
time (sec)	N/A	0.009	0.093	0.027	0.	14.661	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	137	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.108	0.037	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	125	0	0	749	0	0
normalized size	1	1.	1.69	0.	0.	10.12	0.	0.
time (sec)	N/A	0.011	0.045	0.046	0.	6.559	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	91	57	94	95	662	0	0
normalized size	1	1.15	0.72	1.19	1.2	8.38	0.	0.
time (sec)	N/A	0.019	0.026	0.02	0.99	2.31	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	91	54	94	0	645	0	0
normalized size	1	1.23	0.73	1.27	0.	8.72	0.	0.
time (sec)	N/A	0.018	0.019	0.016	0.	2.327	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	91	57	94	0	664	0	0
normalized size	1	1.2	0.75	1.24	0.	8.74	0.	0.
time (sec)	N/A	0.017	0.029	0.014	0.	2.038	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	243	543	0	0	0	0
normalized size	1	1.	0.74	1.66	0.	0.	0.	0.
time (sec)	N/A	0.316	0.448	0.033	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	198	328	0	0	0	0
normalized size	1	1.	0.8	1.32	0.	0.	0.	0.
time (sec)	N/A	0.179	0.238	0.012	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0
normalized size	1	1.	0.42	0.5	0.	0.	0.	0.
time (sec)	N/A	0.093	0.05	0.013	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	133	181	0	0	0	0
normalized size	1	1.	1.58	2.15	0.	0.	0.	0.
time (sec)	N/A	0.017	0.279	0.031	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	243	617	0	0	0	0
normalized size	1	1.	1.03	2.6	0.	0.	0.	0.
time (sec)	N/A	0.12	0.444	0.036	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	285	1411	0	0	0	0
normalized size	1	1.	0.92	4.57	0.	0.	0.	0.
time (sec)	N/A	0.228	0.531	0.042	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	302	780	0	0	0	0
normalized size	1	1.	0.74	1.9	0.	0.	0.	0.
time (sec)	N/A	0.439	0.583	0.018	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	246	545	0	0	0	0
normalized size	1	1.	0.73	1.62	0.	0.	0.	0.
time (sec)	N/A	0.281	0.41	0.014	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	199	330	0	0	0	0
normalized size	1	1.	0.73	1.21	0.	0.	0.	0.
time (sec)	N/A	0.165	0.226	0.014	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	191	332	0	0	0	0
normalized size	1	1.	0.72	1.24	0.	0.	0.	0.
time (sec)	N/A	0.159	0.27	0.021	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	232	607	0	0	0	0
normalized size	1	1.	1.01	2.65	0.	0.	0.	0.
time (sec)	N/A	0.136	0.462	0.021	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	285	1414	0	0	0	0
normalized size	1	1.	0.9	4.49	0.	0.	0.	0.
time (sec)	N/A	0.282	0.58	0.027	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	127	303	0	0	0	0
normalized size	1	1.	0.54	1.29	0.	0.	0.	0.
time (sec)	N/A	0.135	0.107	0.028	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	22	75	0	0	0	0
normalized size	1	1.	0.58	1.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.012	0.026	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	38	0	85	17	23
normalized size	1	1.	0.75	1.9	0.	4.25	0.85	1.15
time (sec)	N/A	0.004	0.002	0.001	0.	1.766	6.094	1.088

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0
normalized size	1	1.	0.2	0.2	0.	0.	0.	0.
time (sec)	N/A	0.081	0.008	0.016	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	78	0	0	0	0
normalized size	1	1.	0.66	0.86	0.	0.	0.	0.
time (sec)	N/A	0.058	0.038	0.022	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	60	53	0	0	0	0
normalized size	1	1.	0.4	0.35	0.	0.	0.	0.
time (sec)	N/A	0.059	0.031	0.025	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	0	34	0
normalized size	1	1.	1.	1.15	0.	0.	1.7	0.
time (sec)	N/A	0.007	0.006	0.015	0.	0.	4.218	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	0	36	0
normalized size	1	1.	1.	0.86	0.	0.	1.71	0.
time (sec)	N/A	0.008	0.004	0.027	0.	0.	4.431	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	0	0	34	0
normalized size	1	1.	1.	1.45	0.	0.	1.7	0.
time (sec)	N/A	0.007	0.005	0.02	0.	0.	4.341	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	0	10	0
normalized size	1	1.	1.	1.25	0.	0.	2.5	0.
time (sec)	N/A	0.005	0.003	0.012	0.	0.	2.138	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	36	0
normalized size	1	1.	1.	0.95	0.	0.	1.8	0.
time (sec)	N/A	0.006	0.004	0.016	0.	0.	4.351	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	0	0	37	0
normalized size	1	1.	1.	0.9	0.	0.	1.76	0.
time (sec)	N/A	0.006	0.004	0.018	0.	0.	4.514	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	36	0
normalized size	1	1.	1.	0.95	0.	0.	1.8	0.
time (sec)	N/A	0.007	0.005	0.023	0.	0.	4.464	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	0	0	0
normalized size	1	1.	0.92	1.08	0.	0.	0.	0.
time (sec)	N/A	0.014	0.003	0.008	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	27	0	0	0	0
normalized size	1	1.	0.87	0.87	0.	0.	0.	0.
time (sec)	N/A	0.02	0.005	0.014	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	0	0	0
normalized size	1	1.	0.77	0.89	0.	0.	0.	0.
time (sec)	N/A	0.021	0.004	0.023	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	0	0	0
normalized size	1	1.	0.77	0.89	0.	0.	0.	0.
time (sec)	N/A	0.02	0.004	0.018	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	27	30	0	0	0	0
normalized size	1	1.	0.21	0.23	0.	0.	0.	0.
time (sec)	N/A	0.041	0.004	0.015	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	27	26	0	0	0	0
normalized size	1	1.	0.2	0.19	0.	0.	0.	0.
time (sec)	N/A	0.044	0.004	0.013	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	27	20	0	0	0	0
normalized size	1	1.	0.18	0.14	0.	0.	0.	0.
time (sec)	N/A	0.05	0.005	0.023	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	32	0	0	0	0
normalized size	1	1.	0.88	0.8	0.	0.	0.	0.
time (sec)	N/A	0.015	0.024	0.01	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	321	852	0	0	0	0
normalized size	1	1.	0.76	2.01	0.	0.	0.	0.
time (sec)	N/A	0.431	1.489	0.023	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	260	615	0	0	0	0
normalized size	1	1.	0.76	1.79	0.	0.	0.	0.
time (sec)	N/A	0.283	0.478	0.017	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	216	399	0	0	0	0
normalized size	1	1.	0.83	1.53	0.	0.	0.	0.
time (sec)	N/A	0.16	0.336	0.015	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0
normalized size	1	1.	0.44	0.81	0.	0.	0.	0.
time (sec)	N/A	0.09	0.05	0.012	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	0	0	0
normalized size	1	1.	0.99	1.15	0.	0.	0.	0.
time (sec)	N/A	0.019	0.055	0.015	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	112	248	0	0	0	0
normalized size	1	1.	0.41	0.91	0.	0.	0.	0.
time (sec)	N/A	0.143	0.236	0.024	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	261	752	0	0	0	0
normalized size	1	1.	1.02	2.95	0.	0.	0.	0.
time (sec)	N/A	0.146	0.592	0.028	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	301	1607	0	0	0	0
normalized size	1	1.	0.9	4.81	0.	0.	0.	0.
time (sec)	N/A	0.278	0.621	0.031	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	318	755	0	0	0	0
normalized size	1	1.	0.71	1.7	0.	0.	0.	0.
time (sec)	N/A	0.405	1.185	0.043	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	256	539	0	0	0	0
normalized size	1	1.	0.74	1.56	0.	0.	0.	0.
time (sec)	N/A	0.266	0.454	0.023	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	196	345	0	0	0	0
normalized size	1	1.	0.76	1.34	0.	0.	0.	0.
time (sec)	N/A	0.152	0.279	0.02	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	136	188	0	0	0	0
normalized size	1	1.	1.62	2.24	0.	0.	0.	0.
time (sec)	N/A	0.017	0.307	0.015	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	112	144	0	0	0	0
normalized size	1	1.	0.58	0.74	0.	0.	0.	0.
time (sec)	N/A	0.152	0.231	0.022	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	224	354	0	0	0	0
normalized size	1	1.	0.93	1.46	0.	0.	0.	0.
time (sec)	N/A	0.12	0.673	0.024	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	337	964	0	0	0	0
normalized size	1	1.	1.04	2.98	0.	0.	0.	0.
time (sec)	N/A	0.264	1.034	0.031	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	0	0	0
normalized size	1	1.	0.99	1.15	0.	0.	0.	0.
time (sec)	N/A	0.019	0.055	0.	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.054	0.061	0.033	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	106	0	0	0	0
normalized size	1	1.	1.02	1.22	0.	0.	0.	0.
time (sec)	N/A	0.052	0.065	0.028	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	108	0	0	0	0
normalized size	1	1.	1.	1.23	0.	0.	0.	0.
time (sec)	N/A	0.057	0.061	0.026	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	0	19	0
normalized size	1	1.	1.	1.17	0.	0.	1.58	0.
time (sec)	N/A	0.007	0.005	0.026	0.	0.	4.443	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	58	14	0	0	0	0
normalized size	1	1.	5.8	1.4	0.	0.	0.	0.
time (sec)	N/A	0.007	0.03	0.023	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	0	19	0
normalized size	1	1.	1.	1.17	0.	0.	1.58	0.
time (sec)	N/A	0.007	0.005	0.015	0.	0.	4.461	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	0	73	0
normalized size	1	1.	1.	1.	0.	0.	7.3	0.
time (sec)	N/A	0.005	0.019	0.018	0.	0.	8.395	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	0	0	19	0
normalized size	1	1.	1.5	1.17	0.	0.	1.58	0.
time (sec)	N/A	0.007	0.019	0.022	0.	0.	2.178	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	0
normalized size	1	1.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.007	0.006	0.022	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	26	8	0	169	22	26
normalized size	1	1.	3.25	1.	0.	21.12	2.75	3.25
time (sec)	N/A	0.002	0.005	0.04	0.	1.985	2.661	1.086

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	17	0
normalized size	1	1.	1.	1.08	0.	0.	1.42	0.
time (sec)	N/A	0.007	0.005	0.017	0.	0.	4.377	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	0	39	0
normalized size	1	1.	1.	1.1	0.	0.	3.9	0.
time (sec)	N/A	0.007	0.009	0.019	0.	0.	8.186	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	17	0
normalized size	1	1.	1.	1.08	0.	0.	1.42	0.
time (sec)	N/A	0.007	0.005	0.025	0.	0.	4.36	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	0	0	0
normalized size	1	1.	0.37	0.33	0.	0.	0.	0.
time (sec)	N/A	0.009	0.02	0.032	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	17	15	0	0	0	0
normalized size	1	1.	0.35	0.31	0.	0.	0.	0.
time (sec)	N/A	0.009	0.028	0.02	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	0	0	0
normalized size	1	1.	0.37	0.33	0.	0.	0.	0.
time (sec)	N/A	0.009	0.02	0.013	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	0	97	8	31
normalized size	1	1.	1.	1.	0.	12.12	1.	3.88
time (sec)	N/A	0.002	0.003	0.027	0.	1.908	2.827	1.197

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	19	15	0	0	0	0
normalized size	1	1.	0.4	0.32	0.	0.	0.	0.
time (sec)	N/A	0.009	0.018	0.022	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	14	0	0	0	0
normalized size	1	1.	1.9	1.4	0.	0.	0.	0.
time (sec)	N/A	0.006	0.018	0.022	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	0	76	0
normalized size	1	1.	1.	1.	0.	0.	7.6	0.
time (sec)	N/A	0.005	0.012	0.006	0.	0.	7.753	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	36	0
normalized size	1	1.	1.	0.95	0.	0.	1.8	0.
time (sec)	N/A	0.007	0.006	0.016	0.	0.	4.814	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	0	41	0
normalized size	1	1.	1.	0.94	0.	0.	2.56	0.
time (sec)	N/A	0.007	0.006	0.019	0.	0.	8.516	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	36	0
normalized size	1	1.	1.	0.95	0.	0.	1.8	0.
time (sec)	N/A	0.009	0.007	0.035	0.	0.	4.562	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	0	0	0
normalized size	1	1.	1.	1.16	0.	0.	0.	0.
time (sec)	N/A	0.014	0.022	0.029	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	0	0	0	0
normalized size	1	1.	1.	1.13	0.	0.	0.	0.
time (sec)	N/A	0.015	0.027	0.029	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	0	0	0
normalized size	1	1.	1.	1.16	0.	0.	0.	0.
time (sec)	N/A	0.013	0.021	0.026	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	46	30	0	0	75	0
normalized size	1	1.	1.84	1.2	0.	0.	3.	0.
time (sec)	N/A	0.011	0.01	0.019	0.	0.	8.348	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	34	0	0	0	0
normalized size	1	1.	1.	1.06	0.	0.	0.	0.
time (sec)	N/A	0.013	0.02	0.021	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	47	28	0	0	0	0
normalized size	1	1.	3.92	2.33	0.	0.	0.	0.
time (sec)	N/A	0.007	0.022	0.02	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	24	0	97	0	0
normalized size	1	1.	1.38	0.83	0.	3.34	0.	0.
time (sec)	N/A	0.003	0.011	0.007	0.	1.83	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	0	37	0
normalized size	1	1.	1.25	0.91	0.	0.	1.16	0.
time (sec)	N/A	0.015	0.023	0.022	0.	0.	4.807	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	0	0	42	0
normalized size	1	1.	1.2	0.9	0.	0.	1.4	0.
time (sec)	N/A	0.015	0.029	0.023	0.	0.	8.983	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	0	37	0
normalized size	1	1.	1.25	0.91	0.	0.	1.16	0.
time (sec)	N/A	0.014	0.024	0.023	0.	0.	4.588	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	0	0	0
normalized size	1	1.	0.74	0.68	0.	0.	0.	0.
time (sec)	N/A	0.01	0.026	0.026	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	37	33	0	0	0	0
normalized size	1	1.	0.73	0.65	0.	0.	0.	0.
time (sec)	N/A	0.01	0.028	0.021	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	0	0	0
normalized size	1	1.	0.74	0.68	0.	0.	0.	0.
time (sec)	N/A	0.009	0.023	0.024	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	24	0	258	0	0
normalized size	1	1.	0.93	0.86	0.	9.21	0.	0.
time (sec)	N/A	0.004	0.01	0.007	0.	1.849	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	33	0	0	0	0
normalized size	1	1.	1.08	0.67	0.	0.	0.	0.
time (sec)	N/A	0.009	0.035	0.019	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	34	0	0	0	0
normalized size	1	1.	1.26	1.1	0.	0.	0.	0.
time (sec)	N/A	0.016	0.025	0.02	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	65	48	30	0	0	73	0
normalized size	1	1.55	1.14	0.71	0.	0.	1.74	0.
time (sec)	N/A	0.012	0.013	0.009	0.	0.	7.93	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	0	0	0
normalized size	1	1.	1.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.016	0.024	0.019	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	0	0	44	0
normalized size	1	1.	1.	0.94	0.	0.	1.22	0.
time (sec)	N/A	0.015	0.03	0.019	0.	0.	8.715	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	0	0	0
normalized size	1	1.	1.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.015	0.027	0.023	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.053	0.056	0.01	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	171	0	0	0	0
normalized size	1	1.	1.	1.9	0.	0.	0.	0.
time (sec)	N/A	0.052	0.048	0.015	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	168	0	0	0	0
normalized size	1	1.	1.	1.91	0.	0.	0.	0.
time (sec)	N/A	0.049	0.05	0.012	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	110	0	0	0	0
normalized size	1	1.	1.	1.21	0.	0.	0.	0.
time (sec)	N/A	0.054	0.043	0.01	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	109	0	0	0	0
normalized size	1	1.	1.	1.24	0.	0.	0.	0.
time (sec)	N/A	0.052	0.056	0.012	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	165	0	0	0	0
normalized size	1	1.	1.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.053	0.051	0.018	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	166	0	0	0	0
normalized size	1	1.	1.	1.87	0.	0.	0.	0.
time (sec)	N/A	0.055	0.047	0.012	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	111	0	0	0	0
normalized size	1	1.	1.	1.23	0.	0.	0.	0.
time (sec)	N/A	0.052	0.046	0.009	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0
normalized size	1	1.	0.44	0.81	0.	0.	0.	0.
time (sec)	N/A	0.084	0.049	0.	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	89	104	0	0	0	0
normalized size	1	1.	0.44	0.51	0.	0.	0.	0.
time (sec)	N/A	0.089	0.045	0.014	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	89	108	0	0	0	0
normalized size	1	1.	0.44	0.53	0.	0.	0.	0.
time (sec)	N/A	0.092	0.051	0.013	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	92	165	0	0	0	0
normalized size	1	1.	0.43	0.78	0.	0.	0.	0.
time (sec)	N/A	0.103	0.044	0.013	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	164	0	0	0	0
normalized size	1	1.	0.47	0.87	0.	0.	0.	0.
time (sec)	N/A	0.125	0.05	0.012	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	109	0	0	0	0
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.
time (sec)	N/A	0.123	0.046	0.014	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	111	0	0	0	0
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.
time (sec)	N/A	0.128	0.054	0.009	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	167	0	0	0	0
normalized size	1	1.	0.47	0.84	0.	0.	0.	0.
time (sec)	N/A	0.131	0.045	0.012	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.048	0.054	0.013	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	171	0	0	0	0
normalized size	1	1.	1.	1.9	0.	0.	0.	0.
time (sec)	N/A	0.052	0.048	0.013	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	168	0	0	0	0
normalized size	1	1.	1.	1.91	0.	0.	0.	0.
time (sec)	N/A	0.047	0.046	0.011	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	110	0	0	0	0
normalized size	1	1.	1.	1.21	0.	0.	0.	0.
time (sec)	N/A	0.05	0.042	0.01	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	164	0	0	0	0
normalized size	1	1.	1.	1.86	0.	0.	0.	0.
time (sec)	N/A	0.052	0.056	0.012	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	110	0	0	0	0
normalized size	1	1.	1.	1.24	0.	0.	0.	0.
time (sec)	N/A	0.051	0.047	0.017	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	110	0	0	0	0
normalized size	1	1.	1.	1.24	0.	0.	0.	0.
time (sec)	N/A	0.051	0.048	0.012	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	167	0	0	0	0
normalized size	1	1.	1.	1.86	0.	0.	0.	0.
time (sec)	N/A	0.052	0.041	0.012	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0
normalized size	1	1.	0.42	0.5	0.	0.	0.	0.
time (sec)	N/A	0.088	0.043	0.001	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	89	161	0	0	0	0
normalized size	1	1.	0.42	0.75	0.	0.	0.	0.
time (sec)	N/A	0.096	0.045	0.013	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	89	162	0	0	0	0
normalized size	1	1.	0.42	0.76	0.	0.	0.	0.
time (sec)	N/A	0.097	0.05	0.013	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	92	111	0	0	0	0
normalized size	1	1.	0.41	0.5	0.	0.	0.	0.
time (sec)	N/A	0.103	0.048	0.012	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	164	0	0	0	0
normalized size	1	1.	0.47	0.87	0.	0.	0.	0.
time (sec)	N/A	0.121	0.05	0.011	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	109	0	0	0	0
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.
time (sec)	N/A	0.122	0.046	0.014	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	111	0	0	0	0
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.
time (sec)	N/A	0.125	0.049	0.012	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	167	0	0	0	0
normalized size	1	1.	0.47	0.84	0.	0.	0.	0.
time (sec)	N/A	0.129	0.045	0.012	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	37	38	0	0	0	0
normalized size	1	1.	0.47	0.49	0.	0.	0.	0.
time (sec)	N/A	0.014	0.012	0.023	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	38	0	0	20	0
normalized size	1	1.	1.03	0.97	0.	0.	0.51	0.
time (sec)	N/A	0.02	0.038	0.022	0.	0.	2.215	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	0	0	0	0
normalized size	1	1.	0.77	0.87	0.	0.	0.	0.
time (sec)	N/A	0.012	0.035	0.017	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	27	25	0	0	0	0
normalized size	1	1.	4.5	4.17	0.	0.	0.	0.
time (sec)	N/A	0.006	0.025	0.016	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	0	0	0
normalized size	1	1.	1.04	1.22	0.	0.	0.	0.
time (sec)	N/A	0.025	0.009	0.015	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0
normalized size	1	1.	0.2	0.2	0.	0.	0.	0.
time (sec)	N/A	0.075	0.009	0.	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	0	0	0	0
normalized size	1	1.	1.84	1.95	0.	0.	0.	0.
time (sec)	N/A	0.007	0.026	0.014	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.108	0.23	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	95	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.087	0.162	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	102	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	0.117	0.159	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	102	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.09	0.115	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	62	23	122	0	0	0	0	0
normalized size	1	0.37	1.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.124	0.04	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	117	0	0	0	0
normalized size	1	1.	1.04	2.54	0.	0.	0.	0.
time (sec)	N/A	0.041	0.082	0.106	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	81	207	0	0	0	0
normalized size	1	1.	1.72	4.4	0.	0.	0.	0.
time (sec)	N/A	0.062	0.147	0.181	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	135	0	0	1520	0	0
normalized size	1	1.	1.05	0.	0.	11.78	0.	0.
time (sec)	N/A	0.019	0.107	0.026	0.	27.794	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	135	0	0	1553	0	0
normalized size	1	1.	1.12	0.	0.	12.94	0.	0.
time (sec)	N/A	0.016	0.121	0.051	0.	28.216	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	144	0	0	2201	0	0
normalized size	1	1.	1.12	0.	0.	17.06	0.	0.
time (sec)	N/A	0.023	0.131	0.037	0.	72.549	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	145	0	0	2229	0	0
normalized size	1	1.	1.17	0.	0.	17.98	0.	0.
time (sec)	N/A	0.02	0.132	0.04	0.	73.024	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	155	0	0	826	0	0
normalized size	1	1.	1.29	0.	0.	6.88	0.	0.
time (sec)	N/A	0.02	0.14	0.043	0.	58.606	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	155	0	0	836	0	0
normalized size	1	1.	1.29	0.	0.	6.97	0.	0.
time (sec)	N/A	0.017	0.153	0.044	0.	62.845	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	165	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.149	0.044	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	162	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.153	0.044	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	281	0	0
normalized size	1	1.	2.08	0.	0.	4.61	0.	0.
time (sec)	N/A	0.009	0.137	0.068	0.	30.872	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	683	0	0
normalized size	1	1.	2.08	0.	0.	11.2	0.	0.
time (sec)	N/A	0.01	0.124	0.027	0.	25.743	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	132	0	0	701	0	0
normalized size	1	1.	1.71	0.	0.	9.1	0.	0.
time (sec)	N/A	0.013	0.158	0.039	0.	77.966	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	137	0	0	675	0	0
normalized size	1	1.	1.73	0.	0.	8.54	0.	0.
time (sec)	N/A	0.014	0.143	0.037	0.	85.168	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	157	0	0	826	0	0
normalized size	1	1.	1.85	0.	0.	9.72	0.	0.
time (sec)	N/A	0.018	0.16	0.042	0.	64.075	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	157	0	0	838	0	0
normalized size	1	1.	1.85	0.	0.	9.86	0.	0.
time (sec)	N/A	0.014	0.147	0.04	0.	54.896	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	163	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.157	0.043	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	168	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.156	0.043	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	115	0	0	254	0	0
normalized size	1	1.	2.17	0.	0.	4.79	0.	0.
time (sec)	N/A	0.008	0.137	0.055	0.	23.335	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	346	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	0.466	0.064	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	348	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.48	0.061	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	161	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.158	0.041	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	160	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.154	0.041	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	160	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.053	0.038	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.037	0.043	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	327	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.269	0.041	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	331	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.274	0.043	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	419	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.382	0.704	0.043	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	431	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.343	0.81	0.041	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	340	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	0.336	0.049	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	341	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.271	0.33	0.048	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	232	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.203	0.046	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	232	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	0.193	0.046	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	392	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.243	0.044	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	336	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.346	0.044	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	380	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.395	0.498	0.044	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	387	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.382	0.538	0.045	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	536	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.552	1.092	0.047	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	419	550	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.484	0.975	0.045	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.221	0.076	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	136	0	0	0	121	0
normalized size	1	1.	0.46	0.	0.	0.	0.41	0.
time (sec)	N/A	0.276	5.063	0.053	0.	0.	52.59	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	88	0
normalized size	1	1.	0.6	0.	0.	0.	0.5	0.
time (sec)	N/A	0.121	5.042	0.043	0.	0.	22.861	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0
normalized size	1	0.91	0.97	0.	0.	0.	0.57	0.
time (sec)	N/A	0.039	0.029	0.03	0.	0.	10.705	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0
normalized size	1	1.	1.	0.	0.	0.	0.5	0.
time (sec)	N/A	0.009	0.003	0.002	0.	0.	2.459	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.178	0.049	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.179	0.051	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.235	0.076	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	182	0	0
normalized size	1	1.	0.98	1.34	0.	3.43	0.	0.
time (sec)	N/A	0.02	0.031	0.004	0.	1.677	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [329] had the largest ratio of [0.4286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	17	0.059
2	A	2	1	1.	17	0.059
3	A	2	1	1.	17	0.059
4	A	2	1	1.	15	0.067
5	A	2	2	1.	17	0.118
6	A	2	2	1.	17	0.118
7	A	3	3	1.	17	0.176
8	A	2	1	1.	19	0.053
9	A	2	1	1.	19	0.053
10	A	2	1	1.	17	0.059
11	A	3	2	1.	19	0.105
12	A	4	3	1.	19	0.158
13	A	3	3	1.	19	0.158
14	A	2	1	1.	19	0.053
15	A	2	1	1.	19	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	2	1	1.	17	0.059
17	A	3	2	1.	19	0.105
18	A	4	3	1.	19	0.158
19	A	5	4	1.	19	0.21
20	A	3	2	1.	19	0.105
21	A	3	2	1.	19	0.105
22	A	3	2	1.	19	0.105
23	A	2	2	1.	17	0.118
24	A	3	2	1.	19	0.105
25	A	4	3	1.	19	0.158
26	A	5	4	1.	19	0.21
27	A	4	3	1.	19	0.158
28	A	4	3	1.	19	0.158
29	A	4	3	1.	19	0.158
30	A	4	3	1.	19	0.158
31	A	2	2	1.	17	0.118
32	A	4	3	1.	19	0.158
33	A	5	4	1.	19	0.21
34	A	6	4	1.	19	0.21
35	A	5	4	1.	19	0.21
36	A	5	4	1.	19	0.21
37	A	5	4	1.	19	0.21
38	A	3	3	1.	19	0.158
39	A	3	3	1.	17	0.176
40	A	5	4	1.	19	0.21
41	A	6	4	1.	19	0.21
42	A	7	4	1.	19	0.21
43	A	3	3	1.	15	0.2
44	A	5	3	1.	15	0.2
45	A	6	6	1.	21	0.286
46	A	5	5	1.	21	0.238
47	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
48	A	3	3	1.	11	0.273
49	A	5	5	1.	21	0.238
50	A	3	3	1.	21	0.143
51	A	4	4	1.	21	0.19
52	A	6	5	1.	21	0.238
53	A	7	6	1.	21	0.286
54	A	6	5	1.	21	0.238
55	A	5	4	1.	19	0.21
56	A	4	3	1.	11	0.273
57	A	6	6	1.	21	0.286
58	A	6	6	1.	21	0.286
59	A	4	3	1.	21	0.143
60	A	5	4	1.	21	0.19
61	A	7	5	1.	21	0.238
62	A	8	6	1.	21	0.286
63	A	7	5	1.	21	0.238
64	A	6	4	1.	19	0.21
65	A	5	3	1.	11	0.273
66	A	7	7	1.	21	0.333
67	A	7	7	1.	21	0.333
68	A	7	7	1.	21	0.333
69	A	5	3	1.	21	0.143
70	A	6	4	1.	21	0.19
71	A	4	4	1.	19	0.21
72	A	4	4	1.	17	0.235
73	A	4	4	1.	21	0.19
74	A	5	5	1.	21	0.238
75	A	4	4	1.	21	0.19
76	A	3	3	1.	19	0.158
77	A	2	2	1.	11	0.182
78	A	2	2	1.	21	0.095
79	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	5	5	1.	21	0.238
81	A	6	5	1.	21	0.238
82	A	5	5	1.	21	0.238
83	A	4	4	1.17	21	0.19
84	A	3	3	1.	19	0.158
85	A	1	1	1.	11	0.091
86	A	3	3	1.	21	0.143
87	A	5	5	1.	21	0.238
88	A	6	5	1.	21	0.238
89	A	6	6	1.	21	0.286
90	A	5	5	1.	21	0.238
91	A	4	4	1.	21	0.19
92	A	2	2	1.	19	0.105
93	A	2	2	1.	11	0.182
94	A	5	5	1.	21	0.238
95	A	6	5	1.	21	0.238
96	A	7	5	1.	21	0.238
97	A	5	3	1.	21	0.143
98	A	4	3	1.	21	0.143
99	A	3	3	1.	19	0.158
100	A	2	2	1.	11	0.182
101	A	3	3	1.	21	0.143
102	A	3	3	1.	21	0.143
103	A	4	4	1.	21	0.19
104	A	5	4	1.	21	0.19
105	A	2	2	1.	26	0.077
106	A	2	2	1.	19	0.105
107	A	2	2	1.	21	0.095
108	A	2	2	1.	15	0.133
109	A	8	8	1.	24	0.333
110	A	7	7	1.	24	0.292
111	A	6	6	1.	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	6	6	1.	24	0.25
113	A	6	6	1.	24	0.25
114	A	8	8	1.	24	0.333
115	A	9	8	1.	24	0.333
116	A	9	8	1.	24	0.333
117	A	8	7	1.	24	0.292
118	A	7	6	1.	22	0.273
119	A	7	7	1.	24	0.292
120	A	7	7	1.	24	0.292
121	A	9	9	1.	24	0.375
122	A	8	7	1.	24	0.292
123	A	7	7	1.	24	0.292
124	A	6	6	1.	24	0.25
125	A	5	5	1.	22	0.227
126	A	1	1	1.	24	0.042
127	A	7	7	1.	24	0.292
128	A	8	8	1.	24	0.333
129	A	7	7	1.	24	0.292
130	A	6	6	1.	24	0.25
131	A	5	5	1.	22	0.227
132	A	7	7	1.	24	0.292
133	A	8	8	1.	24	0.333
134	A	9	8	1.	24	0.333
135	A	8	8	1.	24	0.333
136	A	7	7	1.	24	0.292
137	A	2	2	1.	24	0.083
138	A	6	6	1.	22	0.273
139	A	8	8	1.	24	0.333
140	A	9	8	1.	24	0.333
141	A	1	1	1.	26	0.038
142	A	1	1	1.	24	0.042
143	A	1	1	1.	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	1	1	1.	24	0.042
145	A	1	1	1.	22	0.045
146	A	1	1	1.	19	0.053
147	A	1	1	1.	19	0.053
148	A	1	1	1.	24	0.042
149	A	1	1	1.	22	0.045
150	A	1	1	1.	27	0.037
151	A	1	1	1.	28	0.036
152	A	1	1	1.	29	0.034
153	A	1	1	1.	30	0.033
154	A	1	1	1.	26	0.038
155	A	1	1	1.	26	0.038
156	A	1	1	1.	23	0.043
157	A	1	1	1.	23	0.043
158	A	1	1	1.	23	0.043
159	A	1	1	1.	23	0.043
160	A	1	1	1.	17	0.059
161	A	1	1	1.	21	0.048
162	A	1	1	1.	21	0.048
163	A	3	3	1.15	29	0.103
164	A	3	3	1.23	29	0.103
165	A	3	3	1.2	29	0.103
166	A	6	6	1.	23	0.261
167	A	5	5	1.	23	0.217
168	A	4	4	1.	23	0.174
169	A	1	1	1.	23	0.043
170	A	4	4	1.	23	0.174
171	A	5	5	1.	23	0.217
172	A	7	6	1.	23	0.261
173	A	6	6	1.	23	0.261
174	A	5	5	1.	23	0.217
175	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	4	4	1.	23	0.174
177	A	5	5	1.	23	0.217
178	A	5	5	1.	23	0.217
179	A	3	3	1.	23	0.13
180	A	2	1	1.	23	0.043
181	A	4	4	1.	23	0.174
182	A	5	5	1.	23	0.217
183	A	4	4	1.	21	0.19
184	A	1	1	1.	23	0.043
185	A	1	1	1.	23	0.043
186	A	1	1	1.	23	0.043
187	A	1	1	1.	21	0.048
188	A	1	1	1.	21	0.048
189	A	1	1	1.	21	0.048
190	A	1	1	1.	23	0.043
191	A	4	4	1.	21	0.19
192	A	3	3	1.	23	0.13
193	A	3	3	1.	23	0.13
194	A	3	3	1.	23	0.13
195	A	4	4	1.	21	0.19
196	A	4	4	1.	21	0.19
197	A	4	4	1.	23	0.174
198	A	2	2	1.	23	0.087
199	A	7	6	1.	23	0.261
200	A	6	6	1.	23	0.261
201	A	5	5	1.	23	0.217
202	A	4	4	1.	23	0.174
203	A	1	1	1.	23	0.043
204	A	6	6	1.	23	0.261
205	A	4	4	1.	23	0.174
206	A	5	5	1.	23	0.217
207	A	7	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	6	6	1.	23	0.261
209	A	5	5	1.	23	0.217
210	A	1	1	1.	23	0.043
211	A	6	6	1.	23	0.261
212	A	4	4	1.	23	0.174
213	A	5	5	1.	23	0.217
214	A	1	1	1.	23	0.043
215	A	3	2	1.	24	0.083
216	A	3	2	1.	24	0.083
217	A	3	2	1.	25	0.08
218	A	1	1	1.	23	0.043
219	A	1	1	1.	23	0.043
220	A	1	1	1.	23	0.043
221	A	2	2	1.	23	0.087
222	A	1	1	1.	21	0.048
223	A	1	1	1.	23	0.043
224	A	2	2	1.	23	0.087
225	A	1	1	1.	23	0.043
226	A	1	1	1.	23	0.043
227	A	1	1	1.	23	0.043
228	A	1	1	1.	21	0.048
229	A	1	1	1.	21	0.048
230	A	1	1	1.	21	0.048
231	A	2	2	1.	21	0.095
232	A	1	1	1.	19	0.053
233	A	1	1	1.	21	0.048
234	A	2	2	1.	21	0.095
235	A	1	1	1.	21	0.048
236	A	1	1	1.	21	0.048
237	A	1	1	1.	21	0.048
238	A	2	2	1.	21	0.095
239	A	2	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	2	2	1.	21	0.095
241	A	2	2	1.	21	0.095
242	A	2	2	1.	19	0.105
243	A	1	1	1.	21	0.048
244	A	2	2	1.	21	0.095
245	A	2	2	1.	21	0.095
246	A	2	2	1.	21	0.095
247	A	2	2	1.	21	0.095
248	A	1	1	1.	23	0.043
249	A	1	1	1.	23	0.043
250	A	1	1	1.	23	0.043
251	A	2	2	1.	23	0.087
252	A	1	1	1.	21	0.048
253	A	2	2	1.	23	0.087
254	A	2	2	1.55	23	0.087
255	A	2	2	1.	23	0.087
256	A	2	2	1.	23	0.087
257	A	2	2	1.	23	0.087
258	A	3	3	1.	24	0.125
259	A	3	3	1.	27	0.111
260	A	3	3	1.	25	0.12
261	A	3	3	1.	28	0.107
262	A	3	3	1.	25	0.12
263	A	3	3	1.	26	0.115
264	A	3	3	1.	26	0.115
265	A	3	3	1.	27	0.111
266	A	4	4	1.	23	0.174
267	A	4	4	1.	26	0.154
268	A	4	4	1.	26	0.154
269	A	4	4	1.	29	0.138
270	A	7	6	1.	24	0.25
271	A	7	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	7	6	1.	27	0.222
273	A	7	6	1.	28	0.214
274	A	3	3	1.	24	0.125
275	A	3	3	1.	27	0.111
276	A	3	3	1.	25	0.12
277	A	3	3	1.	28	0.107
278	A	3	3	1.	25	0.12
279	A	3	3	1.	26	0.115
280	A	3	3	1.	26	0.115
281	A	3	3	1.	27	0.111
282	A	4	4	1.	23	0.174
283	A	4	4	1.	26	0.154
284	A	4	4	1.	26	0.154
285	A	4	4	1.	29	0.138
286	A	7	6	1.	24	0.25
287	A	7	6	1.	25	0.24
288	A	7	6	1.	27	0.222
289	A	7	6	1.	28	0.214
290	A	1	1	1.	23	0.043
291	A	2	2	1.	23	0.087
292	A	1	1	1.	21	0.048
293	A	1	1	1.	23	0.043
294	A	4	4	1.	28	0.143
295	A	4	4	1.	23	0.174
296	A	1	1	1.	23	0.043
297	A	1	1	1.	59	0.017
298	A	1	1	1.	59	0.017
299	A	4	4	1.	59	0.068
300	A	3	3	1.	59	0.051
301	C	1	1	0.37	21	0.048
302	A	2	2	1.	26	0.077
303	A	1	1	1.	41	0.024

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	1	1	1.	21	0.048
305	A	1	1	1.	21	0.048
306	A	1	1	1.	21	0.048
307	A	1	1	1.	23	0.043
308	A	1	1	1.	23	0.043
309	A	1	1	1.	23	0.043
310	A	1	1	1.	23	0.043
311	A	1	1	1.	25	0.04
312	A	1	1	1.	21	0.048
313	A	1	1	1.	21	0.048
314	A	1	1	1.	21	0.048
315	A	1	1	1.	23	0.043
316	A	1	1	1.	25	0.04
317	A	1	1	1.	25	0.04
318	A	1	1	1.	25	0.04
319	A	1	1	1.	27	0.037
320	A	1	1	1.	19	0.053
321	A	13	8	1.	21	0.381
322	A	12	8	1.	21	0.381
323	A	8	7	1.	21	0.333
324	A	8	7	1.	21	0.333
325	A	4	3	1.	21	0.143
326	A	5	4	1.	21	0.19
327	A	7	6	1.	21	0.286
328	A	9	8	1.	21	0.381
329	A	10	9	1.	21	0.429
330	A	10	9	1.	21	0.429
331	A	9	8	1.	21	0.381
332	A	9	8	1.	21	0.381
333	A	9	8	1.	21	0.381
334	A	9	8	1.	21	0.381
335	A	9	8	1.	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	9	8	1.	21	0.381
337	A	10	9	1.	21	0.429
338	A	10	9	1.	21	0.429
339	A	11	9	1.	21	0.429
340	A	11	9	1.	21	0.429
341	A	3	2	1.	19	0.105
342	A	5	5	1.	19	0.263
343	A	4	4	1.	19	0.21
344	A	3	3	0.91	17	0.176
345	A	2	2	1.	9	0.222
346	A	2	2	1.	19	0.105
347	A	2	2	1.	19	0.105
348	A	2	2	1.	19	0.105
349	A	1	1	1.	50	0.02

Chapter 3

Listing of integrals

3.1 $\int \left(a+bx^2\right)\left(c+dx^2\right)^4 \, dx$

Optimal. Leaf size=94

$$\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{3}c^3x^3(4ad+bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Rubi [A] time = 0.064207, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{3}c^3x^3(4ad+bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4 dx = \int \left(ac^4+c^3(bc+4ad)x^2+2c^2d(2bc+3ad)x^4+2cd^2(3bc+2ad)x^6+d^3(4bc+ad)x^8+bd^4x\right.$$
$$= ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}$$

Mathematica [A] time = 0.0194268, size = 94, normalized size = 1.

$$\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{3}c^3x^3(4ad+bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Maple [A] time = 0.002, size = 97, normalized size = 1.

$$\frac{bd^4x^{11}}{11}+\frac{\left(ad^4+4\,bcd^3\right)x^9}{9}+\frac{\left(4\,acd^3+6\,bc^2d^2\right)x^7}{7}+\frac{\left(6\,ac^2d^2+4\,bc^3d\right)x^5}{5}+\frac{\left(4\,ac^3d+bc^4\right)x^3}{3}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^4,x)
```

```
[Out] 1/11*b*d^4*x^11+1/9*(a*d^4+4*b*c*d^3)*x^9+1/7*(4*a*c*d^3+6*b*c^2*d^2)*x^7+1
/5*(6*a*c^2*d^2+4*b*c^3*d)*x^5+1/3*(4*a*c^3*d+b*c^4)*x^3+a*c^4*x
```

Maxima [A] time = 1.13958, size = 130, normalized size = 1.38

$$\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\left(4\,bcd^3+ad^4\right)x^9+\frac{2}{7}\left(3\,bc^2d^2+2\,acd^3\right)x^7+ac^4x+\frac{2}{5}\left(2\,bc^3d+3\,ac^2d^2\right)x^5+\frac{1}{3}\left(bc^4+4\,ac^3d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

Fricas [A] time = 1.58752, size = 231, normalized size = 2.46

$$\frac{1}{11}x^{11}d^4b+\frac{4}{9}x^9d^3cb+\frac{1}{9}x^9d^4a+\frac{6}{7}x^7d^2c^2b+\frac{4}{7}x^7d^3ca+\frac{4}{5}x^5dc^3b+\frac{6}{5}x^5d^2c^2a+\frac{1}{3}x^3c^4b+\frac{4}{3}x^3dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^4*b + 4/9*x^9*d^3*c*b + 1/9*x^9*d^4*a + 6/7*x^7*d^2*c^2*b + 4/7
*x^7*d^3*c*a + 4/5*x^5*d*c^3*b + 6/5*x^5*d^2*c^2*a + 1/3*x^3*c^4*b + 4/3*x^
3*d*c^3*a + x*c^4*a
```

Sympy [A] time = 0.075173, size = 107, normalized size = 1.14

$$ac^4x+\frac{bd^4x^{11}}{11}+x^9\left(\frac{ad^4}{9}+\frac{4bcd^3}{9}\right)+x^7\left(\frac{4acd^3}{7}+\frac{6bc^2d^2}{7}\right)+x^5\left(\frac{6ac^2d^2}{5}+\frac{4bc^3d}{5}\right)+x^3\left(\frac{4ac^3d}{3}+\frac{bc^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**7*(4*a*c*d
**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*c**3*d/5) + x**3*(4*
a*c**3*d/3 + b*c**4/3)
```

Giac [A] time = 1.82843, size = 132, normalized size = 1.4

$$\frac{1}{11}bd^4x^{11}+\frac{4}{9}bcd^3x^9+\frac{1}{9}ad^4x^9+\frac{6}{7}bc^2d^2x^7+\frac{4}{7}acd^3x^7+\frac{4}{5}bc^3dx^5+\frac{6}{5}ac^2d^2x^5+\frac{1}{3}bc^4x^3+\frac{4}{3}ac^3dx^3+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2*x^7 + 4/7
*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/3*b*c^4*x^3 + 4/3*a*
c^3*d*x^3 + a*c^4*x
```

3.2 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rubi [A] time = 0.0420841, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^2+3cd(bc+ad)x^4+d^2(3bc+ad)x^6+bd^3x^8\right)dx\\ &= ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9\end{aligned}$$

Mathematica [A] time = 0.0139783, size = 70, normalized size = 1.

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Maple [A] time = 0.002, size = 73, normalized size = 1.

$$\frac{bd^3x^9}{9}+\frac{\left(ad^3+3\,bcd^2\right)x^7}{7}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^5}{5}+\frac{\left(3\,ac^2d+bc^3\right)x^3}{3}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3,x)
```

```
[Out] 1/9*b*d^3*x^9+1/7*(a*d^3+3*b*c*d^2)*x^7+1/5*(3*a*c*d^2+3*b*c^2*d)*x^5+1/3*(
3*a*c^2*d+b*c^3)*x^3+a*c^3*x
```

Maxima [A] time = 0.961674, size = 95, normalized size = 1.36

$$\frac{1}{9}\,bd^3x^9+\frac{1}{7}\left(3\,bcd^2+ad^3\right)x^7+\frac{3}{5}\left(bc^2d+acd^2\right)x^5+ac^3x+\frac{1}{3}\left(bc^3+3\,ac^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Fricas [A] time = 1.55559, size = 169, normalized size = 2.41

$$\frac{1}{9}x^9d^3b+\frac{3}{7}x^7d^2cb+\frac{1}{7}x^7d^3a+\frac{3}{5}x^5dc^2b+\frac{3}{5}x^5d^2ca+\frac{1}{3}x^3c^3b+x^3dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b + 3/7*x^7*d^2*c*b + 1/7*x^7*d^3*a + 3/5*x^5*d*c^2*b + 3/5*x^5
*d^2*c*a + 1/3*x^3*c^3*b + x^3*d*c^2*a + x*c^3*a
```

Sympy [A] time = 0.068978, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^9}{9} + x^7\left(\frac{ad^3}{7} + \frac{3bcd^2}{7}\right) + x^5\left(\frac{3acd^2}{5} + \frac{3bc^2d}{5}\right) + x^3\left(ac^2d + \frac{bc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*(3*a*c*d**
2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

Giac [A] time = 1.14304, size = 99, normalized size = 1.41

$$\frac{1}{9}\,bd^3x^9 + \frac{3}{7}\,bcd^2x^7 + \frac{1}{7}\,ad^3x^7 + \frac{3}{5}\,bc^2dx^5 + \frac{3}{5}\,acd^2x^5 + \frac{1}{3}\,bc^3x^3 + ac^2dx^3 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5 + 3/5*a*c
*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

3.3 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Rubi [A] time = 0.0274234, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^2+d(2bc+ad)x^4+bd^2x^6\right)dx\\ &= ac^2x+\frac{1}{3}c(bc+2ad)x^3+\frac{1}{5}d(2bc+ad)x^5+\frac{1}{7}bd^2x^7\end{aligned}$$

Mathematica [A] time = 0.0108334, size = 50, normalized size = 1.

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^7}{7}+\frac{\left(ad^2+2\,bcd\right)x^5}{5}+\frac{\left(2\,acd+bc^2\right)x^3}{3}+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2,x)
```

```
[Out] 1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c^2*x
```

Maxima [A] time = 1.01954, size = 65, normalized size = 1.3

$$\frac{1}{7}\,bd^2x^7+\frac{1}{5}\left(2\,bcd+ad^2\right)x^5+ac^2x+\frac{1}{3}\left(bc^2+2\,acd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

Fricas [A] time = 1.5615, size = 120, normalized size = 2.4

$$\frac{1}{7}x^7d^2b+\frac{2}{5}x^5dcb+\frac{1}{5}x^5d^2a+\frac{1}{3}x^3c^2b+\frac{2}{3}x^3dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b + 2/5*x^5*d*c*b + 1/5*x^5*d^2*a + 1/3*x^3*c^2*b + 2/3*x^3*d*c
*a + x*c^2*a
```

Sympy [A] time = 0.064517, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^7}{7} + x^5\left(\frac{ad^2}{5} + \frac{2bcd}{5}\right) + x^3\left(\frac{2acd}{3} + \frac{bc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*a*c*d/3 +
b*c**2/3)
```

Giac [A] time = 1.07553, size = 68, normalized size = 1.36

$$\frac{1}{7}\,bd^2x^7 + \frac{2}{5}\,bcdx^5 + \frac{1}{5}\,ad^2x^5 + \frac{1}{3}\,bc^2x^3 + \frac{2}{3}\,acdx^3 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2/3*a*c*d*x
^3 + a*c^2*x
```

3.4 $\int \left(a+bx^2\right)\left(c+dx^2\right) dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rubi [A] time = 0.0128864, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right) dx &= \int \left(ac+(bc+ad)x^2+bdx^4\right) dx\\ &= acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5\end{aligned}$$

Mathematica [A] time = 0.005004, size = 28, normalized size = 1.

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)\,x^3}{3} + \frac{bdx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c),x)
```

```
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Maxima [A] time = 1.46598, size = 32, normalized size = 1.14

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,(bc + ad)x^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Fricas [A] time = 1.51667, size = 66, normalized size = 2.36

$$\frac{1}{5}x^5db + \frac{1}{3}x^3cb + \frac{1}{3}x^3da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b + 1/3*x^3*c*b + 1/3*x^3*d*a + x*c*a
```

Sympy [A] time = 0.054436, size = 26, normalized size = 0.93

$$acx + \frac{bdx^5}{5} + x^3\left(\frac{ad}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c),x)
```

```
[Out] a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

Giac [A] time = 1.08404, size = 35, normalized size = 1.25

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,bcx^3 + \frac{1}{3}\,adx^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

3.5 $\int \frac{a+bx^2}{c+dx^2}\,dx$

Optimal. Leaf size=40

$$\frac{bx}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Rubi [A] time = 0.0198071, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{bx}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^2}\,dx}{d}$$
$$= \frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

Mathematica [A] time = 0.0225555, size = 40, normalized size = 1.

$$\frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Maple [A] time = 0.004, size = 45, normalized size = 1.1

$$\frac{bx}{d} + a\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{bc}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c),x)
```

```
[Out] b*x/d+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-1/d/(c*d)^(1/2)*arctan(x*d/(c
*d)^(1/2))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77492, size = 223, normalized size = 5.58

$$\left[\frac{2\,bcdx + (bc-ad)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{2\,cd^2}, \frac{bcdx-(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{cd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(
d*x^2 + c)))/(c*d^2), (b*c*d*x - (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
))/(c*d^2)]
```

Sympy [B] time = 0.387133, size = 82, normalized size = 2.05

$$\frac{bx}{d} - \frac{\sqrt{-\frac{1}{cd^3}}\,(ad-bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^3}}\,(ad-bc)\log\left(cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3)) + x)/2 + s
qrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3)) + x)/2
```

Giac [A] time = 1.06938, size = 46, normalized size = 1.15

$$\frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

3.6 $\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0202435, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(3/2)*d^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{(c+dx^2)^2} dx = -\frac{(bc-ad)x}{2cd(c+dx^2)} + \frac{(bc+ad)\int \frac{1}{c+dx^2} dx}{2cd}$$
$$= -\frac{(bc-ad)x}{2cd(c+dx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.0443304, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}} - \frac{x(bc-ad)}{2cd(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(3/2)*d^(3/2))
```

Maple [A] time = 0.007, size = 68, normalized size = 1.1

$$\frac{(ad-bc)\,x}{2\,cd\,(dx^2+c)} + \frac{a}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + \frac{b}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+1
/2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62434, size = 381, normalized size = 6.05

$$\left[-\frac{\left(bc^2 + acd + \left(bcd + ad^2\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2 - 2\sqrt{-cd}x - c}{dx^2 + c}\right) + 2\left(bc^2d - acd^2\right)x}{4\left(c^2d^3x^2 + c^3d^2\right)}, \frac{\left(bc^2 + acd + \left(bcd + ad^2\right)x^2\right)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right) - \left(bc^2d - acd^2\right)x}{2\left(c^2d^3x^2 + c^3d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^
2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x
/c) - (b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^2)]
```

Sympy [B] time = 0.516933, size = 112, normalized size = 1.78

$$\frac{x\left(ad - bc\right)}{2c^2d + 2cd^2x^2} - \frac{\sqrt{-\frac{1}{c^3d^3}}\left(ad + bc\right)\log\left(-c^2d\sqrt{-\frac{1}{c^3d^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^3}}\left(ad + bc\right)\log\left(c^2d\sqrt{-\frac{1}{c^3d^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

Giac [A] time = 1.26224, size = 77, normalized size = 1.22

$$\frac{\left(bc + ad\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}cd} - \frac{bcx - adx}{2\left(dx^2 + c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*x - a*d*x)
/((d*x^2 + c)*c*d)
```

3.7 $\int \frac{a+bx^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d\left(c+dx^2\right)}-\frac{x(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2*d*(c + d*
x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Rubi [A] time = 0.0306019, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d\left(c+dx^2\right)}-\frac{x(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2*d*(c + d*
x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\left(c+dx^2\right)^3}\,dx &= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)\int \frac{1}{\left(c+dx^2\right)^2}\,dx}{4cd}\\ &= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\int \frac{1}{c+dx^2}\,dx}{8c^2d}\\ &= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0575071, size = 82, normalized size = 0.89

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x\left(ad\left(5c+3dx^2\right)+bc\left(dx^2-c\right)\right)}{8c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)^2) + ((b*
c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Maple [A] time = 0.007, size = 90, normalized size = 1.

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{(3\,ad+bc)\,x^3}{8\,c^2}+\frac{(5\,ad-bc)\,x}{8\,cd}\right)+\frac{3\,a}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b}{8\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(
1/2)*arctan(x*d/(c*d)^(1/2))*a+1/8/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*
b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77139, size = 621, normalized size = 6.75

$$\left[\frac{2\left(bc^2d^2+3\,acd^3\right)x^3-\left(\left(bcd^2+3\,ad^3\right)x^4+bc^3+3\,ac^2d+2\left(bc^2d+3\,acd^2\right)x^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)-2\left(bc^3d-5\,ac^2\right.}{16\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b*c^2*d^2 + 3*a*c*d^3)*x^3 - ((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 + 3
*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d
)*x - c)/(d*x^2 + c)) - 2*(b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d
^3*x^2 + c^5*d^2), 1/8*((b*c^2*d^2 + 3*a*c*d^3)*x^3 + ((b*c*d^2 + 3*a*d^3)*
x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(c*d)*arctan(sqr
t(c*d)*x/c) - (b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d^3*x^2 + c^5
*d^2)]
```

Sympy [A] time = 0.689287, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad+bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad+bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{x^3\left(3ad^2+bcd\right)+x\left(5acd-bc^2\right)}{8c^4d+16c^3d^2x^2+8c^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3)) + x)/1
6 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*d**3)) + x)
/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d - b*c**2))/(8*c**4*d + 16*c**3*
d**2*x**2 + 8*c**2*d**3*x**4)
```

Giac [A] time = 1.29951, size = 105, normalized size = 1.14

$$\frac{(bc + 3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d} + \frac{bcdx^3 + 3\,ad^2x^3 - bc^2x + 5\,acdx}{8\,\left(dx^2 + c\right)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(b*c*d*x^3
+ 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2*d)
```

3.8 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi [A] time = 0.0735117, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+\right.$$

$$= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7$$

Mathematica [A] time = 0.0217773, size = 122, normalized size = 1.

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A] time = 0.001, size = 125, normalized size = 1.

$$\frac{b^2d^3x^{11}}{11}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^9}{9}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^7}{7}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^5}{5}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A] time = 0.97798, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Fricas [A] time = 1.48247, size = 296, normalized size = 2.43

$$\frac{1}{11}x^{11}d^3b^2 + \frac{1}{3}x^9d^2cb^2 + \frac{2}{9}x^9d^3ba + \frac{3}{7}x^7dc^2b^2 + \frac{6}{7}x^7d^2cba + \frac{1}{7}x^7d^3a^2 + \frac{1}{5}x^5c^3b^2 + \frac{6}{5}x^5dc^2ba + \frac{3}{5}x^5d^2ca^2 + \frac{2}{3}x^3c^3ba + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7*d*c^2*b^2
 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^2 + 6/5*x^5*d*c^2*b*
a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3*d*c^2*a^2 + x*c^3*a^2
```

Sympy [A] time = 0.079024, size = 136, normalized size = 1.11

$$a^2c^3x + \frac{b^2d^3x^{11}}{11} + x^9\left(\frac{2abd^3}{9} + \frac{b^2cd^2}{3}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + x^5\left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^3\left(a^2c^2d + \frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

Giac [A] time = 1.28887, size = 177, normalized size = 1.45

$$\frac{1}{11}b^2d^3x^{11} + \frac{1}{3}b^2cd^2x^9 + \frac{2}{9}abd^3x^9 + \frac{3}{7}b^2c^2dx^7 + \frac{6}{7}abcd^2x^7 + \frac{1}{7}a^2d^3x^7 + \frac{1}{5}b^2c^3x^5 + \frac{6}{5}abc^2dx^5 + \frac{3}{5}a^2cd^2x^5 + \frac{2}{3}abc^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

3.9 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi [A] time = 0.0457626, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2 \, dx &= \int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2x^8\right) dx\\ &= a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9\end{aligned}$$

Mathematica [A] time = 0.0159283, size = 82, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A] time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^9}{9}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^7}{7}+\frac{\left(a^2d^2+4\,abcd+b^2c^2\right)x^5}{5}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^3}{3}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A] time = 0.975871, size = 111, normalized size = 1.35

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A] time = 1.61218, size = 209, normalized size = 2.55

$$\frac{1}{9}x^9d^2b^2+\frac{2}{7}x^7dcb^2+\frac{2}{7}x^7d^2ba+\frac{1}{5}x^5c^2b^2+\frac{4}{5}x^5dcba+\frac{1}{5}x^5d^2a^2+\frac{2}{3}x^3c^2ba+\frac{2}{3}x^3dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2*b^2 + 4/5
*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*x^3*d*c*a^2 + x*c^2*
a^2
```

Sympy [A] time = 0.072224, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^9}{9} + x^7\left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3\left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

Giac [A] time = 1.18338, size = 123, normalized size = 1.5

$$\frac{1}{9}b^2d^2x^9 + \frac{2}{7}b^2cdx^7 + \frac{2}{7}abd^2x^7 + \frac{1}{5}b^2c^2x^5 + \frac{4}{5}abcdx^5 + \frac{1}{5}a^2d^2x^5 + \frac{2}{3}abc^2x^3 + \frac{2}{3}a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

3.10 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rubi [A] time = 0.0279368, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right) \, dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A] time = 0.0068347, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{b^2dx^7}{7}+\frac{\left(2\,abd+b^2c\right)x^5}{5}+\frac{\left(a^2d+2\,abc\right)x^3}{3}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

Maxima [A] time = 0.986889, size = 65, normalized size = 1.3

$$\frac{1}{7}\,b^2dx^7+\frac{1}{5}\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\left(2\,abc+a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Fricas [A] time = 1.47264, size = 120, normalized size = 2.4

$$\frac{1}{7}x^7db^2+\frac{1}{5}x^5cb^2+\frac{2}{5}x^5dba+\frac{2}{3}x^3cba+\frac{1}{3}x^3da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1/3*x^3*d*a
^2 + x*c*a^2
```

Sympy [A] time = 0.06341, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^7}{7} + x^5\left(\frac{2abd}{5} + \frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3} + \frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

Giac [A] time = 1.24141, size = 68, normalized size = 1.36

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abdx^5 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2} \, dx$

Optimal. Leaf size=63

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}+\frac{b^2x^3}{3d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi [A] time = 0.0428152, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^2}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^2)}\right) dx$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

Mathematica [A] time = 0.0482854, size = 59, normalized size = 0.94

$$\frac{bx(6ad-3bc+bdx^2)}{3d^2} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Maple [A] time = 0.003, size = 95, normalized size = 1.5

$$\frac{b^2x^3}{3d} + 2\frac{abx}{d} - \frac{b^2xc}{d^2} + a^2\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - 2\frac{abc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right) + \frac{b^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*x*c+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a
^2-2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*c+1/d^2/(c*d)^(1/2)*arctan(x
*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84056, size = 390, normalized size = 6.19

$$\left[\frac{2\,b^2cd^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)-6\left(b^2c^2d-2\,abcd^2\right)x}{6\,cd^3}, \frac{b^2cd^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

Sympy [B] time = 0.525615, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d}-\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{x\left(2abd-b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) - sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(-c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(
c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d
**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + x*(2*a*b*d - b**2*c)/d**2
```

Giac [A] time = 1.09229, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2d^2x^3 - 3\,b^2cdx + 6\,abd^2x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rubi [A] time = 0.0988627, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2\left(c+dx^2\right)^2}\right)dx$$
$$= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{d^2}$$
$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}$$
$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.0587676, size = 89, normalized size = 1.09

$$-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Maple [A] time = 0.007, size = 129, normalized size = 1.6

$$\frac{b^2x}{d^2}+\frac{xa^2}{2\,c\left(dx^2+c\right)}-\frac{abx}{d\left(dx^2+c\right)}+\frac{cxb^2}{2\,d^2\left(dx^2+c\right)}+\frac{a^2}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,b^2c}{2\,d^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(d*x^2+c)*b
^2+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1/d/(c*d)^(1/2)*arctan(x*d
/(c*d)^(1/2))*a*b-3/2/d^2*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85834, size = 612, normalized size = 7.46

$$\left[\frac{4\,b^2c^2d^2x^3 + \left(3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + \left(3\,b^2c^2d - 2\,abcd^2 - a^2d^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}x - c}{dx^2 + c}\right) + 2\left(3\,b^2c^3d - 2\,abc^2d^2\right.}{4\left(c^2d^4x^2 + c^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

Sympy [B] time = 0.841904, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2} + \frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{2c^2d^2 + 2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad + 3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

Giac [A] time = 1.06507, size = 128, normalized size = 1.56

$$\frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(dx^2 + c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.0722065, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0930836, size = 121, normalized size = 1.04

$$
\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A] time = 0.009, size = 147, normalized size = 1.3

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8d^2c}\right)+\frac{3a^2}{8c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{4cd}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^
2+1/4/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+3/8/d^2/(c*d)^(1/2)*arcta
n(x*d/(c*d)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75657, size = 921, normalized size = 7.94

$$\left[-\frac{2\left(5b^2c^3d^2-2abc^2d^3-3a^2cd^4\right)x^3+\left(3b^2c^4+2abc^3d+3a^2c^2d^2+\left(3b^2c^2d^2+2abcd^3+3a^2d^4\right)x^4+2\left(3b^2c^3d+2\right.\right.}{16\left(c^3d^5x^4+2c^4d^4x^2+c^5d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
```

```
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

Sympy [B] time = 1.16113, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

Giac [A] time = 1.08346, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2}-\frac{5\,b^2c^2dx^3-2\,abcd^2x^3-3\,a^2d^3x^3+3\,b^2c^3x+2\,abc^2dx-5\,a^2cd^2x}{8\left(dx^2+c\right)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

3.14 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=154

$$\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+$$

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^2*
c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*
d^3*x^13)/13
```

Rubi [A] time = 0.103268, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^2*
c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*
d^3*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx = \int \left(a^3c^3+3a^2c^2(bc+ad)x^2+3ac\left(b^2c^2+3abcd+a^2d^2\right)x^4+(bc+ad)\left(b^2c^2+8abcd+a\right.\right.$$

$$= a^3c^3x+a^2c^2(bc+ad)x^3+\frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{7}(bc+ad)\left(b^2c^2+8abcd+a^2\right.$$

Mathematica [A] time = 0.0284419, size = 161, normalized size = 1.05

$$\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7\left(9a^2bcd^2+a^3d^3+9ab^2c^2d+b^3c^3\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7)/7 + (b
*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11
 + (b^3*d^3*x^13)/13
```

Maple [A] time = 0.001, size = 177, normalized size = 1.2

$$\frac{b^3d^3x^{13}}{13}+\frac{\left(3\,ab^2d^3+3\,b^3cd^2\right)x^{11}}{11}+\frac{\left(3\,a^2bd^3+9\,ab^2cd^2+3\,b^3c^2d\right)x^9}{9}+\frac{\left(a^3d^3+9\,a^2bcd^2+9\,ab^2c^2d+b^3c^3\right)x^7}{7}+\frac{\left(3\,a^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^3*d^3*x^13+1/11*(3*a*b^2*d^3+3*b^3*c*d^2)*x^11+1/9*(3*a^2*b*d^3+9*a*
b^2*c*d^2+3*b^3*c^2*d)*x^9+1/7*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3
)*x^7+1/5*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^5+1/3*(3*a^3*c^2*d+3*a^
2*b*c^3)*x^3+a^3*c^3*x
```

Maxima [A] time = 1.08075, size = 225, normalized size = 1.46

$$\frac{1}{13}\,b^3d^3x^{13}+\frac{3}{11}\left(b^3cd^2+ab^2d^3\right)x^{11}+\frac{1}{3}\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9+\frac{1}{7}\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
```

```
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Fricas [A] time = 1.50852, size = 414, normalized size = 2.69

$$\frac{1}{13}x^{13}d^3b^3 + \frac{3}{11}x^{11}d^2cb^3 + \frac{3}{11}x^{11}d^3b^2a + \frac{1}{3}x^9dc^2b^3 + x^9d^2cb^2a + \frac{1}{3}x^9d^3ba^2 + \frac{1}{7}x^7c^3b^3 + \frac{9}{7}x^7dc^2b^2a + \frac{9}{7}x^7d^2cba^2 + \frac{1}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^3 + 3/11*x^11*d^2*c*b^3 + 3/11*x^11*d^3*b^2*a + 1/3*x^9*d*c
^2*b^3 + x^9*d^2*c*b^2*a + 1/3*x^9*d^3*b*a^2 + 1/7*x^7*c^3*b^3 + 9/7*x^7*d*
c^2*b^2*a + 9/7*x^7*d^2*c*b*a^2 + 1/7*x^7*d^3*a^3 + 3/5*x^5*c^3*b^2*a + 9/5
*x^5*d*c^2*b*a^2 + 3/5*x^5*d^2*c*a^3 + x^3*c^3*b*a^2 + x^3*d*c^2*a^3 + x*c^
3*a^3
```

Sympy [A] time = 0.086279, size = 189, normalized size = 1.23

$$a^3c^3x + \frac{b^3d^3x^{13}}{13} + x^{11}\left(\frac{3ab^2d^3}{11} + \frac{3b^3cd^2}{11}\right) + x^9\left(\frac{a^2bd^3}{3} + ab^2cd^2 + \frac{b^3c^2d}{3}\right) + x^7\left(\frac{a^3d^3}{7} + \frac{9a^2bcd^2}{7} + \frac{9ab^2c^2d}{7} + \frac{b^3c^3}{7}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b**3*c*d**2/
11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d/3) + x**7*(a**3*d**
3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 + b**3*c**3/7) + x**5*(3*a**3*c
*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b**2*c**3/5) + x**3*(a**3*c**2*d + a**2*b
*c**3)
```

Giac [A] time = 1.09512, size = 252, normalized size = 1.64

$$\frac{1}{13}b^3d^3x^{13} + \frac{3}{11}b^3cd^2x^{11} + \frac{3}{11}ab^2d^3x^{11} + \frac{1}{3}b^3c^2dx^9 + ab^2cd^2x^9 + \frac{1}{3}a^2bd^3x^9 + \frac{1}{7}b^3c^3x^7 + \frac{9}{7}ab^2c^2dx^7 + \frac{9}{7}a^2bcd^2x^7 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1/3*b^3*c^2
*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*c^3*x^7 + 9/7*a*b^2*
c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3*x^7 + 3/5*a*b^2*c^3*x^5 + 9/5
*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 + a^3*
c^3*x
```

3.15 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+a^3c^2x+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3$$

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rubi [A] time = 0.0700041, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+a^3c^2x+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^2\,dx=\int \left(a^3c^2+a^2c(3bc+2ad)x^2+a\left(3b^2c^2+6abcd+a^2d^2\right)x^4+b\left(b^2c^2+6abcd+3a^2d^2\right)x^6\right.$$
$$=a^3c^2x+\frac{1}{3}a^2c(3bc+2ad)x^3+\frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)$$

Mathematica [A] time = 0.020559, size = 122, normalized size = 1.

$$\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+a^3c^2x+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Maple [A] time = 0.002, size = 125, normalized size = 1.

$$\frac{b^3d^2x^{11}}{11}+\frac{\left(3\,ab^2d^2+2\,b^3cd\right)x^9}{9}+\frac{\left(3\,a^2bd^2+6\,ab^2cd+b^3c^2\right)x^7}{7}+\frac{\left(a^3d^2+6\,a^2bcd+3\,ab^2c^2\right)x^5}{5}+\frac{\left(2\,a^3cd+3\,a^2bc^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^3*d^2*x^11+1/9*(3*a*b^2*d^2+2*b^3*c*d)*x^9+1/7*(3*a^2*b*d^2+6*a*b^2*
c*d+b^3*c^2)*x^7+1/5*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^5+1/3*(2*a^3*c*d+3
*a^2*b*c^2)*x^3+a^3*c^2*x
```

Maxima [A] time = 0.982753, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

Fricas [A] time = 1.47619, size = 296, normalized size = 2.43

$$\frac{1}{11}x^{11}d^2b^3 + \frac{2}{9}x^9dcb^3 + \frac{1}{3}x^9d^2b^2a + \frac{1}{7}x^7c^2b^3 + \frac{6}{7}x^7dcb^2a + \frac{3}{7}x^7d^2ba^2 + \frac{3}{5}x^5c^2b^2a + \frac{6}{5}x^5dcba^2 + \frac{1}{5}x^5d^2a^3 + x^3c^2ba^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^3 + 2/9*x^9*d*c*b^3 + 1/3*x^9*d^2*b^2*a + 1/7*x^7*c^2*b^3 +
 6/7*x^7*d*c*b^2*a + 3/7*x^7*d^2*b*a^2 + 3/5*x^5*c^2*b^2*a + 6/5*x^5*d*c*b*
a^2 + 1/5*x^5*d^2*a^3 + x^3*c^2*b*a^2 + 2/3*x^3*d*c*a^3 + x*c^2*a^3
```

Sympy [A] time = 0.080502, size = 136, normalized size = 1.11

$$a^3c^2x + \frac{b^3d^2x^{11}}{11} + x^9\left(\frac{ab^2d^2}{3} + \frac{2b^3cd}{9}\right) + x^7\left(\frac{3a^2bd^2}{7} + \frac{6ab^2cd}{7} + \frac{b^3c^2}{7}\right) + x^5\left(\frac{a^3d^2}{5} + \frac{6a^2bcd}{5} + \frac{3ab^2c^2}{5}\right) + x^3\left(\frac{2a^3cd}{3} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c*d/9) + x*
*7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x**5*(a**3*d**2/5 + 6
*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a**3*c*d/3 + a**2*b*c**2)
```

Giac [A] time = 1.08105, size = 177, normalized size = 1.45

$$\frac{1}{11}b^3d^2x^{11} + \frac{2}{9}b^3cdx^9 + \frac{1}{3}ab^2d^2x^9 + \frac{1}{7}b^3c^2x^7 + \frac{6}{7}ab^2cdx^7 + \frac{3}{7}a^2bd^2x^7 + \frac{3}{5}ab^2c^2x^5 + \frac{6}{5}a^2bcdx^5 + \frac{1}{5}a^3d^2x^5 + a^2bc^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3*c^2*x^7 +
 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*x^5 + 6/5*a^2*b*c*d*
x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a^3*c*d*x^3 + a^3*c^2*x
```

3.16 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=70

$$\frac{1}{3}a^2x^3(ad+3bc)+a^3cx+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rubi [A] time = 0.0437831, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{3}a^2x^3(ad+3bc)+a^3cx+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\, dx &= \int \left(a^3c+a^2(3bc+ad)x^2+3ab(bc+ad)x^4+b^2(bc+3ad)x^6+b^3dx^8\right) dx\\ &= a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9\end{aligned}$$

Mathematica [A] time = 0.0116448, size = 70, normalized size = 1.

$$\frac{1}{3}a^2x^3(ad+3bc)+a^3cx+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$\frac{b^3 d x^9}{9} + \frac{\left(3\,a b^2 d + b^3 c\right) x^7}{7} + \frac{\left(3\,a^2 b d + 3\,a b^2 c\right) x^5}{5} + \frac{\left(a^3 d + 3\,a^2 b c\right) x^3}{3} + a^3 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(d*x^2+c),x)
```

```
[Out] 1/9*b^3*d*x^9+1/7*(3*a*b^2*d+b^3*c)*x^7+1/5*(3*a^2*b*d+3*a*b^2*c)*x^5+1/3*(
a^3*d+3*a^2*b*c)*x^3+a^3*c*x
```

Maxima [A] time = 0.949192, size = 95, normalized size = 1.36

$$\frac{1}{9}\,b^3 d x^9 + \frac{1}{7}\left(b^3 c + 3\,a b^2 d\right)x^7 + \frac{3}{5}\left(a b^2 c + a^2 b d\right)x^5 + a^3 c x + \frac{1}{3}\left(3\,a^2 b c + a^3 d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Fricas [A] time = 1.41907, size = 169, normalized size = 2.41

$$\frac{1}{9}x^9 d b^3 + \frac{1}{7}x^7 c b^3 + \frac{3}{7}x^7 d b^2 a + \frac{3}{5}x^5 c b^2 a + \frac{3}{5}x^5 d b a^2 + x^3 c b a^2 + \frac{1}{3}x^3 d a^3 + x c a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^3 + 1/7*x^7*c*b^3 + 3/7*x^7*d*b^2*a + 3/5*x^5*c*b^2*a + 3/5*x^5
*d*b*a^2 + x^3*c*b*a^2 + 1/3*x^3*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.068256, size = 76, normalized size = 1.09

$$a^3cx + \frac{b^3dx^9}{9} + x^7\left(\frac{3ab^2d}{7} + \frac{b^3c}{7}\right) + x^5\left(\frac{3a^2bd}{5} + \frac{3ab^2c}{5}\right) + x^3\left(\frac{a^3d}{3} + a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(d*x**2+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*(3*a**2*b*
d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

Giac [A] time = 1.10157, size = 99, normalized size = 1.41

$$\frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bdx^5 + a^2bcx^3 + \frac{1}{3}\,a^3dx^3 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5 + 3/5*a^2
*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2}\,dx$

Optimal. Leaf size=98

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}+\frac{b^3x^5}{5d}$$

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d^
2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*d^(7/2))
```

Rubi [A] time = 0.0637283, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}+\frac{b^3x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d^
2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*d^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{c+dx^2}\,dx = \int \left(\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^4}{d} + \frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3d^3}{d^3\left(c+dx^2\right)}\right) dx$$

$$= \frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3\int \frac{1}{c+dx^2}\,dx}{d^3}$$

$$= \frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

Mathematica [A] time = 0.0599627, size = 93, normalized size = 0.95

$$\frac{bx\left(45a^2d^2+15abd\left(dx^2-3c\right)+b^2\left(15c^2-5cdx^2+3d^2x^4\right)\right)}{15d^3} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*x^2 + 3*d^
2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^
(7/2))
```

Maple [A] time = 0.003, size = 161, normalized size = 1.6

$$\frac{b^3x^5}{5\,d} + \frac{ab^2x^3}{d} - \frac{b^3x^3c}{3\,d^2} + 3\,\frac{a^2bx}{d} - 3\,\frac{ab^2cx}{d^2} + \frac{b^3c^2x}{d^3} + a^3\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - 3\,\frac{a^2bc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right) + 3\,\frac{ab^2c^2}{d^2\sqrt{cd}}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] 1/5*b^3*x^5/d+b^2/d*x^3*a-1/3*b^3/d^2*x^3*c+3*b/d*a^2*x-3*b^2/d^2*a*c*x+b^3
/d^3*c^2*x+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^3-3/d/(c*d)^(1/2)*arctan
(x*d/(c*d)^(1/2))*a^2*b*c+3/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b^2*c
^2-1/d^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^3*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75112, size = 613, normalized size = 6.26

$$\left[\frac{6\,b^3cd^3x^5-10\left(b^3c^2d^2-3\,ab^2cd^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+30\left(b^3c^3d-3\right.}{30\,cd^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^3*c*d^3*x^5 - 10*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(
-c*d)*x - c)/(d*x^2 + c)) + 30*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
)*x)/(c*d^4), 1/15*(3*b^3*c*d^3*x^5 - 5*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 -
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*arctan(sq
rt(c*d)*x/c) + 15*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*x)/(c*d^4)]
```

Sympy [B] time = 0.671182, size = 240, normalized size = 2.45

$$\frac{b^3x^5}{5d}-\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{x^3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d**3*sqrt(-1/(c*d
**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*
c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(c*d**3*sqrt(-1/(c*d**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 + x**3*(3*a*b**2*d - b**3*c)/(3*d**2) + x*(3*a**2*b*d**2 - 3*a*b**
2*c*d + b**3*c**2)/d**3
```

Giac [A] time = 1.28352, size = 176, normalized size = 1.8

$$-\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^3d^4x^5 - 5\,b^3cd^3x^3 + 15\,ab^2d^4x^3 + 15\,b^3c^2d^2x - 45\,ab^2cd^3x + 45\,a}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3 + 15*a*b^2*d^4*x^3
+ 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^2*b*d^4*x)/d^5
```

3.18 $\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Rubi [A] time = 0.0961711, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x^2}{d^2}+\frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{d^3(c+dx^2)^2}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}+\frac{\int \frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{(c+dx^2)^2}\,dx}{d^3}\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3(c+dx^2)}+\frac{\left((bc-ad)^2(5bc+ad)\right)\int \frac{1}{c+dx^2}\,dx}{2cd^3}\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3(c+dx^2)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0582583, size = 107, normalized size = 1.

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3(c+dx^2)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Maple [B] time = 0.01, size = 205, normalized size = 1.9

$$\frac{b^3x^3}{3\,d^2}+3\,\frac{ab^2x}{d^2}-2\,\frac{b^3xc}{d^3}+\frac{xa^3}{2\,c\left(dx^2+c\right)}-\frac{3\,a^2bx}{2\,d\left(dx^2+c\right)}+\frac{3\,acxb^2}{2\,d^2\left(dx^2+c\right)}-\frac{c^2xb^3}{2\,d^3\left(dx^2+c\right)}+\frac{a^3}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^3*x^3/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c+1/2/c*x/(d*x^2+c)*a^3-3/2/d*x/(
d*x^2+c)*a^2*b+3/2/d^2*c*x/(d*x^2+c)*a*b^2-1/2/d^3*c^2*x/(d*x^2+c)*b^3+1/2/
c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^3+3/2/d/(c*d)^(1/2)*arctan(x*d/(c*d
)^(1/2))*a^2*b-9/2/d^2*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b^2+5/2/d^3*
c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79082, size = 896, normalized size = 8.37

$$\left[\frac{4\,b^3c^2d^3x^5 - 4\left(5\,b^3c^3d^2 - 9\,ab^2c^2d^3\right)x^3 - 3\left(5\,b^3c^4 - 9\,ab^2c^3d + 3\,a^2bc^2d^2 + a^3cd^3 + \left(5\,b^3c^3d - 9\,ab^2c^2d^2 + 3\,a^2bcd^3 + \right.\right.}{12\left(c^2d^5x^2 + c^3d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^3*c^2*d^3*x^5 - 4*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 - 3*(5*b
^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-
c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d
^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4), 1/6*(2*b^3*c^2*d^3*x^5 - 2*(5*b
^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 + 3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*
d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^3*c^4*d - 9*a*b^2*c^3*d^
```

```
2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4)]
```

Sympy [B] time = 1.21979, size = 313, normalized size = 2.93

$$\frac{b^3x^3}{3d^2} + \frac{x\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3\right)}{2c^2d^3 + 2cd^4x^2} - \frac{\sqrt{-\frac{1}{c^3d^7}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^7}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**
3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2) - sqrt(-1/(c**3*d**7))*(a*d - b*c)**2
*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*
b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 +
 sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c
**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*
b**2*c**2*d + 5*b**3*c**3) + x)/4 + x*(3*a*b**2*d - 2*b**3*c)/d**3
```

Giac [A] time = 1.63629, size = 205, normalized size = 1.92

$$\frac{\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^3} - \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\left(dx^2+c\right)cd^3} + \frac{b^3d^4x^3 - 6\,b^3cd^3x + 9\,ab^2}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c
*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*d^4*x^3 - 6*b^3*c*d^3*x + 9*a*
b^2*d^4*x)/d^6
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Rubi [A] time = 0.164726, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {390, 1157, 385, 205}

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3}\,dx &= \int \left(\frac{b^3}{d^3}-\frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{d^3\left(c+dx^2\right)^3}\right)dx\\
&= \frac{b^3x}{d^3}-\frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{\left(c+dx^2\right)^3}\,dx}{d^3}\\
&= \frac{b^3x}{d^3}-\frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2}+\frac{\int \frac{-3(bc-ad)(bc+ad)^2-12b^2cd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd^3}\\
&= \frac{b^3x}{d^3}-\frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2}+\frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)}-\frac{\left(3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\right)\int \frac{1}{c+dx^2}\,dx}{8c^2d^3}\\
&= \frac{b^3x}{d^3}-\frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2}+\frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)}-\frac{3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0782675, size = 141, normalized size = 1.08

$$-\frac{3\left(-a^2bcd^2-a^3d^3-3ab^2c^2d+5b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^3 - 3*a*b^2*c^2*d -
a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Maple [B] time = 0.01, size = 266, normalized size = 2.1

$$\frac{b^3x}{d^3}+\frac{3\,dx^3a^3}{8\left(dx^2+c\right)^2c^2}+\frac{3\,x^3a^2b}{8\left(dx^2+c\right)^2c}-\frac{15\,ab^2x^3}{8\,d\left(dx^2+c\right)^2}+\frac{9\,cx^3b^3}{8\,d^2\left(dx^2+c\right)^2}+\frac{5\,xa^3}{8\left(dx^2+c\right)^2c}-\frac{3\,a^2bx}{8\,d\left(dx^2+c\right)^2}-\frac{9\,acx}{8\,d^2\left(dx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] b^3*x/d^3+3/8*d/(d*x^2+c)^2/c^2*x^3*a^3+3/8/(d*x^2+c)^2/c*x^3*a^2*b-15/8/d/
(d*x^2+c)^2*x^3*a*b^2+9/8/d^2/(d*x^2+c)^2*c*x^3*b^3+5/8/(d*x^2+c)^2/c*x*a^3
-3/8/d/(d*x^2+c)^2*x*a^2*b-9/8/d^2/(d*x^2+c)^2*c*x*a*b^2+7/8/d^3/(d*x^2+c)^
2*c^2*x*b^3+3/8/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^3+3/8/d/c/(c*d)^(
1/2)*arctan(x*d/(c*d)^(1/2))*a^2*b+9/8/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/
2))*a*b^2-15/8/d^3*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93027, size = 1224, normalized size = 9.42

$$\left[\frac{16\,b^3c^3d^3x^5+2\left(25\,b^3c^4d^2-15\,ab^2c^3d^3+3\,a^2bc^2d^4+3\,a^3cd^5\right)x^3+3\left(5\,b^3c^5-3\,ab^2c^4d-a^2bc^3d^2-a^3c^2d^3+\left(5\,b^3c^3d^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^3*c^3*d^3*x^5 + 2*(25*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 3*a^2*b*
c^2*d^4 + 3*a^3*c*d^5)*x^3 + 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 -
 a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)*x^
4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^3*c^5*d - 9*
a*b^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^
5*x^2 + c^5*d^4), 1/8*(8*b^3*c^3*d^3*x^5 + (25*b^3*c^4*d^2 - 15*a*b^2*c^3*d
^3 + 3*a^2*b*c^2*d^4 + 3*a^3*c*d^5)*x^3 - 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^
2*b*c^3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4
- a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d
^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (15*b^3*c^5*d - 9*a*b^2*c^4*d^2
- 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^5*x^2 + c^5*d^
4)]
```

Sympy [B] time = 2.18939, size = 422, normalized size = 3.25

$$\frac{b^3x}{d^3}-\frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\left(a^2d^2+2abcd+5b^2c^2\right)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{c^5d^7}}\,(ad-bc)\left(a^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5
*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 +
```

```
2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**2*d -
 15*b**3*c**3) + x)/16 + 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*
a*b*c*d + 5*b**2*c**2)*log(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a*
*2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**
2*c**2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d**3 - 1
5*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a**2*b*c**2*d**2
 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 + 16*c**3*d**4*x**2 + 8*c**
2*d**5*x**4)
```

Giac [A] time = 1.10929, size = 243, normalized size = 1.87

$$\frac{b^3x}{d^3} - \frac{3\left(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3} + \frac{9\,b^3c^3dx^3 - 15\,ab^2c^2d^2x^3 + 3\,a^2bcd^3x^3 + 3\,a^3d^4x^3 + 7\,b^3c^4x}{8\left(dx^2 + c\right)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan(
d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x^3 - 15*a*b^2*c^2*d^
2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7*b^3*c^4*x - 9*a*b^2*c^3*d*x -
 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)/((d*x^2 + c)^2*c^2*d^3)
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2} \, dx$

Optimal. Leaf size=142

$$\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}+\frac{d^4x^7}{7b}$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) + (d
^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2
))
```

Rubi [A] time = 0.093036, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}+\frac{d^4x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2),x]
```


```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) + (d
^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2
))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^2}{b^3} + \frac{d^3(4bc-ad)x^4}{b^2} + \frac{d^4x^6}{b} + \right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2} + \frac{d^4x^7}{7b} + \frac{(b}{}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2} + \frac{d^4x^7}{7b} + \frac{(b}{}$$

Mathematica [A] time = 0.0855929, size = 136, normalized size = 0.96

$$\frac{dx\left(35a^2bd^2\left(12c+dx^2\right)-105a^3d^3-7ab^2d\left(90c^2+20cdx^2+3d^2x^4\right)+3b^3\left(70c^2dx^2+140c^3+28cd^2x^4+5d^3x^6\right)\right)}{105b^4} + \frac{(b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*c^2 + 20*c
*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 28*c*d^2*x^4 + 5*d^3*
x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(
9/2))
```

Maple [A] time = 0.004, size = 246, normalized size = 1.7

$$\frac{d^4x^7}{7\,b} - \frac{d^4x^5a}{5\,b^2} + \frac{4\,d^3x^5c}{5\,b} + \frac{d^4x^3a^2}{3\,b^3} - \frac{4\,d^3x^3ac}{3\,b^2} + 2\,\frac{d^2x^3c^2}{b} - \frac{d^4a^3x}{b^4} + 4\,\frac{a^2d^3cx}{b^3} - 6\,\frac{ac^2d^2x}{b^2} + 4\,\frac{dc^3x}{b} + \frac{a^4d^4}{b^4}\arctan\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a),x)
```

```
[Out] 1/7*d^4*x^7/b-1/5*d^4/b^2*x^5*a+4/5*d^3/b*x^5*c+1/3*d^4/b^3*x^3*a^2-4/3*d^3
/b^2*x^3*a*c+2*d^2/b*x^3*c^2-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^
2*x+4*d/b*c^3*x+1/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^4*d^4-4/b^3/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*c*d^3+6/b^2/(a*b)^(1/2)*arctan(b*x/(a*
```

```
b)^(1/2))*a^2*c^2*d^2-4/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*c^3*d+1/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85122, size = 892, normalized size = 6.28

$$\left[\frac{30\,ab^4d^4x^7+42\left(4\,ab^4cd^3-a^2b^3d^4\right)x^5+70\left(6\,ab^4c^2d^2-4\,a^2b^3cd^3+a^3b^2d^4\right)x^3-105\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a\right.}{210\,ab^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*a*b^4*d^4*x^7 + 42*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 70*(6*a*b
^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3*b^2*d^4)*x^3 - 105*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(4*a*b^4*c^3*d - 6*a^2*b^3*c^2*d^2 +
 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5), 1/105*(15*a*b^4*d^4*x^7 + 21*(4*a
*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 35*(6*a*b^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3
*b^2*d^4)*x^3 + 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(4*a*b^4*c^3*d - 6*a
^2*b^3*c^2*d^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5)]
```

Sympy [B] time = 0.881547, size = 323, normalized size = 2.27

$$-\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^4\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)
**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d
+ b**4*c**4) + x)/2 + sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(a*b**4*sqrt(-1/(
a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + d**4*x**7/(7*b) - x**5*(a*d**4 -
4*b*c*d**3)/(5*b**2) + x**3*(a**2*d**4 - 4*a*b*c*d**3 + 6*b**2*c**2*d**2)/(
3*b**3) - x*(a**3*d**4 - 4*a**2*b*c*d**3 + 6*a*b**2*c**2*d**2 - 4*b**3*c**3
*d)/b**4
```

Giac [A] time = 1.11394, size = 267, normalized size = 1.88

$$\frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^4x^7 + 84\,b^6cd^3x^5 - 21\,ab^5d^4x^5 + 210\,b^6c^2d^2x^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*x^7 + 84*b^6*c*d^3*x
^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 - 140*a*b^5*c*d^3*x^3 + 35*a^2*
b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3*x -
 105*a^3*b^3*d^4*x)/b^7
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^3x^5}{5b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rubi [A] time = 0.0594187, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^2\right)}\right)dx$$
$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int \frac{1}{a+bx^2}\,dx}{b^3}$$
$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Mathematica [A] time = 0.0659506, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

Maple [A] time = 0.003, size = 161, normalized size = 1.6

$$\frac{d^3x^5}{5\,b}-\frac{d^3x^3a}{3\,b^2}+\frac{d^2x^3c}{b}+\frac{a^2d^3x}{b^3}-3\,\frac{acd^2x}{b^2}+3\,\frac{dc^2x}{b}-\frac{a^3d^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+3\,\frac{a^2cd^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\,\frac{ac^2d}{b\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5*d^3*x^5/b-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c*x+3
*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a^3*d^3+3/b^2/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*a^2*c*d^2-3/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*a*c^2*d+1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69763, size = 613, normalized size = 6.26

$$\left[\frac{6\,ab^3d^3x^5 + 10\left(3\,ab^3cd^2 - a^2b^2d^3\right)x^3 + 15\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 30\left(3\,ab^3c^2d - \right.}{30\,ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

Sympy [B] time = 0.69904, size = 240, normalized size = 2.45

$$\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x^5}{5b} - \frac{x^3\left(ad\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*
*3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 - sqr
t(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)**3/(
a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x*
*5/(5*b) - x**3*(a*d**3 - 3*b*c*d**2)/(3*b**2) + x*(a**2*d**3 - 3*a*b*c*d**
2 + 3*b**2*c**2*d)/b**3
```

Giac [A] time = 1.16894, size = 174, normalized size = 1.78

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^{3}}+\frac{3\,b^{4}d^{3}x^{5}+15\,b^{4}cd^{2}x^{3}-5\,ab^{3}d^{3}x^{3}+45\,b^{4}c^{2}dx-45\,ab^{3}cd^{2}x+15\,a^{2}}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2} \, dx$

Optimal. Leaf size=63

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{d^2x^3}{3b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0409947, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^2)}\right)dx$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\int\frac{1}{a+bx^2}\,dx}{b^2}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

Mathematica [A] time = 0.0491206, size = 59, normalized size = 0.94

$$\frac{dx\left(-3ad+6bc+bdx^2\right)}{3b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.003, size = 95, normalized size = 1.5

$$\frac{d^2x^3}{3\,b}-\frac{ad^2x}{b^2}+2\,\frac{dxc}{b}+\frac{a^2d^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{acd}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^2*x^3/b-d^2/b^2*a*x+2*d/b*x*c+1/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))*a^2*d^2-2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*c*d+1/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08632, size = 390, normalized size = 6.19

$$\left[\frac{2\,ab^2d^2x^3 - 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 6\left(2\,ab^2cd - a^2bd^2\right)x}{6\,ab^3}, \frac{ab^2d^2x^3 + 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

Sympy [B] time = 0.537803, size = 172, normalized size = 2.73

$$-\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}-\frac{x\left(ad^2-2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a*b**5))*(a*d - b
*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
 b**2*c**2) + x)/2 + d**2*x**3/(3*b) - x*(a*d**2 - 2*b*c*d)/b**2
```

Giac [A] time = 1.12288, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.23 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0153834, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}$$
$$= \frac{dx}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Mathematica [A] time = 0.0251749, size = 40, normalized size = 1.03

$$\frac{dx}{b} - \frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.003, size = 45, normalized size = 1.2

$$\frac{dx}{b} - \frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] d*x/b-1/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*a*d+1/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09903, size = 223, normalized size = 5.72

$$\left[\frac{2\,abdx+\sqrt{-ab}(bc-ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\frac{abdx+\sqrt{ab}(bc-ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] time = 0.400374, size = 82, normalized size = 2.1

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

Giac [A] time = 1.10009, size = 45, normalized size = 1.15

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi [A] time = 0.0270253, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Mathematica [A] time = 0.0437391, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A] time = 0.005, size = 55, normalized size = 0.8

$$\frac{d}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{b}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] d/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-b/(a*d-b*c)/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07481, size = 608, normalized size = 8.69

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)}, -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

Sympy [B] time = 2.2155, size = 712, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
```

```
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

Giac [B] time = 1.22475, size = 257, normalized size = 3.67

$$-\frac{2\sqrt{cd}b|d|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{2\sqrt{abd}|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -2*sqrt(c*d)*b*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt(-4*a*b*c*
d + (b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d^2*abs(b*c - a*d) +
(b*c - a*d)^2*d) + 2*sqrt(a*b)*d*abs(b)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*
d - sqrt(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b*d
*abs(b*c - a*d) - (b*c - a*d)^2*b)
```

3.25 $\int \frac{1}{(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.0837109, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{\int \frac{2bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.16838, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{dx(ad-bc)}{c(c+dx^2)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.009, size = 144, normalized size = 1.3

$$\frac{d^2xa}{2\,(ad-bc)^2\,c\left(dx^2+c\right)}-\frac{bdx}{2\,(ad-bc)^2\left(dx^2+c\right)}+\frac{ad^2}{2\,(ad-bc)^2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)*b+1/2*d^2
/(a*d-b*c)^2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-3/2*d/(a*d-b*c)^2/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.89347, size = 1451, normalized size = 13.31

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)},\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
```

```
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

Sympy [B] time = 15.0329, size = 2033, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*a*c**2*d - 2*b*c**3 + x**2*(2*a*c*d**2 - 2*b*c**2*d)) - sqrt(-b**3/a
)*log(x + (-4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 28*a**6*b*c*
*4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**2*c**5*d**4*(-b**3/a)*
*(3/2)/(a*d - b*c)**6 - a**5*d**5*sqrt(-b**3/a)/(a*d - b*c)**2 + 56*a**4*b*
*3*c**6*d**3*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 9*a**4*b*c*d**4*sqrt(-b**3/a
)/(a*d - b*c)**2 - 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 -
27*a**3*b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 - 20*a**2*b**5*c**8*d*(
-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**3*c**3*d**2*sqrt(-b**3/a)/(a*d
- b*c)**2 + 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 8*b**5*c**5*sqr
t(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2
*d))/(2*(a*d - b*c)**2) + sqrt(-b**3/a)*log(x + (4*a**7*c**3*d**6*(-b**3/a)
**(3/2)/(a*d - b*c)**6 - 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**
6 + 64*a**5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 + a**5*d**5*sqrt
(-b**3/a)/(a*d - b*c)**2 - 56*a**4*b**3*c**6*d**3*(-b**3/a)**(3/2)/(a*d - b
*c)**6 - 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**2 + 4*a**3*b**4*c**7*d*
*2*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sqrt(-b**3/a)/(
a*d - b*c)**2 + 20*a**2*b**5*c**8*d*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a*
*2*b**3*c**3*d**2*sqrt(-b**3/a)/(a*d - b*c)**2 - 8*a*b**6*c**9*(-b**3/a)**(
3/2)/(a*d - b*c)**6 - 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*
d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) - sqrt(-d/c**3
)*(a*d - 3*b*c)*log(x + (-a**7*c**3*d**6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/
(2*(a*d - b*c)**6) + 7*a**6*b*c**4*d**5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(
2*(a*d - b*c)**6) - 8*a**5*b**2*c**5*d**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3
/(a*d - b*c)**6 - a**5*d**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2)
+ 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 +
9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a**3*b**4*
```

```
c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 27*a**3*b*
*2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 5*a**2*b**5*c
**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 27*a**2*b**3*c
**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + a*b**6*c**9*(-d/c
**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + 4*b**5*c**5*sqrt(-d/c**3)*(a*
d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2
*d))/(4*(a*d - b*c)**2) + sqrt(-d/c**3)*(a*d - 3*b*c)*log(x + (a**7*c**3*d*
*6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 7*a**6*b*c**4*d**
5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 8*a**5*b**2*c**5*d
**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + a**5*d**5*sqrt(-d/c*
*3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/
2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3
*b*c)/(2*(a*d - b*c)**2) + a**3*b**4*c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*
c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*
c)/(2*(a*d - b*c)**2) + 5*a**2*b**5*c**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**
3/(2*(a*d - b*c)**6) - 27*a**2*b**3*c**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c)/(
2*(a*d - b*c)**2) - a*b**6*c**9*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*
c)**6 - 4*b**5*c**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*
d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(4*(a*d - b*c)**2)
```

Giac [A] time = 1.16176, size = 165, normalized size = 1.51

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{\left(3\,bcd-ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{cd}}-\frac{dx}{2\left(bc^2-acd\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.191967, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}-\frac{\left(d\left(15b^2c^2-10abcd+\right.\right.}{8c^2(bc-}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{\sqrt{d}\left(15b^2c^2-10abc\right.}{8c^{5/2}(}$$

Mathematica [A] time = 0.233332, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}+\frac{dx(3ad-7bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{2dx}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

Maple [B] time = 0.01, size = 310, normalized size = 1.9

$$\frac{3\,d^4x^3a^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2c^2}-\frac{5\,d^3x^3ab}{4\,(ad-bc)^3\left(dx^2+c\right)^2c}+\frac{7\,d^2x^3b^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{5\,d^3xa^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2c}-\frac{7\,a}{4\,(ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(d*x^2+c)^2
/c*x^3*a*b+7/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+5/8*d^3/(a*d-b*c)^3/(d*x
^2+c)^2/c*x*a^2-7/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x*a*b+9/8*d/(a*d-b*c)^3/(d*
x^2+c)^2*c*x*b^2+3/8*d^3/(a*d-b*c)^3/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)
)*a^2-5/4*d^2/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+15/8*d/
(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-b^3/(a*d-b*c)^3/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.80695, size = 3217, normalized size = 20.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21122, size = 293, normalized size = 1.83

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3\right)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2dx \cdots}{8\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right) \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.27 $\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=192

$$\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^4}{}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Rubi [A] time = 0.162539, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^2}{b^4}+\frac{d^4(5bc-2ad)}{b^3}\right.$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4}+\frac{d^4(5bc-2ad)x^5}{5b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4}+\frac{d^4(5bc-2ad)x^5}{5b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{3b^4}+\frac{d^4(5bc-2ad)x^5}{5b^3}$$

Mathematica [A] time = 0.0979393, size = 192, normalized size = 1.

$$\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
```

(11/2))

Maple [B] time = 0.009, size = 402, normalized size = 2.1

$$\frac{d^5x^7}{7\,b^2}-\frac{2\,d^5x^5a}{5\,b^3}+\frac{d^4x^5c}{b^2}+\frac{d^5x^3a^2}{b^4}-\frac{10\,d^4x^3ac}{3\,b^3}+\frac{10\,d^3x^3c^2}{3\,b^2}-4\,\frac{a^3d^5x}{b^5}+15\,\frac{a^2cd^4x}{b^4}-20\,\frac{ac^2d^3x}{b^3}+10\,\frac{c^3d^2x}{b^2}-\frac{a^4xd^5}{2\,b^5\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^5*x^7/b^2-2/5*d^5/b^3*x^5*a+d^4/b^2*x^5*c+d^5/b^4*x^3*a^2-10/3*d^4/b^
3*x^3*a*c+10/3*d^3/b^2*x^3*c^2-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-20*d^3/b^
3*a*c^2*x+10*d^2/b^2*c^3*x-1/2/b^5*a^4*x/(b*x^2+a)*d^5+5/2/b^4*a^3*x/(b*x^2
+a)*c*d^4-5/b^3*a^2*x/(b*x^2+a)*c^2*d^3+5/b^2*a*x/(b*x^2+a)*c^3*d^2-5/2/b*x
/(b*x^2+a)*c^4*d+1/2/a*x/(b*x^2+a)*c^5+9/2/b^5*a^4/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))*d^5-35/2/b^4*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^4+25/b
^3*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d^3-15/b^2*a/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2))*c^3*d^2+5/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^4*
d+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81095, size = 1675, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*a^2*b^5*d^5*x^9 + 12*(35*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 28
*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*a^4*b^3*d^5)*x^5 + 140*(30*a^2*
b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3 -
105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3
- 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c^3*d^
2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(a*b^6*c^5 - 5*a^2*b^5*c^
4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a^6*b*
d^5)*x)/(a^2*b^7*x^2 + a^3*b^6), 1/210*(30*a^2*b^5*d^5*x^9 + 6*(35*a^2*b^5*
c*d^4 - 9*a^3*b^4*d^5)*x^7 + 14*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*
a^4*b^3*d^5)*x^5 + 70*(30*a^2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3
*c*d^4 - 9*a^5*b^2*d^5)*x^3 + 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3
*c^3*d^2 + 50*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a
*b^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9
*a^5*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(a*b^6*c^5 - 5*a^2*b
^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a
^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [B] time = 2.2884, size = 498, normalized size = 2.59

$$-\frac{x\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{2a^2b^5+2ab^6x^2}-\frac{\sqrt{-\frac{1}{a^3b^{11}}}\,(ad-bc)^4\,(9ad+bc)\log\left(-\frac{a^2b^5\sqrt{\cdots}}{9a^5d^5-35a^4bcd^4+50\cdots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**
3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)/(2*a**2*b**5 + 2*a*b**6*x**2) - sqrt(
-1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(-a**2*b**5*sqrt(-1/(a**3*
b**11))*(a*d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a
**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5)
+ x)/4 + sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*s
qrt(-1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*
c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d
+ b**5*c**5) + x)/4 + d**5*x**7/(7*b**2) - x**5*(2*a*d**5 - 5*b*c*d**4)/(5*
b**3) + x**3*(3*a**2*d**5 - 10*a*b*c*d**4 + 10*b**2*c**2*d**3)/(3*b**4) - x
*(4*a**3*d**5 - 15*a**2*b*c*d**4 + 20*a*b**2*c**2*d**3 - 10*b**3*c**3*d**2)
```

/b**5

Giac [A] time = 1.169, size = 413, normalized size = 2.15

$$\frac{\left(b^5c^5 + 5\,ab^4c^4d - 30\,a^2b^3c^3d^2 + 50\,a^3b^2c^2d^3 - 35\,a^4bcd^4 + 9\,a^5d^5\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^5} + \frac{b^5c^5x - 5\,ab^4c^4dx + 10\,a^2b^3c^3d^2x - 1}{2\left(bx^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 - 35
*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^5) + 1/2*(b^
5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5
*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^7 +
105*b^12*c*d^4*x^5 - 42*a*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*b^11*
c*d^4*x^3 + 105*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*c^2*d^
3*x + 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=142

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{d^4x^5}{5b^2}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.120468, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4}+\frac{2d^3(2bc-ad)x^2}{b^3}+\frac{d^4x^4}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{b^4(a+bx^2)^2}\right.$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{\int\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{(a+bx^2)^2}dx}{b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4(a+bx^2)}+\frac{((bc-ad)^3(bc+7ad))\int}{2ab^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4(a+bx^2)}+\frac{(bc-ad)^3(bc+7ad)\tan}{2a^{3/2}b^{9/2}}$$

Mathematica [A] time = 0.0865084, size = 142, normalized size = 1.

$$\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{x(bc-ad)^4}{2ab^4(a+bx^2)}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Maple [B] time = 0.009, size = 296, normalized size = 2.1

$$\frac{d^4x^5}{5\,b^2}-\frac{2\,d^4x^3a}{3\,b^3}+\frac{4\,d^3x^3c}{3\,b^2}+3\,\frac{a^2d^4x}{b^4}-8\,\frac{acd^3x}{b^3}+6\,\frac{c^2d^2x}{b^2}+\frac{a^3xd^4}{2\,b^4(bx^2+a)}-2\,\frac{a^2cxd^3}{b^3(bx^2+a)}+3\,\frac{ac^2xd^2}{b^2(bx^2+a)}-2\,\frac{xc^3d}{b(bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^4*x^5/b^2-2/3*d^4/b^3*x^3*a+4/3*d^3/b^2*x^3*c+3*d^4/b^4*a^2*x-8*d^3/b
^3*a*c*x+6*d^2/b^2*c^2*x+1/2/b^4*a^3*x/(b*x^2+a)*d^4-2/b^3*a^2*x/(b*x^2+a)*
c*d^3+3/b^2*a*x/(b*x^2+a)*c^2*d^2-2/b*x/(b*x^2+a)*c^3*d+1/2/a*x/(b*x^2+a)*c
^4-7/2/b^4*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^4+10/b^3*a^2/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))*c*d^3-9/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)*c^2*d^2+2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3*d+1/2/a/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78316, size = 1261, normalized size = 8.88

$$\left[\frac{12\,a^2b^4d^4x^7 + 4\left(20\,a^2b^4cd^3 - 7\,a^3b^3d^4\right)x^5 + 20\left(18\,a^2b^4c^2d^2 - 20\,a^3b^3cd^3 + 7\,a^4b^2d^4\right)x^3 + 15\left(ab^4c^4 + 4\,a^2b^3c^3d - 18\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a^2*b^4*d^4*x^7 + 4*(20*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 20*(
18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^4
+ 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*
c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*
x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*b^5*c
^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^4)
*x)/(a^2*b^6*x^2 + a^3*b^5), 1/30*(6*a^2*b^4*d^4*x^7 + 2*(20*a^2*b^4*c*d^3
```

```
- 7*a^3*b^3*d^4)*x^5 + 10*(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^
2*d^4)*x^3 + 15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*
b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^
3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(a*b^5
*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^
4)*x)/(a^2*b^6*x^2 + a^3*b^5)]
```

Sympy [B] time = 1.70845, size = 398, normalized size = 2.8

$$\frac{x\left(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4\right)}{2a^2b^4 + 2ab^5x^2} + \frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\,(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d +
b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2) + sqrt(-1/(a**3*b**9))*(a*d - b*c)
**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d
 + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 - 4*a*b**3
*c**3*d - b**4*c**4) + x)/4 - sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d +
b*c)*log(a**2*b**4*sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**
4*d**4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4
*c**4) + x)/4 + d**4*x**5/(5*b**2) - x**3*(2*a*d**4 - 4*b*c*d**3)/(3*b**3)
+ x*(3*a**2*d**4 - 8*a*b*c*d**3 + 6*b**2*c**2*d**2)/b**4
```

Giac [A] time = 1.12159, size = 297, normalized size = 2.09

$$\frac{\left(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^4} + \frac{b^4c^4x - 4\,ab^3c^3dx + 6\,a^2b^2c^2d^2x - 4\,a^3bcd^3x + a^4d^4}{2\left(bx^2+a\right)ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*
d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4*c^4*x - 4*a*b^3*c^3
```

```
*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^2 + a)*a*b^
4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8*c^2
*d^2*x - 120*a*b^7*c*d^3*x + 45*a^2*b^6*d^4*x)/b^10
```

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.0932229, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3(a+bx^2)^2}\right)dx\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{(a+bx^2)^2}\,dx}{b^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3(a+bx^2)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0604665, size = 106, normalized size = 1.

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Maple [B] time = 0.008, size = 205, normalized size = 1.9

$$\frac{d^3x^3}{3\,b^2}-2\,\frac{ad^3x}{b^3}+3\,\frac{d^2xc}{b^2}-\frac{a^2xd^3}{2\,b^3(bx^2+a)}+\frac{3\,acxd^2}{2\,b^2(bx^2+a)}-\frac{3\,xc^2d}{2\,b(bx^2+a)}+\frac{xc^3}{2\,a(bx^2+a)}+\frac{5\,a^2d^3}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^3*x^3/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/2/b^3*a^2*x/(b*x^2+a)*d^3+3/2
/b^2*a*x/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2/a*x/(b*x^2+a)*c^3+5/2/
b^3*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3-9/2/b^2*a/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*c*d^2+3/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d+1/
2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81831, size = 896, normalized size = 8.45

$$\left[\frac{4\,a^2b^3d^3x^5 + 4\left(9\,a^2b^3cd^2 - 5\,a^3b^2d^3\right)x^3 - 3\left(ab^3c^3 + 3\,a^2b^2c^2d - 9\,a^3bcd^2 + 5\,a^4d^3 + \left(b^4c^3 + 3\,ab^3c^2d - 9\,a^2b^2cd^2 + 5\,a^3\right.\right.}{12\left(a^2b^5x^2 + a^3b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
```

```
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] time = 1.23759, size = 313, normalized size = 2.95

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\,(ad-bc)^2\,(5ad+bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\,(ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*a**2*b**3
 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(-
a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 -
9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7)
)*(a*d - b*c)**2*(5*a*d + b*c)*log(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*
c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3
*c**3) + x)/4 + d**3*x**3/(3*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

Giac [A] time = 1.15905, size = 205, normalized size = 1.93

$$\frac{\left(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^3}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\left(bx^2+a\right)ab^3}+\frac{b^4d^3x^3+9\,b^4cd^2x-6\,a}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

3.30 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.103651, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2\left(a+bx^2\right)^2}\right)dx\\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{b^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0630599, size = 88, normalized size = 1.07

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Maple [A] time = 0.008, size = 129, normalized size = 1.6

$$\frac{d^2x}{b^2}+\frac{axd^2}{2\,b^2\left(bx^2+a\right)}-\frac{cxd}{b\left(bx^2+a\right)}+\frac{xc^2}{2\,a\left(bx^2+a\right)}-\frac{3\,ad^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{cd}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^2}{2\,a}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] d^2*x/b^2+1/2/b^2*a*x/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2/a*x/(b*x^2+a)*c
^2-3/2/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+1/b/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))*c*d+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79625, size = 612, normalized size = 7.46

$$\left[\frac{4\,a^2b^2d^2x^3 + \left(ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + \left(b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right) + 2\left(ab^3c^2 - 2\,a^2b^2cd + \right.}{4\left(a^2b^4x^2 + a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] time = 0.861887, size = 236, normalized size = 2.88

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{2a^2b^2+2ab^3x^2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)\,(3ad+bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

Giac [A] time = 1.15406, size = 127, normalized size = 1.55

$$\frac{d^2x}{b^2}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(bx^2+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

3.31 $\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0214587, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)x}{2ab\left(a+bx^2\right)} + \frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.0440173, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} - \frac{x(ad-bc)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(b*c) + a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.005, size = 68, normalized size = 1.1

$$-\frac{(ad-bc)\,x}{2\,ab\left(bx^2+a\right)} + \frac{d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+
1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84744, size = 381, normalized size = 6.05

$$\left[-\frac{\left(abc + a^2d + \left(b^2c + abd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) - 2\left(ab^2c - a^2bd\right)x}{4\left(a^2b^3x^2 + a^3b^2\right)}, \frac{\left(abc + a^2d + \left(b^2c + abd\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a}\right.}{2\left(a^2b^3x^2 + a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] time = 0.532893, size = 112, normalized size = 1.78

$$-\frac{x\left(ad - bc\right)}{2a^2b + 2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad + bc\right)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad + bc\right)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A] time = 1.17154, size = 77, normalized size = 1.22

$$\frac{\left(bc + ad\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} + \frac{bcx - adx}{2\left(bx^2 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

3.32 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi [A] time = 0.081232, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}-\frac{\int \frac{-bc+2ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}\\ &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}+\frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2}\\ &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.144614, size = 109, normalized size = 1.01

$$-\frac{\sqrt{b}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}-\frac{bx}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(b*x)/(2*a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A] time = 0.008, size = 144, normalized size = 1.3

$$\frac{d^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{bdx}{2\,(ad-bc)^2\left(bx^2+a\right)}+\frac{b^2xc}{2\,(ad-bc)^2\,a\left(bx^2+a\right)}-\frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*b/(a*d-b*c)^2*x/(b*
x^2+a)*d+1/2*b^2/(a*d-b*c)^2/a*x/(b*x^2+a)*c-3/2*b/(a*d-b*c)^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))*d+1/2*b^2/(a*d-b*c)^2/a/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.42114, size = 1451, normalized size = 13.44

$$\left[-\frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(b^2c-abd\right)x\ \ 4}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
```

```
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

Sympy [B] time = 15.3868, size = 2033, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -b*x/(2*a**3*d - 2*a**2*b*c + x**2*(2*a**2*b*d - 2*a*b**2*c)) + sqrt(-b/a**
3)*(3*a*d - b*c)*log(x + (-a**9*c*d**6*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a
*d - b*c)**6 + 5*a**8*b*c**2*d**5*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d
 - b*c)**6) + a**7*b**2*c**3*d**4*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d
 - b*c)**6) - 7*a**6*b**3*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d
- b*c)**6 + 8*a**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d -
b*c)**6 - 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d - b*c)/(a*d - b*c)**2 - 7*a**4*b
**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + a**3*b**6
*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 27*a**3*b**2*c
**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 27*a**2*b**3*c**3
*d**2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 9*a*b**4*c**4*d*sqrt
(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + b**5*c**5*sqrt(-b/a**3)*(3*a*d
 - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(4*(a*d - b*c)**2) - sqrt(-b/a**3)*(3*a*d - b*c)*log(x + (a**9*c*d**6
*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 5*a**8*b*c**2*d**5*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - a**7*b**2*c**3*d**4*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 7*a**6*b**3*c**4*d**3*(-
b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 8*a**5*b**4*c**5*d**2*(-b/
a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 4*a**5*d**5*sqrt(-b/a**3)*(3
*a*d - b*c)/(a*d - b*c)**2 + 7*a**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b
*c)**3/(2*(a*d - b*c)**6) - a**3*b**6*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**
3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(
2*(a*d - b*c)**2) - 27*a**2*b**3*c**3*d**2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(
a*d - b*c)**2) + 9*a*b**4*c**4*d*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)
**2) - b**5*c**5*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b
*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d**2))/(4*(a*d - b*c)**2) + sqrt(-d**3/
```

```
c)*log(x + (-8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 20*a**8*b*c**2
*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**7*b**2*c**3*d**4*(-d**3/c)**(3
/2)/(a*d - b*c)**6 - 56*a**6*b**3*c**4*d**3*(-d**3/c)**(3/2)/(a*d - b*c)**6
 + 64*a**5*b**4*c**5*d**2*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 8*a**5*d**5*sqr
t(-d**3/c)/(a*d - b*c)**2 - 28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c
)**6 + 4*a**3*b**6*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 27*a**3*b**2*c**2
*d**3*sqrt(-d**3/c)/(a*d - b*c)**2 + 27*a**2*b**3*c**3*d**2*sqrt(-d**3/c)/(
a*d - b*c)**2 - 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2 + b**5*c**5*sq
rt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(2*(a*d - b*c)**2) - sqrt(-d**3/c)*log(x + (8*a**9*c*d**6*(-d**3/c)**
(3/2)/(a*d - b*c)**6 - 20*a**8*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6
- 4*a**7*b**2*c**3*d**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 56*a**6*b**3*c**4
*d**3*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**4*c**5*d**2*(-d**3/c)**(
3/2)/(a*d - b*c)**6 + 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2 + 28*a**4*b*
*5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**3*b**6*c**7*(-d**3/c)**(3/
2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sqrt(-d**3/c)/(a*d - b*c)**2 - 2
7*a**2*b**3*c**3*d**2*sqrt(-d**3/c)/(a*d - b*c)**2 + 9*a*b**4*c**4*d*sqrt(-
d**3/c)/(a*d - b*c)**2 - b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b
*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2)
```

Giac [A] time = 1.11065, size = 163, normalized size = 1.51

$$\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{\left(b^2c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{ab}}+\frac{bx}{2\left(abc-a^2d\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

3.33 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rubi [A] time = 0.201135, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-bc+2ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2\left(b^2c^2-4abcd+a^2d^2\right)-2bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}}{4ac(bc-ad)^2}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(bc-5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^3} + \frac{(}{}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} +$$

Mathematica [A] time = 0.314488, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}+\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A] time = 0.013, size = 238, normalized size = 1.4

$$\frac{d^3xa}{2\,(ad-bc)^3\,c\left(dx^2+c\right)}-\frac{bd^2x}{2\,(ad-bc)^3\left(dx^2+c\right)}+\frac{ad^3}{2\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{5\,d^2b}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*b+1/2*d
^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-5/2*d^2/(a*d-b*c)^3/
(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2
*b^3/(a*d-b*c)^3/a*x/(b*x^2+a)*c+5/2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*d-1/2*b^3/(a*d-b*c)^3/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.50868, size = 3294, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
```

```
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [B] time = 141.098, size = 3662, normalized size = 21.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-b**3/a**3)*(5*a*d - b*c)*log(x + (-a**12*c**3*d**9*(-b**3/a**3)**(3/
2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 11*a**11*b*c**4*d**8*(-b**3/a**3)**(3/
2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 40*a**10*b**2*c**5*d**7*(-b**3/a**3)**
(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 64*a**9*b**3*c**6*d**6*(-b**3/a**3)
**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 34*a**8*b**4*c**7*d**5*(-b**3/a**
3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - a**8*d**8*sqrt(-b**3/a**3)*(5*a
*d - b*c)/(a*d - b*c)**3 - 34*a**7*b**5*c**8*d**4*(-b**3/a**3)**(3/2)*(5*a*
d - b*c)**3/(a*d - b*c)**9 + 15*a**7*b*c*d**7*sqrt(-b**3/a**3)*(5*a*d - b*c
)/(a*d - b*c)**3 + 64*a**6*b**6*c**9*d**3*(-b**3/a**3)**(3/2)*(5*a*d - b*c)
**3/(a*d - b*c)**9 - 75*a**6*b**2*c**2*d**6*sqrt(-b**3/a**3)*(5*a*d - b*c)/
(a*d - b*c)**3 - 40*a**5*b**7*c**10*d**2*(-b**3/a**3)**(3/2)*(5*a*d - b*c)*
*3/(a*d - b*c)**9 + 125*a**5*b**3*c**3*d**5*sqrt(-b**3/a**3)*(5*a*d - b*c)/
(a*d - b*c)**3 + 11*a**4*b**8*c**11*d*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/
(a*d - b*c)**9 - a**3*b**9*c**12*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d
- b*c)**9 + 125*a**3*b**5*c**5*d**3*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b
*c)**3 - 75*a**2*b**6*c**6*d**2*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)*
*3 + 15*a*b**7*c**7*d*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - b**8*
c**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3)/(5*a**4*b**2*d**6 - 61*
a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3*d**3 + 5*b**6*c
**4*d**2))/(4*(a*d - b*c)**3) + sqrt(-b**3/a**3)*(5*a*d - b*c)*log(x + (a**
12*c**3*d**9*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 11*a**11
*b*c**4*d**8*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 40*a**10
*b**2*c**5*d**7*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 64*a*
*9*b**3*c**6*d**6*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 34*
a**8*b**4*c**7*d**5*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + a
**8*d**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 34*a**7*b**5*c**8*
d**4*(-b**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 15*a**7*b*c*d**7
*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - 64*a**6*b**6*c**9*d**3*(-b
**3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + 75*a**6*b**2*c**2*d**6*s
qrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 40*a**5*b**7*c**10*d**2*(-b*
*3/a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 125*a**5*b**3*c**3*d**5*s
qrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 - 11*a**4*b**8*c**11*d*(-b**3/
```

```
a**3)**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 + a**3*b**9*c**12*(-b**3/a**3)
**(3/2)*(5*a*d - b*c)**3/(a*d - b*c)**9 - 125*a**3*b**5*c**5*d**3*sqrt(-b**
3/a**3)*(5*a*d - b*c)/(a*d - b*c)**3 + 75*a**2*b**6*c**6*d**2*sqrt(-b**3/a*
*3)*(5*a*d - b*c)/(a*d - b*c)**3 - 15*a*b**7*c**7*d*sqrt(-b**3/a**3)*(5*a*d
 - b*c)/(a*d - b*c)**3 + b**8*c**8*sqrt(-b**3/a**3)*(5*a*d - b*c)/(a*d - b*
c)**3)/(5*a**4*b**2*d**6 - 61*a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 -
61*a*b**5*c**3*d**3 + 5*b**6*c**4*d**2))/(4*(a*d - b*c)**3) - sqrt(-d**3/c*
*3)*(a*d - 5*b*c)*log(x + (-a**12*c**3*d**9*(-d**3/c**3)**(3/2)*(a*d - 5*b*
c)**3/(a*d - b*c)**9 + 11*a**11*b*c**4*d**8*(-d**3/c**3)**(3/2)*(a*d - 5*b*
c)**3/(a*d - b*c)**9 - 40*a**10*b**2*c**5*d**7*(-d**3/c**3)**(3/2)*(a*d - 5
*b*c)**3/(a*d - b*c)**9 + 64*a**9*b**3*c**6*d**6*(-d**3/c**3)**(3/2)*(a*d -
 5*b*c)**3/(a*d - b*c)**9 - 34*a**8*b**4*c**7*d**5*(-d**3/c**3)**(3/2)*(a*d
 - 5*b*c)**3/(a*d - b*c)**9 - a**8*d**8*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d
 - b*c)**3 - 34*a**7*b**5*c**8*d**4*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a
*d - b*c)**9 + 15*a**7*b*c*d**7*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)*
*3 + 64*a**6*b**6*c**9*d**3*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c
)**9 - 75*a**6*b**2*c**2*d**6*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3
 - 40*a**5*b**7*c**10*d**2*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)
**9 + 125*a**5*b**3*c**3*d**5*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3
 + 11*a**4*b**8*c**11*d*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9
 - a**3*b**9*c**12*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 12
5*a**3*b**5*c**5*d**3*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 - 75*a*
*2*b**6*c**6*d**2*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 + 15*a*b**7
*c**7*d*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 - b**8*c**8*sqrt(-d**
3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3)/(5*a**4*b**2*d**6 - 61*a**3*b**3*c*d*
*5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3*d**3 + 5*b**6*c**4*d**2))/(4*
(a*d - b*c)**3) + sqrt(-d**3/c**3)*(a*d - 5*b*c)*log(x + (a**12*c**3*d**9*(
-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 11*a**11*b*c**4*d**8*(
-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 40*a**10*b**2*c**5*d**
7*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 64*a**9*b**3*c**6*d
**6*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 34*a**8*b**4*c**7
*d**5*(-d**3/c**3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + a**8*d**8*sqrt(
-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3 + 34*a**7*b**5*c**8*d**4*(-d**3/c*
*3)**(3/2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 15*a**7*b*c*d**7*sqrt(-d**3/c*
*3)*(a*d - 5*b*c)/(a*d - b*c)**3 - 64*a**6*b**6*c**9*d**3*(-d**3/c**3)**(3/
2)*(a*d - 5*b*c)**3/(a*d - b*c)**9 + 75*a**6*b**2*c**2*d**6*sqrt(-d**3/c**3
)*(a*d - 5*b*c)/(a*d - b*c)**3 + 40*a**5*b**7*c**10*d**2*(-d**3/c**3)**(3/2
)*(a*d - 5*b*c)**3/(a*d - b*c)**9 - 125*a**5*b**3*c**3*d**5*sqrt(-d**3/c**3
)*(a*d - 5*b*c)/(a*d - b*c)**3 - 11*a**4*b**8*c**11*d*(-d**3/c**3)**(3/2)*(
a*d - 5*b*c)**3/(a*d - b*c)**9 + a**3*b**9*c**12*(-d**3/c**3)**(3/2)*(a*d -
 5*b*c)**3/(a*d - b*c)**9 - 125*a**3*b**5*c**5*d**3*sqrt(-d**3/c**3)*(a*d -
 5*b*c)/(a*d - b*c)**3 + 75*a**2*b**6*c**6*d**2*sqrt(-d**3/c**3)*(a*d - 5*b
*c)/(a*d - b*c)**3 - 15*a*b**7*c**7*d*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d -
 b*c)**3 + b**8*c**8*sqrt(-d**3/c**3)*(a*d - 5*b*c)/(a*d - b*c)**3)/(5*a**4
*b**2*d**6 - 61*a**3*b**3*c*d**5 + 192*a**2*b**4*c**2*d**4 - 61*a*b**5*c**3
```

```
*d**3 + 5*b**6*c**4*d**2))/(4*(a*d - b*c)**3) + (x**3*(a*b*d**2 + b**2*c*d)
 + x*(a**2*d**2 + b**2*c**2))/(2*a**4*c**2*d**2 - 4*a**3*b*c**3*d + 2*a**2*
b**2*c**4 + x**4*(2*a**3*b*c*d**3 - 4*a**2*b**2*c**2*d**2 + 2*a*b**3*c**3*d
) + x**2*(2*a**4*c*d**3 - 2*a**3*b*c**2*d**2 - 2*a**2*b**2*c**3*d + 2*a*b**
3*c**4))
```

Giac [B] time = 1.50058, size = 1928, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(c*d)*a*b^5*c^5*abs(d) - 12*sqrt(c*d)*a^2*b^4*c^4*d*abs(d) + 22*s
qrt(c*d)*a^3*b^3*c^3*d^2*abs(d) - 12*sqrt(c*d)*a^4*b^2*c^2*d^3*abs(d) + sqr
t(c*d)*a^5*b*c*d^4*abs(d) - sqrt(c*d)*b^2*c*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d
 + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*abs(d) - sqrt(c*d)*a*b*d*abs(a*b^3*c^4 - 3*
a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*abs(d))*arctan(2*sqrt(1/2)*x/s
qrt((a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3 + sqrt((a*b^3*c^
4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)^2 - 4*(a^2*b^2*c^4 - 2*a^3*b
*c^3*d + a^4*c^2*d^2)*(a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*
b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*b^3*c^4*d*abs(a*b^3*c^4 -
 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - a^2*b^2*c^3*d^2*abs(a*b^3
*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - a^3*b*c^2*d^3*abs(a
*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + a^4*c*d^4*abs(a
*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + (a*b^3*c^4 - 3*
a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)^2*d) + 1/2*(sqrt(a*b)*a*b^4*c^
5*d*abs(b) - 12*sqrt(a*b)*a^2*b^3*c^4*d^2*abs(b) + 22*sqrt(a*b)*a^3*b^2*c^3
*d^3*abs(b) - 12*sqrt(a*b)*a^4*b*c^2*d^4*abs(b) + sqrt(a*b)*a^5*c*d^5*abs(b
) + sqrt(a*b)*b*c*d*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4
*c*d^3)*abs(b) + sqrt(a*b)*a*d^2*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*
c^2*d^2 - a^4*c*d^3)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c^4 - a^2*b^2
*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3 - sqrt((a*b^3*c^4 - a^2*b^2*c^3*d - a^3*
b*c^2*d^2 + a^4*c*d^3)^2 - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(a
*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)))/(a*b^3*c^3*d - 2*a^2*b^2*c^
2*d^2 + a^3*b*c*d^3)))/(a*b^4*c^4*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b
*c^2*d^2 - a^4*c*d^3) - a^2*b^3*c^3*d*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a
^3*b*c^2*d^2 - a^4*c*d^3) - a^3*b^2*c^2*d^2*abs(a*b^3*c^4 - 3*a^2*b^2*c^3*d
 + 3*a^3*b*c^2*d^2 - a^4*c*d^3) + a^4*b*c*d^3*abs(a*b^3*c^4 - 3*a^2*b^2*c^3
*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3) - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*
c^2*d^2 - a^4*c*d^3)^2*b) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^
```

```
2*d^2*x)/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*
x^2 + a*c))
```

3.34 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{}{2a\left(a+bx^2\right)(c}$$

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.30876, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{}{2a\left(a+bx^2\right)(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{(a+bx^2)(c+dx^2)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{-2(2b^2c^2-8abcd+3a^2d^2)-6bd(2}{(a+bx^2)(c+dx^2)^2}}{8ac(bc-ad)^2}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3(c+dx^2)} -$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3(c+dx^2)} +$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3(c+dx^2)} +$$

Mathematica [A] time = 0.414394, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{d^{3/2}(3a^2d^2-14abcd+35b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4} + \frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} - \frac{4b^3x}{a(a+bx^2)(ad-bc)^3} + \frac{d^2x(11bc-}{c^2(c+dx^2)(b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

Maple [A] time = 0.014, size = 403, normalized size = 1.8

$$\frac{3d^5x^3a^2}{8(ad-bc)^4(dx^2+c)^2c^2} - \frac{7d^4x^3ab}{4(ad-bc)^4(dx^2+c)^2c} + \frac{11d^3x^3b^2}{8(ad-bc)^4(dx^2+c)^2} + \frac{5d^4xa^2}{8(ad-bc)^4(dx^2+c)^2c} - \frac{9d}{4(ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(d*x^2+c)^2
/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+5/8*d^4/(a*d-b*c)^4/(d*
x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x*a*b+13/8*d^2/(a*d-b*c)^4
/(d*x^2+c)^2*c*x*b^2+3/8*d^4/(a*d-b*c)^4/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(
1/2))*a^2-7/4*d^3/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+35/
8*d^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/2*b^3/(a*d-b*c)
^4*x/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x/(b*x^2+a)*c-7/2*b^3/(a*d-b*c)^4/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d+1/2*b^4/(a*d-b*c)^4/a/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 22.8343, size = 6472, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
```

(70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
(a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
+ a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
(a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
+ a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^

```
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12254, size = 448, normalized size = 1.95

$$\frac{b^3x}{2\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)\left(bx^2+a\right)}+\frac{\left(b^4c-7ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^4c^4-4a^2b^3c^3d+6a^3b^2c^2d^2-4a^4bcd^3+a^5d^4\right)\sqrt{ab}}+\frac{(35}{8\left(b^4c^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.35 $\int \frac{(c+dx^2)^5}{(a+bx^2)^3} dx$

Optimal. Leaf size=196

$$\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^4x^3(}{}$$

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Rubi [A] time = 0.226919, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {390, 1157, 385, 205}

$$\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^4x^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3}\,dx = \int \left(\frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)}{b^5}+\frac{d^4(5bc-3ad)x^2}{b^4}+\frac{d^5x^4}{b^3}+\frac{(bc-ad)^3\left(b^2c^2+3abcd+6a^2d^2\right)+5\ldots}{b^5\ldots}\right.$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}+\frac{d^5x^5}{5b^3}+\frac{\int \frac{(bc-ad)^3\left(b^2c^2+3abcd+6a^2d^2\right)+5bd(bc-ad)^3(bc+3a\ldots}{\left(a+bx^2\right)^3}}{b^5}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}+\frac{d^5x^5}{5b^3}+\frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2}-\frac{\int \frac{-(bc-ad)^3\left(3b^2c^2+14abc\ldots\right.}{(a\ldots}}{4\ldots}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}+\frac{d^5x^5}{5b^3}+\frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2}+\frac{(bc-ad)^4(3bc+17a\ldots}{8a^2b^5\left(a+bx^2\right)}$$

$$= \frac{d^3\left(10b^2c^2-15abcd+6a^2d^2\right)x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}+\frac{d^5x^5}{5b^3}+\frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2}+\frac{(bc-ad)^4(3bc+17a\ldots}{8a^2b^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.122091, size = 196, normalized size = 1.

$$\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^4x^3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Maple [B] time = 0.013, size = 484, normalized size = 2.5

$$\frac{d^5x^5}{5\,b^3}-\frac{d^5x^3a}{b^4}+\frac{5\,d^4x^3c}{3\,b^3}+6\,\frac{a^2d^5x}{b^5}-15\,\frac{acd^4x}{b^4}+10\,\frac{c^2d^3x}{b^3}+\frac{17\,a^3x^3d^5}{8\,b^4\left(bx^2+a\right)^2}-\frac{65\,a^2x^3cd^4}{8\,b^3\left(bx^2+a\right)^2}+\frac{45\,ax^3c^2d^3}{4\,b^2\left(bx^2+a\right)^2}-\frac{25}{4\,b\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^5/(b*x^2+a)^3,x)
```

```
[Out] 1/5*d^5*x^5/b^3-d^5/b^4*x^3*a+5/3*d^4/b^3*x^3*c+6*d^5/b^5*a^2*x-15*d^4/b^4*
a*c*x+10*d^3/b^3*c^2*x+17/8/b^4/(b*x^2+a)^2*a^3*x^3*d^5-65/8/b^3/(b*x^2+a)^
2*a^2*x^3*c*d^4+45/4/b^2/(b*x^2+a)^2*a*x^3*c^2*d^3-25/4/b/(b*x^2+a)^2*x^3*c
^3*d^2+5/8/(b*x^2+a)^2/a*x^3*c^4*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^5+15/8/b^5/(
b*x^2+a)^2*x*a^4*d^5-55/8/b^4/(b*x^2+a)^2*x*a^3*c*d^4+35/4/b^3/(b*x^2+a)^2*
x*a^2*c^2*d^3-15/4/b^2/(b*x^2+a)^2*x*a*c^3*d^2-5/8/b/(b*x^2+a)^2*x*c^4*d+5/
8/(b*x^2+a)^2*x/a*c^5-63/8/b^5*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^5+
175/8/b^4*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^4-75/4/b^3*a/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*c^2*d^3+15/4/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))*c^3*d^2+5/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^4*d+3/8/a^2/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60851, size = 2195, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a^3*b^5*d^5*x^9 + 16*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 16
*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 10*(9*a*b
^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875
*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d
 + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5
+ (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 1
75*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d +
 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^
5)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(5*a^
2*b^6*c^5 - 5*a^3*b^5*c^4*d - 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 17
5*a^6*b^2*c*d^4 + 63*a^7*b*d^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6),
 1/120*(24*a^3*b^5*d^5*x^9 + 8*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 8*(
150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 5*(9*a*b^7*
c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875*a^
5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d +
30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (
3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 175*
a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d + 30
*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^5)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(5*a^2*b^6*c^5 - 5*a^3*b^5*c^4*d
- 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 175*a^6*b^2*c*d^4 + 63*a^7*b*d
^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [B] time = 6.5243, size = 614, normalized size = 3.13

$$\frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16} - \frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c*
*2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*
a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**4*b*c*d**4 + 150*a**3*b**2*c*
*2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d - 3*b**5*c**5) + x)/16 -
 sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c
**2)*log(a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*
a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**4*b*c*d**4 + 150*a**3*b**2*c*
*2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d - 3*b**5*c**5) + x)/16 +
 (x**3*(17*a**5*b*d**5 - 65*a**4*b**2*c*d**4 + 90*a**3*b**3*c**2*d**3 - 50*
a**2*b**4*c**3*d**2 + 5*a*b**5*c**4*d + 3*b**6*c**5) + x*(15*a**6*d**5 - 55
*a**5*b*c*d**4 + 70*a**4*b**2*c**2*d**3 - 30*a**3*b**3*c**3*d**2 - 5*a**2*b
**4*c**4*d + 5*a*b**5*c**5))/(8*a**4*b**5 + 16*a**3*b**6*x**2 + 8*a**2*b**7
*x**4) + d**5*x**5/(5*b**3) - x**3*(3*a*d**5 - 5*b*c*d**4)/(3*b**4) + x*(6*
a**2*d**5 - 15*a*b*c*d**4 + 10*b**2*c**2*d**3)/b**5
```

Giac [A] time = 1.13319, size = 459, normalized size = 2.34

$$\frac{\left(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^5}+\frac{3\,b^6c^5x^3+5\,ab^5c^4dx^3-50\,a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 +
 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5) +
1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a^2*b^4*c^3*d^2*x^3 + 90*a^3*b^
3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x - 5
*a^2*b^4*c^4*d*x - 30*a^3*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^5*b*c
```

```
*d^4*x + 15*a^6*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5 + 25*
b^12*c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11*c*d^4*
x + 90*a^2*b^10*d^5*x)/b^15
```

3.36 $\int \frac{(c+dx^2)^4}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=160

$$\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4}{3}$$

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rubi [A] time = 0.197019, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {390, 1157, 385, 205}

$$\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx = \int \left(\frac{d^3(4bc-3ad)}{b^4}+\frac{d^4x^2}{b^3}+\frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{b^4\left(a+bx^2\right)^3}\right.$$

$$=\frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{\int \frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{\left(a+bx^2\right)^3}\,dx}{b^4}$$

$$=\frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2}-\frac{\int \frac{-(bc-ad)^2\left(3b^2c^2+10abcd+11a^2d^2\right)-24abd^2(bc-ad)^2x^2}{\left(a+bx^2\right)^2}\,dx}{4ab^4}$$

$$=\frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2}+\frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)}+\frac{\left((bc-ad)^2\left(3b^2c^2+10abcd+\right.\right.}{8a^2b^4}$$

$$=\frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2}+\frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)}+\frac{(bc-ad)^2\left(3b^2c^2+10abcd+\right.}{8a^{5/2}b^{9/2}}$$

Mathematica [A] time = 0.0948017, size = 160, normalized size = 1.

$$\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Maple [B] time = 0.01, size = 367, normalized size = 2.3

$$\frac{d^4x^3}{3\,b^3}-3\,\frac{ad^4x}{b^4}+4\,\frac{d^3xc}{b^3}-\frac{13\,a^2x^3d^4}{8\,b^3\left(bx^2+a\right)^2}+\frac{9\,ax^3cd^3}{2\,b^2\left(bx^2+a\right)^2}-\frac{15\,x^3c^2d^2}{4\,b\left(bx^2+a\right)^2}+\frac{x^3c^3d}{2\,\left(bx^2+a\right)^2a}+\frac{3\,bx^3c^4}{8\,\left(bx^2+a\right)^2a^2}-\frac{1}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^3,x)
```

```
[Out] 1/3*d^4*x^3/b^3-3*d^4/b^4*a*x+4*d^3/b^3*x*c-13/8/b^3/(b*x^2+a)^2*a^2*x^3*d^
4+9/2/b^2/(b*x^2+a)^2*a*x^3*c*d^3-15/4/b/(b*x^2+a)^2*x^3*c^2*d^2+1/2/(b*x^2
+a)^2/a*x^3*c^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^4-11/8/b^4/(b*x^2+a)^2*x*a^3*
d^4+7/2/b^3/(b*x^2+a)^2*x*a^2*c*d^3-9/4/b^2/(b*x^2+a)^2*x*a*c^2*d^2-1/2/b/(
b*x^2+a)^2*x*c^3*d+5/8/(b*x^2+a)^2*x/a*c^4+35/8/b^4*a^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*d^4-15/2/b^3*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^3+9
/4/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2*d^2+1/2/b/a/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*c^3*d+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7646, size = 1689, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*b^4*d^4*x^7 + 16*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + 2*(
9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 1
75*a^5*b^2*d^4)*x^3 - 3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d
^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*
c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b
^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*a^2*b^5*c^4 - 4*
a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d^4)*x)/(a
^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5), 1/24*(8*a^3*b^4*d^4*x^7 + 8*(12*a^3*
b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + (9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b
^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 175*a^5*b^2*d^4)*x^3 + 3*(3*a^2*b^4*c^4 +
4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6
*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d
^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^
2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*a^2*b^5
*c^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d
^4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5)]
```

Sympy [B] time = 3.71684, size = 513, normalized size = 3.21

$$-\frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\left(35a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c*
*2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a
*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3*b*c*d**3 + 18*a**2*b**2*c**2*
d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) + x)/16 + sqrt(-1/(a**5*b**9))*(a*d -
 b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a
**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**
4*d**4 - 60*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b*
*4*c**4) + x)/16 - (x**3*(13*a**4*b*d**4 - 36*a**3*b**2*c*d**3 + 30*a**2*b*
*3*c**2*d**2 - 4*a*b**4*c**3*d - 3*b**5*c**4) + x*(11*a**5*d**4 - 28*a**4*b
*c*d**3 + 18*a**3*b**2*c**2*d**2 + 4*a**2*b**3*c**3*d - 5*a*b**4*c**4))/(8*
a**4*b**4 + 16*a**3*b**5*x**2 + 8*a**2*b**6*x**4) + d**4*x**3/(3*b**3) - x*
(3*a*d**4 - 4*b*c*d**3)/b**4
```

Giac [A] time = 1.10662, size = 343, normalized size = 2.14

$$\frac{\left(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4}+\frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a
^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8*(3*b^5*c^4*x^3 + 4*
a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36*a^3*b^2*c*d^3*x^3 - 13*a^4*b*
d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28*a^4
*b*c*d^3*x - 11*a^5*d^4*x)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3 + 12*
b^6*c*d^3*x - 9*a*b^5*d^4*x)/b^9
```

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=130

$$\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rubi [A] time = 0.165763, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {390, 1157, 385, 205}

$$\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx = \int \left(\frac{d^3}{b^3}+\frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{b^3\left(a+bx^2\right)^3}\right)dx$$

$$= \frac{d^3x}{b^3}+\frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{\left(a+bx^2\right)^3}\,dx}{b^3}$$

$$= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}-\frac{\int \frac{-3(bc-ad)(bc+ad)^2-12abd^2(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{4ab^3}$$

$$= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\left(3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^3}$$

$$= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)}+\frac{3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

Mathematica [A] time = 0.080454, size = 139, normalized size = 1.07

$$\frac{3\left(3a^2bcd^2-5a^3d^3+ab^2c^2d+b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Maple [B] time = 0.009, size = 266, normalized size = 2.1

$$\frac{d^3x}{b^3}+\frac{9\,ax^3d^3}{8\,b^2\left(bx^2+a\right)^2}-\frac{15\,x^3cd^2}{8\,b\left(bx^2+a\right)^2}+\frac{3\,x^3c^2d}{8\left(bx^2+a\right)^2a}+\frac{3\,bx^3c^3}{8\left(bx^2+a\right)^2a^2}+\frac{7\,xa^2d^3}{8\,b^3\left(bx^2+a\right)^2}-\frac{9\,acxd^2}{8\,b^2\left(bx^2+a\right)^2}-\frac{3\,xc}{8\,b\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^3,x)
```

```
[Out] d^3*x/b^3+9/8/b^2/(b*x^2+a)^2*a*x^3*d^3-15/8/b/(b*x^2+a)^2*x^3*c*d^2+3/8/(b
*x^2+a)^2/a*x^3*c^2*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^3+7/8/b^3/(b*x^2+a)^2*x*a
^2*d^3-9/8/b^2/(b*x^2+a)^2*x*a*c*d^2-3/8/b/(b*x^2+a)^2*x*c^2*d+5/8/(b*x^2+a
)^2*x/a*c^3-15/8/b^3*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^3+9/8/b^2/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d^2+3/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))*c^2*d+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86477, size = 1224, normalized size = 9.42

$$\left[\frac{16\,a^3b^3d^3x^5 + 2\left(3\,ab^5c^3 + 3\,a^2b^4c^2d - 15\,a^3b^3cd^2 + 25\,a^4b^2d^3\right)x^3 + 3\left(a^2b^3c^3 + a^3b^2c^2d + 3\,a^4bcd^2 - 5\,a^5d^3 + \left(b^5c^3 + \right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*d^3*x^5 + 2*(3*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 15*a^3*b^3*c
*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2
 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^4*c^3 - 3
*a^3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^
5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*d^3*x^5 + (3*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
 15*a^3*b^3*c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d +
3*a^4*b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^
3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d
^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^
4)]
```

Sympy [B] time = 2.18924, size = 422, normalized size = 3.25

$$\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*lo
g(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d +
```

```
b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*c**3)
 + x)/16 - 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b*
*2*c**2)*log(3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*
a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*
b**3*c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*a*b**3*
c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3*a**2*b**2*c**2
*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**5*x**4) +
 d**3*x/b**3
```

Giac [A] time = 1.10709, size = 240, normalized size = 1.85

$$\frac{d^3x}{b^3} + \frac{3\left(b^3c^3 + ab^2c^2d + 3\,a^2bcd^2 - 5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4c^3x^3 + 3\,ab^3c^2dx^3 - 15\,a^2b^2cd^2x^3 + 9\,a^3bd^3x^3 + 5\,ab^3c^3x}{8\left(bx^2 + a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3 + 3*a*b^3*c^2*d*x^3
 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5*a*b^3*c^3*x - 3*a^2*b^2*c^2*d
*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)/((b*x^2 + a)^2*a^2*b^3)
```

3.38 $\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d*x^2))/(4*
a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi [A] time = 0.0769482, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d*x^2))/(4*
a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}\,dx &= \frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2}+\frac{\int \frac{c(3bc+ad)+d(bc+3ad)x^2}{\left(a+bx^2\right)^2}\,dx}{4ab}\\ &= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\left(a+bx^2\right)}+\frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^2}\\ &= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\left(a+bx^2\right)}+\frac{(bc-ad)x\left(c+dx^2\right)}{4ab\left(a+bx^2\right)^2}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0945662, size = 124, normalized size = 1.07

$$\frac{x\left(-a^2bd\left(2c+5dx^2\right)-3a^3d^2+ab^2c\left(5c+2dx^2\right)+3b^3c^2x^2\right)}{8a^2b^2\left(a+bx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (x*(-3*a^3*d^2 + 3*b^3*c^2*x^2 + a*b^2*c*(5*c + 2*d*x^2) - a^2*b*d*(2*c + 5
*d*x^2)))/(8*a^2*b^2*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Maple [A] time = 0.008, size = 147, normalized size = 1.3

$$\frac{1}{\left(bx^2+a\right)^2}\left(-\frac{\left(5\,a^2d^2-2\,abcd-3\,b^2c^2\right)x^3}{8\,a^2b}-\frac{\left(3\,a^2d^2+2\,abcd-5\,b^2c^2\right)x}{8\,ab^2}\right)+\frac{3\,d^2}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{cd}{4\,ab}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^3,x)
```

```
[Out] (-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*a*b*c*d-5*
b^2*c^2)/a/b^2*x)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d
^2+1/4/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d+3/8/a^2/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87662, size = 918, normalized size = 7.91

$$\left[\frac{2\left(3\,ab^4c^2+2\,a^2b^3cd-5\,a^3b^2d^2\right)x^3-\left(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+\left(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2\right)x^4+2\left(3\,ab^3c^2+2\,a^2\right.\right.}{16\left(a^3b^5x^4+2\,a^4b^4x^2+a^5b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 - (3*a^2*b^2*c^2
 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4
+ 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a
```

```
^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*((3*a*b^4*c^2 + 2
*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^
2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*
b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^3*c^
2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3)
]
```

Sympy [B] time = 1.19055, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}-\frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**
2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 - (x**3*(5*a*
*2*b*d**2 - 2*a*b**2*c*d - 3*b**3*c**2) + x*(3*a**3*d**2 + 2*a**2*b*c*d - 5
*a*b**2*c**2))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

Giac [A] time = 1.18004, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}+\frac{3\,b^3c^2x^3+2\,ab^2cdx^3-5\,a^2bd^2x^3+5\,ab^2c^2x-2\,a^2bcdx-3\,a^3d^2x}{8\left(bx^2+a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b*d^2*x^3 + 5*a*b^2*c
^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=92

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\left(a+bx^2\right)}+\frac{x(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0328971, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\left(a+bx^2\right)}+\frac{x(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^3}\,dx = \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)\int \frac{1}{\left(a+bx^2\right)^2}\,dx}{4ab}$$
$$= \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bc+ad)\int \frac{1}{a+bx^2}\,dx}{8a^2b}$$
$$= \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.0633886, size = 84, normalized size = 0.91

$$\frac{x\left(a^2(-d)+ab\left(5c+dx^2\right)+3b^2cx^2\right)}{8a^2b\left(a+bx^2\right)^2}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Maple [A] time = 0.007, size = 89, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(ad+3\,bc)\,x^3}{8\,a^2}-\frac{(ad-5\,bc)\,x}{8\,ab}\right)+\frac{d}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8/a/b/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))*d+3/8/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60605, size = 621, normalized size = 6.75

$$\left[\frac{2\left(3\,ab^3c + a^2b^2d\right)x^3 - \left(\left(3\,b^3c + ab^2d\right)x^4 + 3\,a^2bc + a^3d + 2\left(3\,ab^2c + a^2bd\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) + 2\left(5\,a^2b^2c - \right.}{16\left(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*c + a^2*b^2*d)*x^3 - ((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*c
 + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*c + a^2*b^2*d)*x^3 + ((3*b^3*c + a*b^2*d)*
x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [A] time = 0.713771, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad + 3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad + 3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}} + x\right)}{16} + \frac{x^3\left(abd + 3b^2c\right) + x\left(-a^2d + 5a\right.}{8a^4b + 16a^3b^2x^2 + 8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d + 5*a*b*c))/(8*a**4*b + 16*a**3
*b**2*x**2 + 8*a**2*b**3*x**4)
```

Giac [A] time = 1.19545, size = 105, normalized size = 1.14

$$\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}+\frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(3*b^2*c*x^
3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2*b)
```

3.40 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)} \, dx$

Optimal. Leaf size=161

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}+\frac{bx(3bc-7ad)}{8a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a\left(a+bx^2\right)^2(bc-ad)}$$

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*a^2*(b*c -
 a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3)
```

Rubi [A] time = 0.197347, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}+\frac{bx(3bc-7ad)}{8a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*a^2*(b*c -
 a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} - \frac{\int \frac{-3bc+4ad-3bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} + \frac{\int \frac{3b^2c^2-7abcd+8a^2d^2+bd(3bc-7ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8a^2(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{\left(b\left(3b^2c^2-10abcd+15\right.\right.}{8a^2(bc-\ldots}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} + \frac{\sqrt{b}\left(3b^2c^2-10abcd+15a^2d^2\right)\tan^{-1}\left(\ldots\right.}{8a^{5/2}(bc-ad)^3}$$

Mathematica [A] time = 0.264827, size = 158, normalized size = 0.98

$$\frac{1}{8}\left(-\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3} + \frac{bx(3bc-7ad)}{a^2\left(a+bx^2\right)(bc-ad)^2} - \frac{8d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3} - \frac{2bx}{a\left(a+bx^2\right)^2(ad-bc)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] ((-2*b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(a^2*(b*
c - a*d)^2*(a + b*x^2)) - (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (8*d^(5/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3))/8
```

Maple [B] time = 0.01, size = 309, normalized size = 1.9

$$\frac{d^3}{(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,b^2x^3d^2}{8\,(ad-bc)^3\left(bx^2+a\right)^2}+\frac{5\,b^3x^3cd}{4\,(ad-bc)^3\left(bx^2+a\right)^2a}-\frac{3\,b^4x^3c^2}{8\,(ad-bc)^3\left(bx^2+a\right)^2a^2}-\frac{}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] d^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-7/8*b^2/(a*d-b*c)^3/(b*
x^2+a)^2*x^3*d^2+5/4*b^3/(a*d-b*c)^3/(b*x^2+a)^2/a*x^3*c*d-3/8*b^4/(a*d-b*c
)^3/(b*x^2+a)^2/a^2*x^3*c^2-9/8*b/(a*d-b*c)^3/(b*x^2+a)^2*x*a*d^2+7/4*b^2/(
a*d-b*c)^3/(b*x^2+a)^2*x*c*d-5/8*b^3/(a*d-b*c)^3/(b*x^2+a)^2*x/a*c^2-15/8*b
/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+5/4*b^2/(a*d-b*c)^3/a/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d-3/8*b^3/(a*d-b*c)^3/a^2/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.13419, size = 3212, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 -
10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4
 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*
x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) +
 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2
*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b
^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*
c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)
*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)*arctan(x*
sqrt(d/c)) - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a
*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b
*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(
5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2
*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c
*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^
2 - a^6*b*d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 +
(3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15
*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - 4*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^
2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^3*c^
2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6
*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5
*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b^2
*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d
^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(b/a)*ar
ctan(x*sqrt(b/a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/
c)*arctan(x*sqrt(d/c)) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a
^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a
^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*
b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14794, size = 294, normalized size = 1.83

$$-\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{cd}} + \frac{\left(3\,b^3c^2 - 10\,ab^2cd + 15\,a^2bd^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3\right)\sqrt{ab}} + \frac{3\,b^3cx^3 - 7\,ab^2dx^3 + 5\,ab^2c}{8\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/((a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)^2)
```

3.41 $\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=236

$$\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}+\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\left(c+dx^2\right)(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}-\frac{d^{5/2}(7bc-}{2c^{3/}}$$

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^2)) + (b*
x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(b*c - 3*a*d)*x)/(8*a
^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*
d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d
^(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.310513, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}+\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\left(c+dx^2\right)(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}-\frac{d^{5/2}(7bc-}{2c^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```


```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^2)) + (b*
x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(b*c - 3*a*d)*x)/(8*a
^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*
d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d
^(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} - \frac{\int \frac{-3bc+4ad-5bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\int \frac{3b^2c^2-5abcd+8a^2d^2 \ldots}{\left(a+bx^2\right)(c \ldots}}{8a^2(bc \ldots}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)(c \ldots}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)(c \ldots}$$

$$= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)(c \ldots}$$

Mathematica [A] time = 0.41711, size = 197, normalized size = 0.83

$$\frac{1}{8}\left(\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4}+\frac{b^2x(11ad-3bc)}{a^2\left(a+bx^2\right)(ad-bc)^3}+\frac{2b^2x}{a\left(a+bx^2\right)^2(bc-ad)^2}+\frac{4d^{5/2}(ad-7bc)\tan^{-1}}{c^{3/2}(bc-ad)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] ((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*d)*x)/(a^2
*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*d)^3*(c + d*x^2)) +
(b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(3/2)*(b*c - a*d)^4))/8
```

Maple [A] time = 0.014, size = 403, normalized size = 1.7

$$\frac{d^4xa}{2\,(ad-bc)^4\,c\left(dx^2+c\right)}-\frac{d^3xb}{2\,(ad-bc)^4\left(dx^2+c\right)}+\frac{ad^4}{2\,(ad-bc)^4\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,d^3b}{2\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/(a*d-b*c)^4/c*x/(d*x^2+c)*a-1/2*d^3/(a*d-b*c)^4*x/(d*x^2+c)*b+1/2*d
^4/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-7/2*d^3/(a*d-b*c)^4/
(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+11/8*b^3/(a*d-b*c)^4/(b*x^2+a)^2*x^3*
d^2-7/4*b^4/(a*d-b*c)^4/(b*x^2+a)^2/a*x^3*c*d+3/8*b^5/(a*d-b*c)^4/(b*x^2+a)
^2/a^2*x^3*c^2+13/8*b^2/(a*d-b*c)^4/(b*x^2+a)^2*x*a*d^2-9/4*b^3/(a*d-b*c)^4
/(b*x^2+a)^2*x*c*d+5/8*b^4/(a*d-b*c)^4/(b*x^2+a)^2*x/a*c^2+35/8*b^2/(a*d-b*
c)^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2-7/4*b^3/(a*d-b*c)^4/a/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))*c*d+3/8*b^4/(a*d-b*c)^4/a^2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 23.1117, size = 6472, normalized size = 27.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)
*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 +
 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 +
(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*
b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a
^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 +
 (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a
^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^
3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^4*b*c^2*d^2 - a^5*c
*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^
2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 3
5*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
 (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (
6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
```

```
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*
b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2
*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3
*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)
*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x
^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)
*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 2*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a
^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2
*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(
-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^4*c^4 - 18*a
^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*
c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 +
 (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4
 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2
+ 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c
^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*
d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d
^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 +
5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35
*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
(3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6
*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*
c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2*a^4*b*
d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 -
4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^
4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2
+ 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6
 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*
d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4
*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13542, size = 450, normalized size = 1.91

$$-\frac{d^{3}x}{2\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+3\,a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)\left(dx^{2}+c\right)}+\frac{\left(3\,b^{4}c^{2}-14\,ab^{3}cd+35\,a^{2}b^{2}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4}\right)\sqrt{ab}}-\frac{}{2\left(b^{4}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2
+ c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b
))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^
6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*
d)) + 1/8*(3*b^4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*x)/((a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^2)
```

3.42 $\int \frac{1}{(a+bx^2)^3 (c+dx^2)^3} \, dx$

Optimal. Leaf size=315

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}+\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5}-\frac{3d^{5/}}{}$$

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^
2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + (b*(3*b*c - 1
1*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)^2) + (3*d*(b*c + a*d
)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2))
+ (3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(5/2)*(b*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rubi [A] time = 0.450865, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 205}

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}+\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5}-\frac{3d^{5/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^
2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + (b*(3*b*c - 1
1*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)^2) + (3*d*(b*c + a*d
)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2))
+ (3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(5/2)*(b*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+4ad-7bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\int \frac{3b^2c^2-3abcd+8a^2d}{\left(a+bx^2\right)(}}{8a^2(bc}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad}{8a^2(bc-ad)^2\left(a+bx^2\right.}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad}{8a^2(bc-ad)^2\left(a+bx^2\right.}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad}{8a^2(bc-ad)^2\left(a+bx^2\right.}$$

$$= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad}{8a^2(bc-ad)^2\left(a+bx^2\right.}$$

<u>Mathematica</u> [A] time = 0.914943, size = 233, normalized size = 0.74

$$\frac{1}{8}\left(\frac{x(bc-ad)\left(\frac{3b^4c}{a^2\left(a+bx^2\right)} + \frac{b^3\left(-17ad+2bc-15bdx^2\right)}{a\left(a+bx^2\right)^2} - \frac{d^3\left(-2ad+17bc+15bdx^2\right)}{c\left(c+dx^2\right)^2} + \frac{3ad^4}{c^2\left(c+dx^2\right)}\right) - \frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}}}{(bc-ad)^5} - \frac{3b^{5/2}\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] ((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c)/(a^2*(a + b*x^2))
+ (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c - 17*a*d - 15*b*d*x^2))/(a*(a +
 b*x^2)^2) - (d^3*(17*b*c - 2*a*d + 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (3*d^
(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(5/
2))/(b*c - a*d)^5)/8
```

Maple [A] time = 0.015, size = 568, normalized size = 1.8

$$\frac{3\,d^{6}x^{3}a^{2}}{8\,(ad-bc)^{5}\left(dx^{2}+c\right)^{2}c^{2}}-\frac{9\,d^{5}x^{3}ab}{4\,(ad-bc)^{5}\left(dx^{2}+c\right)^{2}c}+\frac{15\,d^{4}x^{3}b^{2}}{8\,(ad-bc)^{5}\left(dx^{2}+c\right)^{2}}+\frac{5\,d^{5}xa^{2}}{8\,(ad-bc)^{5}\left(dx^{2}+c\right)^{2}c}-\frac{11\,d}{4\,(ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^6/(a*d-b*c)^5/(d*x^2+c)^2/c^2*x^3*a^2-9/4*d^5/(a*d-b*c)^5/(d*x^2+c)^2
/c*x^3*a*b+15/8*d^4/(a*d-b*c)^5/(d*x^2+c)^2*x^3*b^2+5/8*d^5/(a*d-b*c)^5/(d*
x^2+c)^2/c*x*a^2-11/4*d^4/(a*d-b*c)^5/(d*x^2+c)^2*x*a*b+17/8*d^3/(a*d-b*c)^
5/(d*x^2+c)^2*c*x*b^2+3/8*d^5/(a*d-b*c)^5/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^
(1/2))*a^2-9/4*d^4/(a*d-b*c)^5/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+63
/8*d^3/(a*d-b*c)^5/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-15/8*b^4/(a*d-b*
c)^5/(b*x^2+a)^2*x^3*d^2+9/4*b^5/(a*d-b*c)^5/(b*x^2+a)^2/a*x^3*c*d-3/8*b^6/
(a*d-b*c)^5/(b*x^2+a)^2/a^2*x^3*c^2-17/8*b^3/(a*d-b*c)^5/(b*x^2+a)^2*x*a*d^
2+11/4*b^4/(a*d-b*c)^5/(b*x^2+a)^2*x*c*d-5/8*b^5/(a*d-b*c)^5/(b*x^2+a)^2*x/
a*c^2-63/8*b^3/(a*d-b*c)^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*d^2+9/4*b^4/
(a*d-b*c)^5/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c*d-3/8*b^5/(a*d-b*c)^5/a
^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 72.7818, size = 9997, normalized size = 31.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^
7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d
^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 2
9*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 - 3
*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b
^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*
a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^
2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6
 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(21*a^4*b^2*c^4*d^2
- 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a
^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*
d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2
*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^
2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*
b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*
b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8
*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^
5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 +
 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d
^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d
- 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c
^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*
d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*
x^2), 1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^
6)*x^7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*
c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*
d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^
3 - 6*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2
*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*
b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a
^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a
^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) - 3*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*
c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6
*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (
b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b
^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*
a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2
*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*
```

d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5
 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c
^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^
3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 -
 a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25
*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 -
 a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 -
5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/16*(6*(b^6*c^4*d
^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + 2*(6*b^6*c^5*d
- 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d
^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 +
 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 + 6*(a^2*b^4*c^6 - 6*a
^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2
*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 2
1*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d
+ 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a
)) - 3*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^
2*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3
*b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*
a^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*
a^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^6 -
 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*
d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 -
 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*
a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d
^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^
5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a
^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^
6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a
^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^
8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/8*(3*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*
a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + (6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*
b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + (3*
b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b
*c*d^5 - 3*a^6*d^6)*x^3 + 3*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4
*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^
5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6
*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*
c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4
*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(21*a^4*b^2*c^4*d^2 -
6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a^4
*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*d^
5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2*c

```
^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^2*
c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) +
 (5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3
+ 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^
6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b
^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 +
5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7
*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^
2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*
c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*
d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2
*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.88356, size = 4545, normalized size = 14.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/8*(sqrt(c*d)*a^2*b^9*c^10*abs(d) - 10*sqrt(c*d)*a^3*b^8*c^9*d*abs(d) + 7
2*sqrt(c*d)*a^4*b^7*c^8*d^2*abs(d) - 214*sqrt(c*d)*a^5*b^6*c^7*d^3*abs(d) +
 302*sqrt(c*d)*a^6*b^5*c^6*d^4*abs(d) - 214*sqrt(c*d)*a^7*b^4*c^5*d^5*abs(d
) + 72*sqrt(c*d)*a^8*b^3*c^4*d^6*abs(d) - 10*sqrt(c*d)*a^9*b^2*c^3*d^7*abs(
d) + sqrt(c*d)*a^10*b*c^2*d^8*abs(d) - sqrt(c*d)*b^4*c^3*abs(a^2*b^5*c^7 -
5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4
 - a^7*c^2*d^5)*abs(d) + 5*sqrt(c*d)*a*b^3*c^2*d*abs(a^2*b^5*c^7 - 5*a^3*b^
4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c
```

^2*d^5)*abs(d) + 5*sqrt(c*d)*a^2*b^2*c*d^2*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*
d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5
)*abs(d) - sqrt(c*d)*a^3*b*d^3*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b
^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*abs(d))*ar
ctan(2*sqrt(1/2)*x/sqrt((a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2
+ 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5 + sqrt((a^2*b^5*c^7 - 3
*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 +
a^7*c^2*d^5)^2 - 4*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a
^6*b*c^4*d^3 + a^7*c^3*d^4)*(a^2*b^5*c^6*d - 4*a^3*b^4*c^5*d^2 + 6*a^4*b^3*
c^4*d^3 - 4*a^5*b^2*c^3*d^4 + a^6*b*c^2*d^5)))/(a^2*b^5*c^6*d - 4*a^3*b^4*c
^5*d^2 + 6*a^4*b^3*c^4*d^3 - 4*a^5*b^2*c^3*d^4 + a^6*b*c^2*d^5)))/(a^2*b^5*
c^7*d*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c
^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) - 3*a^3*b^4*c^6*d^2*abs(a^2*b^5*c^7
 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*
d^4 - a^7*c^2*d^5) + 2*a^4*b^3*c^5*d^3*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d +
10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) +
2*a^5*b^2*c^4*d^4*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 -
10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) - 3*a^6*b*c^3*d^5*abs(a
^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*
a^6*b*c^3*d^4 - a^7*c^2*d^5) + a^7*c^2*d^6*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*
d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5
) + (a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^
3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)^2*d) + 3/8*(sqrt(a*b)*a^2*b^8*c^10*d*abs
(b) - 10*sqrt(a*b)*a^3*b^7*c^9*d^2*abs(b) + 72*sqrt(a*b)*a^4*b^6*c^8*d^3*ab
s(b) - 214*sqrt(a*b)*a^5*b^5*c^7*d^4*abs(b) + 302*sqrt(a*b)*a^6*b^4*c^6*d^5
*abs(b) - 214*sqrt(a*b)*a^7*b^3*c^5*d^6*abs(b) + 72*sqrt(a*b)*a^8*b^2*c^4*d
^7*abs(b) - 10*sqrt(a*b)*a^9*b*c^3*d^8*abs(b) + sqrt(a*b)*a^10*c^2*d^9*abs(
b) + sqrt(a*b)*b^3*c^3*d*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5
*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*abs(b) - 5*sqrt(
a*b)*a*b^2*c^2*d^2*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 -
 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*abs(b) - 5*sqrt(a*b)*a
^2*b*c*d^3*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*
b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*abs(b) + sqrt(a*b)*a^3*d^4*abs
(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 +
5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*abs(b))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5*c
^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*
d^4 + a^7*c^2*d^5 - sqrt((a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2
 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)^2 - 4*(a^3*b^4*c^7 -
4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*(a^2*b
^5*c^6*d - 4*a^3*b^4*c^5*d^2 + 6*a^4*b^3*c^4*d^3 - 4*a^5*b^2*c^3*d^4 + a^6*
b*c^2*d^5)))/(a^2*b^5*c^6*d - 4*a^3*b^4*c^5*d^2 + 6*a^4*b^3*c^4*d^3 - 4*a^5
*b^2*c^3*d^4 + a^6*b*c^2*d^5)))/(a^2*b^6*c^7*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^
6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d
^5) - 3*a^3*b^5*c^6*d*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^
2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) + 2*a^4*b^4*c^5*d^2

```
*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^
3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) + 2*a^5*b^3*c^4*d^3*abs(a^2*b^5*c^7 - 5*
a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 -
 a^7*c^2*d^5) - 3*a^6*b^2*c^3*d^4*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^
4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) + a^7*b
*c^2*d^5*abs(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^
2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5) - (a^2*b^5*c^7 - 5*a^3*b^4*c^6*d
 + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)
^2*b) + 1/8*(3*b^5*c^3*d^2*x^7 - 15*a*b^4*c^2*d^3*x^7 - 15*a^2*b^3*c*d^4*x^
7 + 3*a^3*b^2*d^5*x^7 + 6*b^5*c^4*d*x^5 - 25*a*b^4*c^3*d^2*x^5 - 34*a^2*b^3
*c^2*d^3*x^5 - 25*a^3*b^2*c*d^4*x^5 + 6*a^4*b*d^5*x^5 + 3*b^5*c^5*x^3 - 5*a
*b^4*c^4*d*x^3 - 34*a^2*b^3*c^3*d^2*x^3 - 34*a^3*b^2*c^2*d^3*x^3 - 5*a^4*b*
c*d^4*x^3 + 3*a^5*d^5*x^3 + 5*a*b^4*c^5*x - 17*a^2*b^3*c^4*d*x - 17*a^4*b*c
^2*d^3*x + 5*a^5*c*d^4*x)/((a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d
^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)^2)
```

3.43 $\int \frac{(-1+x^2)^3}{(1+x^2)^4} \, dx$

Optimal. Leaf size=34

$$-\frac{x(1-x^2)^2}{3(x^2+1)^3} - \frac{2x}{3(x^2+1)}$$

```
[Out] -(x*(1 - x^2)^2)/(3*(1 + x^2)^3) - (2*x)/(3*(1 + x^2))
```

Rubi [A] time = 0.0095149, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {413, 21, 383}

$$-\frac{x(1-x^2)^2}{3(x^2+1)^3} - \frac{2x}{3(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^3/(1 + x^2)^4,x]
```

```
[Out] -(x*(1 - x^2)^2)/(3*(1 + x^2)^3) - (2*x)/(3*(1 + x^2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4}\,dx &= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}+\frac{1}{6}\int \frac{\left(-1+x^2\right)\left(4+4x^2\right)}{\left(1+x^2\right)^3}\,dx\\ &= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}+\frac{2}{3}\int \frac{-1+x^2}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3}-\frac{2x}{3\left(1+x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0071504, size = 24, normalized size = 0.71

$$-\frac{x\left(3x^4+2x^2+3\right)}{3\left(x^2+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]
```

```
[Out] -(x*(3 + 2*x^2 + 3*x^4))/(3*(1 + x^2)^3)
```

Maple [A] time = 0.005, size = 23, normalized size = 0.7

$$\frac{1}{\left(x^2+1\right)^3}\left(-x^5-\frac{2\,x^3}{3}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3/(x^2+1)^4,x)
```

```
[Out] (-x^5-2/3*x^3-x)/(x^2+1)^3
```

Maxima [A] time = 1.02401, size = 45, normalized size = 1.32

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,\left(x^6+3\,x^4+3\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Fricas [A] time = 1.48485, size = 73, normalized size = 2.15

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,\left(x^6+3\,x^4+3\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] time = 0.126587, size = 31, normalized size = 0.91

$$-\frac{3x^5+2x^3+3x}{3x^6+9x^4+9x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3/(x**2+1)**4,x)
```

```
[Out] -(3*x**5 + 2*x**3 + 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

<u>Giac</u> [A] time = 1.09326, size = 27, normalized size = 0.79

$$-\frac{3\left(x+\frac{1}{x}\right)^{2}-4}{3\left(x+\frac{1}{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

3.44 $\int \frac{(-1+x^2)^4}{(1+x^2)^5} dx$

Optimal. Leaf size=47

$$\frac{x(1-x^2)^3}{4(x^2+1)^4}+\frac{3x(1-x^2)}{8(x^2+1)^2}+\frac{3}{8}\tan^{-1}(x)$$

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2) + (3*ArcT
an[x])/8
```

Rubi [A] time = 0.0163951, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {413, 21, 203}

$$\frac{x(1-x^2)^3}{4(x^2+1)^4}+\frac{3x(1-x^2)}{8(x^2+1)^2}+\frac{3}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2) + (3*ArcT
an[x])/8
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^2\right)^4}{\left(1+x^2\right)^5}\,dx &= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{1}{8}\int \frac{\left(-1+x^2\right)^2\left(6+6x^2\right)}{\left(1+x^2\right)^4}\,dx\\
&= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3}{4}\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^3}\,dx\\
&= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{16}\int \frac{2+2x^2}{\left(1+x^2\right)^2}\,dx\\
&= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{x\left(1-x^2\right)^3}{4\left(1+x^2\right)^4}+\frac{3x\left(1-x^2\right)}{8\left(1+x^2\right)^2}+\frac{3}{8}\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0124781, size = 41, normalized size = 0.87

$$\frac{-5x^7+3x^5-3x^3+3\left(x^2+1\right)^4\tan^{-1}(x)+5x}{8\left(x^2+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x^2)^4)
```

Maple [A] time = 0.007, size = 33, normalized size = 0.7

$$\frac{1}{\left(x^{2}+1\right)^{4}}\left(-\frac{5 x^{7}}{8}+\frac{3 x^{5}}{8}-\frac{3 x^{3}}{8}+\frac{5 x}{8}\right)+\frac{3 \arctan \left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^4/(x^2+1)^5,x)
```

```
[Out] (-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

Maxima [A] time = 1.46019, size = 65, normalized size = 1.38

$$-\frac{5 x^{7}-3 x^{5}+3 x^{3}-5 x}{8 \left(x^{8}+4 x^{6}+6 x^{4}+4 x^{2}+1\right)}+\frac{3}{8} \arctan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1) + 3/8*
arctan(x)
```

Fricas [A] time = 1.53216, size = 159, normalized size = 3.38

$$-\frac{5 x^{7}-3 x^{5}+3 x^{3}-3 \left(x^{8}+4 x^{6}+6 x^{4}+4 x^{2}+1\right) \arctan \left(x\right)-5 x}{8 \left(x^{8}+4 x^{6}+6 x^{4}+4 x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="fricas")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)*arctan(x)
 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

Sympy [A] time = 0.156707, size = 46, normalized size = 0.98

$$-\frac{5x^7 - 3x^5 + 3x^3 - 5x}{8x^8 + 32x^6 + 48x^4 + 32x^2 + 8} + \frac{3\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**4/(x**2+1)**5,x)
```

```
[Out] -(5*x**7 - 3*x**5 + 3*x**3 - 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 32*x**2 + 8
) + 3*atan(x)/8
```

Giac [A] time = 1.44375, size = 73, normalized size = 1.55

$$\frac{3}{32}\,\pi\mathrm{sgn}\,(x) - \frac{5\left(x - \frac{1}{x}\right)^3 + 12\,x - \frac{12}{x}}{8\left(\left(x - \frac{1}{x}\right)^2 + 4\right)^2} + \frac{3}{16}\,\arctan\left(\frac{x^2 - 1}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="giac")
```

```
[Out] 3/32*pi*sgn(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2 + 4)^2 + 3/
16*arctan(1/2*(x^2 - 1)/x)
```

3.45 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=231

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(24a^2bcd^2-5a^3d^3-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(24a^2bcd^2-5a^3\right.}{}$$

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^
2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(3/
2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(48*b^
2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^
2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*b^(7/2))
```

Rubi [A] time = 0.178677, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(24a^2bcd^2-5a^3d^3-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(24a^2bcd^2-5a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^
2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(3/
2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(48*b^
2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^
2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*b^(7/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx = \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(c(8bc-ad)+d(12bc-5ad)x^2\right)dx}{8b}$$
$$= \frac{d(12bc-5ad)x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{48b^2} + \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b} + \frac{\int \sqrt{a+bx^2}\left(c\left(48b^2c\right.\right.}{}$$
$$= \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{48b^2} + \frac{dx}{}$$
$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x\left(a\right.}{192b^3}$$
$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x\left(a\right.}{192b^3}$$
$$= \frac{\left(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{128b^3} + \frac{d\left(72b^2c^2-52abcd+15a^2d^2\right)x\left(a\right.}{192b^3}$$

Mathematica [A] time = 5.1079, size = 181, normalized size = 0.78

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(-2a^2bd^2\left(36c+5dx^2\right)+15a^3d^3+8ab^2d\left(18c^2+6cdx^2+d^2x^4\right)+48b^3\left(6c^2dx^2+4c^3+4cd^2x^4+d^3x^6\right)\right)}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x^2) + 8*a
*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d
^2*x^4 + d^3*x^6)) - 3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5
*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(384*b^(7/2))
```

Maple [A] time = 0.013, size = 310, normalized size = 1.3

$$\frac{d^3x^5}{8\,b}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5\,ad^3x^3}{48\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{5\,a^2d^3x}{64\,b^3}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{5\,a^3d^3x}{128\,b^3}\sqrt{bx^2+a} - \frac{5\,d^3a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}} + \frac{ca}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/8*d^3*x^5*(b*x^2+a)^(3/2)/b-5/48*d^3/b^2*a*x^3*(b*x^2+a)^(3/2)+5/64*d^3/b
^3*a^2*x*(b*x^2+a)^(3/2)-5/128*d^3/b^3*a^3*x*(b*x^2+a)^(1/2)-5/128*d^3/b^(7
/2)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c*d^2*x^3*(b*x^2+a)^(3/2)/b-3/8*c
*d^2/b^2*a*x*(b*x^2+a)^(3/2)+3/16*c*d^2/b^2*a^2*x*(b*x^2+a)^(1/2)+3/16*c*d^
2/b^(5/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/4*c^2*d*x*(b*x^2+a)^(3/2)/b-3
/8*c^2*d/b*a*x*(b*x^2+a)^(1/2)-3/8*c^2*d/b^(3/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))+1/2*c^3*x*(b*x^2+a)^(1/2)+1/2*c^3*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33364, size = 886, normalized size = 3.84

$$\left[-\frac{3\left(64\,ab^3c^3 - 48\,a^2b^2c^2d + 24\,a^3bcd^2 - 5\,a^4d^3\right)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) - 2\left(48\,b^4d^3x^7 + 8\left(24\,b^4cd^2 + ab^3\right.\right.}{768\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*d^3*x^7
+ 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 - 5*
a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a^2*b^2*c*d^2 + 5*a^
3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*
d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (48*b^4*d^3*x^7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d +
24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a
^2*b^2*c*d^2 + 5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] time = 17.9835, size = 484, normalized size = 2.1

$$\frac{5a^{\frac{7}{2}}d^3x}{128b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{5}{2}}cd^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}d^3x^3}{384b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{3}{2}}c^2dx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}cd^2x^3}{16b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}d^3x^5}{192b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}c^3x\sqrt{1+\frac{bx^2}{a}}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(7/2)*d**3*x/(128*b**3*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x/(16*b
**2*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*d**3*x**3/(384*b**2*sqrt(1 + b*x**2/a)
) + 3*a**(3/2)*c**2*d*x/(8*b*sqrt(1 + b*x**2/a)) - a**(3/2)*c*d**2*x**3/(16
*b*sqrt(1 + b*x**2/a)) - a**(3/2)*d**3*x**5/(192*b*sqrt(1 + b*x**2/a)) + sq
rt(a)*c**3*x*sqrt(1 + b*x**2/a)/2 + 9*sqrt(a)*c**2*d*x**3/(8*sqrt(1 + b*x**
2/a)) + 5*sqrt(a)*c*d**2*x**5/(8*sqrt(1 + b*x**2/a)) + 7*sqrt(a)*d**3*x**7/
(48*sqrt(1 + b*x**2/a)) - 5*a**4*d**3*asinh(sqrt(b)*x/sqrt(a))/(128*b**(7/2
)) + 3*a**3*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) - 3*a**2*c**2*d*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + a*c**3*asinh(sqrt(b)*x/sqrt(a))/(2*s
qrt(b)) + 3*b*c**2*d*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b*c*d**2*x**7/(2
*sqrt(a)*sqrt(1 + b*x**2/a)) + b*d**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.80805, size = 271, normalized size = 1.17

$$\frac{1}{384}\left(2\left(4\left(6\,d^3x^2+\frac{24\,b^6cd^2+ab^5d^3}{b^6}\right)x^2+\frac{144\,b^6c^2d+24\,ab^5cd^2-5\,a^2b^4d^3}{b^6}\right)x^2+\frac{3\left(64\,b^6c^3+48\,ab^5c^2d-24\,a^2b^4cd^2\right.}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (144*b^6*c^2
*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b^6*c^3 + 48*a*b^5*c^
2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(64*
a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.46 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^2 dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+b\right.}{}$$

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (d*(8*b*c
- 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)
)/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.0879085, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (d*(8*b*c
- 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)
)/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(5/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}+\frac{\int \sqrt{a+bx^2}\left(c(6bc-ad)+d(8bc-3ad)x^2\right)\,dx}{6b}$$

$$= \frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}+\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\int \sqrt{a+b}}{8b^2}$$

$$= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right.}{6b}$$

$$= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right.}{6b}$$

$$= \frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right.}{6b}$$

Mathematica [C] time = 2.47574, size = 160, normalized size = 1.07

$$\frac{x\sqrt{a+bx^2}\left(2bx^2\left(c+dx^2\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)+4bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(\frac{1}{2},\frac{3}{2};\frac{9}{2};-\frac{b}{}\right.\right.}{105a\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2F1
[-1/2, 1/2, 7/2, -((b*x^2)/a)] + 4*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hype
rgeometric2F1[1/2, 3/2, 9/2, -((b*x^2)/a)] + 2*b*x^2*(c + d*x^2)^2*Hypergeo
metricPFQ[{1/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a*Sqrt[1 + (b*x^2)/
a])
```

Maple [A] time = 0.006, size = 190, normalized size = 1.3

$$\frac{d^2x^3}{6b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{ad^2x}{8b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{a^2d^2x}{16b^2}\sqrt{bx^2+a}+\frac{a^3d^2}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{2b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{acdx}{4b}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/6*d^2*x^3*(b*x^2+a)^(3/2)/b-1/8*d^2/b^2*a*x*(b*x^2+a)^(3/2)+1/16*d^2/b^2*
a^2*x*(b*x^2+a)^(1/2)+1/16*d^2/b^(5/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/
2*c*d*x*(b*x^2+a)^(3/2)/b-1/4*c*d/b*a*x*(b*x^2+a)^(1/2)-1/4*c*d/b^(3/2)*a^2
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c^2*x*(b*x^2+a)^(1/2)+1/2*c^2*a/b^(1/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81612, size = 585, normalized size = 3.93

$$\left[\frac{3\left(8\,ab^2c^2 - 4\,a^2bcd + a^3d^2\right)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) + 2\left(8\,b^3d^2x^5 + 2\left(12\,b^3cd + ab^2d^2\right)x^3 + 3\left(8\,b^3c^2 + \cdots\right.\right.}{96\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)
*x^3 + 3*(8*b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/
48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)*x^3 + 3*(8*b^3*c
^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] time = 10.1787, size = 291, normalized size = 1.95

$$-\frac{a^{\frac{5}{2}}d^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}cdx}{4b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{3}{2}}d^2x^3}{48b\sqrt{1+\frac{bx^2}{a}}} + \frac{\sqrt{a}c^2x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{3\sqrt{a}cdx^3}{4\sqrt{1+\frac{bx^2}{a}}} + \frac{5\sqrt{a}d^2x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{a^3d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}} - \frac{a^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

```
[Out] -a**(5/2)*d**2*x/(16*b**2*sqrt(1 + b*x**2/a)) + a**(3/2)*c*d*x/(4*b*sqrt(1
+ b*x**2/a)) - a**(3/2)*d**2*x**3/(48*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c**2*
x*sqrt(1 + b*x**2/a)/2 + 3*sqrt(a)*c*d*x**3/(4*sqrt(1 + b*x**2/a)) + 5*sqrt
(a)*d**2*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*d**2*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(5/2)) - a**2*c*d*asinh(sqrt(b)*x/sqrt(a))/(4*b**(3/2)) + a*c**2*asi
nh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*c*d*x**5/(2*sqrt(a)*sqrt(1 + b*x**2/a
)) + b*d**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.14444, size = 174, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,d^2x^2+\frac{12\,b^4cd+ab^3d^2}{b^4}\right)x^2+\frac{3\left(8\,b^4c^2+4\,ab^3cd-a^2b^2d^2\right)}{b^4}\right)\sqrt{bx^2+a}x-\frac{\left(8\,ab^2c^2-4\,a^2bcd+a^3d^2\right)\log\left(\left|-\sqrt{b}x\right.\right.}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4*c^2 + 4*a
*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(8*a*b^2*c^2 - 4*a^2*
b*c*d + a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.47 $\int \sqrt{a+bx^2}\left(c+dx^2\right) dx$

Optimal. Leaf size=87

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] time = 0.0276762, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\left(c+dx^2\right)dx &= \frac{dx\left(a+bx^2\right)^{3/2}}{4b}-\frac{(-4bc+ad)\int \sqrt{a+bx^2}\,dx}{4b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}+\frac{(a(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}+\frac{(a(4bc-ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\ &= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.154802, size = 85, normalized size = 0.98

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(ad+4bc+2bdx^2\right)-\frac{\sqrt{a}(ad-4bc)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(4*b*c + a*d + 2*b*d*x^2) - (Sqrt[a]*(-4*b*c +
a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(8*b^(3/2))
```

Maple [A] time = 0.005, size = 96, normalized size = 1.1

$$\frac{dx}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{adx}{8\,b}\sqrt{bx^2+a}-\frac{da^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{cx}{2}\sqrt{bx^2+a}+\frac{ac}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c),x)
```

```
[Out] 1/4*d*x*(b*x^2+a)^(3/2)/b-1/8*d/b*a*x*(b*x^2+a)^(1/2)-1/8*d/b^(3/2)*a^2*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c*x*(b*x^2+a)^(1/2)+1/2*c*a/b^(1/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6509, size = 371, normalized size = 4.26

$$\left[-\frac{\left(4\,abc-a^2d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(2\,b^2dx^3+\left(4\,b^2c+abd\right)x\right)\sqrt{bx^2+a}}{16\,b^2},-\frac{\left(4\,abc-a^2d\right)\sqrt{-b}\,\mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*a*b*c - a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*b^2*d*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/8*(
(4*a*b*c - a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d*x^
3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] time = 5.13809, size = 144, normalized size = 1.66

$$\frac{a^{\frac{3}{2}}dx}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}dx^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2d\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bdx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] a**(3/2)*d*x/(8*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c*x*sqrt(1 + b*x**2/a)/2 +
3*sqrt(a)*d*x**3/(8*sqrt(1 + b*x**2/a)) - a**2*d*asinh(sqrt(b)*x/sqrt(a))/(
8*b**(3/2)) + a*c*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + b*d*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.14035, size = 95, normalized size = 1.09

$$\frac{1}{8}\sqrt{bx^2+a}\left(2\,dx^2+\frac{4\,b^2c+abd}{b^2}\right)x-\frac{\left(4\,abc-a^2d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*a*b*c - a^
2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.48 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rubi [A] time = 0.0103711, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0197803, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]

[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A] time = 0., size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{bx^2+a}+\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x)

[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57063, size = 232, normalized size = 5.04

$$\left[\frac{2\sqrt{bx^2+a}bx + a\sqrt{b}\log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{b}x - a\right)}{4b}, \frac{\sqrt{bx^2+a}bx - a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

Sympy [A] time = 1.79875, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A] time = 1.10883, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2} \, dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh
[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

Rubi [A] time = 0.0542935, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {402, 217, 206, 377, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh
[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx &= \frac{b\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d}\\
&= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d}\\
&= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}
\end{aligned}
$$

Mathematica [A] time = 0.0446294, size = 84, normalized size = 1.02

$$
\frac{\sqrt{ad-bc}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}+\frac{\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(Sqrt[c]*d) + (Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/d
```

Maple [B] time = 0.033, size = 932, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)+1/2*b^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)
/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2))-1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d
)^(1/2)/d))*a+1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1
/2)/d))*b*c-1/2/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/2*b^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x
+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x+(-c*d)^(1/2)/d))*a-1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*
d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
/(x+(-c*d)^(1/2)/d))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91823, size = 1288, normalized size = 15.71

$$\left[\frac{2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2-4\left(ac^2x+\left(2\,bc^2-acd\right)x^3\right)\sqrt{bx^2+a}\sqrt{}}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt((b*c
 - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*
c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*
sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d, -1/4*(4*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x
 + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2
*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^
2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
 - a^2*d)*x)) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/d,
 -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(-(b*c - a*d)/c)
*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/
((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

Giac [A] time = 1.1362, size = 150, normalized size = 1.83

$$\frac{\left(b^{\frac{3}{2}}c - a\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 d + 2\,bc - ad}{2\,\sqrt{-b^2c^2 + abcd}}\right)}{\sqrt{-b^2c^2 + abcd}d} - \frac{\sqrt{b}\log\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^(3/2)*c - a*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2
*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*d) - 1/2*sq
rt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d
```

3.50 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{a\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0344839, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{a\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx &= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2c}\\
&= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c}\\
&= \frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [B] time = 0.231145, size = 165, normalized size = 2.01

$$\frac{x\sqrt{a+bx^2}\left(\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}+\sqrt{\frac{dx^2}{c}+1}\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)\right)}{2c^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))] + Sqrt[1 + (d*x^2)/c]*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x
^2)/c]]))/(2*c^2*Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c])
```

Maple [B] time = 0.024, size = 2521, normalized size = 30.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)*(
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)+1/4/d*b^(3/2)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^
(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))-1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*
d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
/(x+(-c*d)^(1/2)/d))*a+1/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/
2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^
(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-
2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/4/c*b^(1/2)/(a
*d-b*c)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2
)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a+1/4/(-
c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2)+1/4/c*b^(1/2)/d*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d
))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2))-1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d
)^(1/2)/d))*a+1/4/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1
/2)/d))*b+1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/4/c/d*b*(-c*d)^(1/2)/(
a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)+1/4/d*b^(3/2)/(a*d-b*c)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/
2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2))+1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*
ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/
2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2))/(x-(-c*d)^(1/2)/d))*a-1/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*
c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d
-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/4/c*b
^(1/2)/(a*d-b*c)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*
a-1/4/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
```

```
1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/c*b^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d
)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2))+1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(a*
d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))
/(x+(-c*d)^(1/2)/d))*a-1/4/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+
(-c*d)^(1/2)/d))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\left(dx^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

Fricas [B] time = 2.13738, size = 765, normalized size = 9.33

$$\left[\frac{4\left(bc^2 - acd\right)\sqrt{bx^2 + a}x + \left(adx^2 + ac\right)\sqrt{bc^2 - acd}\log\left(\frac{\left(8\,b^2c^2 - 8\,abcd + a^2d^2\right)x^4 + a^2c^2 + 2\left(4\,abc^2 - 3\,a^2cd\right)x^2 + 4\left((2\,bc - ad)x^3 + acx\right)\sqrt{bc^2 - acd}\sqrt{b}}{d^2x^4 + 2\,cdx^2 + c^2}\right.}{8\left(bc^4 - ac^3d + \left(bc^3d - ac^2d^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*sqrt(b*c^2 - a*
c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*
d^2)*x^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x - (a*d*x^2 + a*c)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(b*c^4 -
```

```
a*c^3*d + (b*c^3*d - a*c^2*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

Giac [B] time = 3.03052, size = 293, normalized size = 3.57

$$-\frac{a\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}c}+\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d+a^2\sqrt{b}d}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)
/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c) + (2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d +
 a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d)
```

3.51 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3} dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (a*(4*b*c - 3*a*d)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d
)^(3/2))
```

Rubi [A] time = 0.0940901, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {382, 378, 377, 208}

$$\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -(d*x*(a + b*x^2)^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (a*(4*b*c - 3*a*d)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d
)^(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx = -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{(4bc-3ad)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)}+\frac{(a(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)}+\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\left(c+dx^2\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.527832, size = 176, normalized size = 1.18

$$\frac{x\left(c\left(a^2d\left(5c+3dx^2\right)+ab\left(-4c^2+3cdx^2+3d^2x^4\right)-2b^2cx^2\left(2c+dx^2\right)\right)+\frac{a\left(c+dx^2\right)^2(3ad-4bc)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}\right)}{8c^3\sqrt{a+bx^2}\left(c+dx^2\right)^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] (x*(c*(-2*b^2*c*x^2*(2*c + d*x^2) + a^2*d*(5*c + 3*d*x^2) + a*b*(-4*c^2 + 3
*c*d*x^2 + 3*d^2*x^4)) + (a*(-4*b*c + 3*a*d)*(c + d*x^2)^2*ArcTanh[Sqrt[((b
*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))
/(8*c^3*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2)
```

Maple [B] time = 0.022, size = 5101, normalized size = 34.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\left(dx^2 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

Fricas [B] time = 2.61643, size = 1439, normalized size = 9.66

$$\left[\frac{\left(4\,abc^3 - 3\,a^2c^2d + \left(4\,abcd^2 - 3\,a^2d^3\right)x^4 + 2\left(4\,abc^2d - 3\,a^2cd^2\right)x^2\right)\sqrt{bc^2 - acd}\,\log\left(\frac{\left(8\,b^2c^2 - 8\,abcd + a^2d^2\right)x^4 + a^2c^2 + 2\left(4\,abc^2 - 3\,a^2c\right.}{d^2x^4 + 2\,c}\right.}{32\left(b^2c^7 - 2\,abc^6d + a^2c^5d^2 + \left(b^2c^5d^2 - 2\,abc^4d^3 -\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*
c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*
x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^
2)) + 4*((2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a
*b*c^3*d + 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*
c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d -
2*a*b*c^5*d^2 + a^2*c^4*d^3)*x^2), -1/16*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-b*c^2 +
a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2
 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3*d
- 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a*b*c^3*d + 5*a^2*c^2*d
^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2
 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^
4*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**3, x)
```

Giac [B] time = 2.52096, size = 657, normalized size = 4.41

$$-\frac{\left(4\,ab^{\frac{3}{2}}c-3\,a^{2}\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}d+2\,bc-ad}{2\,\sqrt{-b^{2}c^{2}+abcd}}\right)}{8\,\sqrt{-b^{2}c^{2}+abcd}\left(bc^{3}-ac^{2}d\right)}-\frac{4\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}ab^{\frac{3}{2}}cd^{2}-3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}a^{2}\sqrt{b}d^{3}-16\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a*b^(3/2)*c - 3*a^2*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*(b*c^3 - a*c^2*d)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d
^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3 - 16*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*b^(7/2)*c^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)
*c^2*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*a^2*b^(5/2)*c^2*d + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 -
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^3 - 2*a^4*b^(3/2)*c*d^2 +
3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*(b*c
^3*d - a*c^2*d^2))
```

3.52 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4} \, dx$

Optimal. Leaf size=208

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a\ \cdots}}{6c\left(c+\cdots\right.}$$

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt[a + b*x^2
])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*
Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d*x^2)) + (a*(8*b^2*c^2 - 12*a*
b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(16*c^(7/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.213217, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {412, 527, 12, 377, 208}

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a\ \cdots}}{6c\left(c+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt[a + b*x^2
])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*
Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d*x^2)) + (a*(8*b^2*c^2 - 12*a*
b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(16*c^(7/2)*(b*c - a*d)^(5/2))
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4}\,dx = \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} - \frac{\int \frac{-5a-4bx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{6c}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{-a(16bc-15ad)-2b(4bc-5ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{24c^2(bc-ad)}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} - \frac{\int -\frac{3a\left(8b^2c^2-12abcd+5a^2\ldots\right.}{\sqrt{a+bx^2}\left(c+dx^2\right)}}{48c^3(bc-ad)^2}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a\left(8b^2c^2-12abcd+\ldots\right.\right.}{16c^3\ldots}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a\left(8b^2c^2-12abcd+\ldots\right.\right.}{\ldots}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{a\left(8b^2c^2-12abcd+\ldots\right.}{16c^{7/2}\ldots}$$

<u>Mathematica</u> [A] time = 0.957091, size = 227, normalized size = 1.09

$$\frac{x\sqrt{a+bx^2}\left((bc-ad)\left(a^2d^2\left(33c^2+40cdx^2+15d^2x^4\right)-2abcd\left(30c^2+35cdx^2+13d^2x^4\right)+8b^2c^2\left(3c^2+3cdx^2+d^2x^4\right)\right)+\ldots\right.}{48c^3\left(c+dx^2\right)^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((b*c - a*d)*(8*b^2*c^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) -
2*a*b*c*d*(30*c^2 + 35*c*d*x^2 + 13*d^2*x^4) + a^2*d^2*(33*c^2 + 40*c*d*x^2
 + 15*d^2*x^4)) + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a
 + b*x^2))]])/x^2))/(48*c^3*(b*c - a*d)^3*(c + d*x^2)^3)
```

Maple [B] time = 0.026, size = 7922, normalized size = 38.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(dx^2 + c)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

Fricas [B] time = 6.19139, size = 2485, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3
- 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 +
 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^
2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4*d^2
 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*
d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^
```

```
6 - 28*a*b^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))
/(b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 -
 3*a*b^2*c^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*
a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2
*c^8*d^2 + 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2), -1/96*(3*(8*a*b^2*c^5 - 12*
a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5
)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b
^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arct
an(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2
*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4*d^2 - 34*a*b^
2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*d - 47*a*b
^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^6 - 28*a*b
^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^10
 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 - 3*a*b^2*c
^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*a*b^2*c^7*
d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2*c^8*d^2 +
 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 21.9838, size = 1293, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(8*a*b^(5/2)*c^2 - 12*a^2*b^(3/2)*c*d + 5*a^3*sqrt(b)*d^2)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d)
)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/24*(
```

```
24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*d^3 - 36*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*c*d^4 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
3*sqrt(b)*d^5 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 480
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*d^3 + 330*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*
sqrt(b)*d^5 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 + 1216*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d - 2016*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*c^3*d^2 + 1736*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 - 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 + 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 384*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 1392*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/
2)*c^3*d^2 - 1608*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 + 780
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7
/2)*c^3*d^2 + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 - 300
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^7*sqrt(b)*d^5 - 8*a^6*b^(5/2)*c^2*d^3 + 26*a^7*b^(3/2)*c*d^4
 - 15*a^8*sqrt(b)*d^5)/((b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*((sqrt(b)
*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

3.53 $\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=272

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3/2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)\left(\right.}{256}$$

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*(a + b*x^2)^(3/
2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*x*(a + b*x^2)^(5/2
))/(160*b^3) + (d*(14*b*c - 5*a*d)*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2
) + (d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b
^(7/2))
```

Rubi [A] time = 0.217773, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3/2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)\left(\right.}{256}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```


```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*(a + b*x^2)^(3/
2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*x*(a + b*x^2)^(5/2
))/(160*b^3) + (d*(14*b*c - 5*a*d)*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2
) + (d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b
^(7/2))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 dx = \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b} + \frac{\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(c(10bc-ad)+d(14bc-5ad)x^2\right) dx}{10b}$$

$$= \frac{d(14bc-5ad)x\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{80b^2} + \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b} + \frac{\int \left(a+bx^2\right)^{3/2}\left(c\left(80\right.\right.}{}$$

$$= \frac{d\left(36b^2c^2-20abcd+5a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{80b^2} + \frac{dx}{}$$

$$= \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{128b^3} + \frac{d\left(36b^2c^2-20abcd+5a^2d^2\right)x\left(a+bx\right.}{160b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\left(a\right.}{128b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\left(a\right.}{128b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\left(a\right.}{128b^3}$$

Mathematica [A] time = 5.12393, size = 220, normalized size = 0.81

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(4a^2b^2d\left(60c^2+15cdx^2+2d^2x^4\right)-10a^3bd^2\left(9c+dx^2\right)+15a^4d^3+16ab^3\left(70c^2dx^2+50c^3+45cd^2x^4+11d\right.\right.}{1280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^2) + 4*a^2
*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(10*c^3 + 20*c^2*d*x^
2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*c^3 + 70*c^2*d*x^2 + 45*c*d^2*
x^4 + 11*d^3*x^6)) - 15*a^2*(-4*b*c + a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(1280*b^(7/2))
```

Maple [A] time = 0.012, size = 393, normalized size = 1.4

$$\frac{d^3x^5}{10\,b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{ad^3x^3}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{a^2d^3x}{32\,b^3}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{a^3d^3x}{128\,b^3}\left(bx^2+a\right)^{\frac{3}{2}} - \frac{3\,d^3a^4x}{256\,b^3}\sqrt{bx^2+a} - \frac{3\,d^3a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/10*d^3*x^5*(b*x^2+a)^(5/2)/b-1/16*d^3/b^2*a*x^3*(b*x^2+a)^(5/2)+1/32*d^3/
b^3*a^2*x*(b*x^2+a)^(5/2)-1/128*d^3/b^3*a^3*x*(b*x^2+a)^(3/2)-3/256*d^3/b^3
*a^4*x*(b*x^2+a)^(1/2)-3/256*d^3/b^(7/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+
3/8*c*d^2*x^3*(b*x^2+a)^(5/2)/b-3/16*c*d^2/b^2*a*x*(b*x^2+a)^(5/2)+3/64*c*d
^2/b^2*a^2*x*(b*x^2+a)^(3/2)+9/128*c*d^2/b^2*a^3*x*(b*x^2+a)^(1/2)+9/128*c*
d^2/b^(5/2)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c^2*d*x*(b*x^2+a)^(5/2)/b
-1/8*c^2*d/b*a*x*(b*x^2+a)^(3/2)-3/16*c^2*d/b*a^2*x*(b*x^2+a)^(1/2)-3/16*c^
2*d/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*c^3*x*(b*x^2+a)^(3/2)+3/8
*c^3*a*x*(b*x^2+a)^(1/2)+3/8*c^3*a^2/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.01576, size = 1102, normalized size = 4.05

$$\left[-\frac{15\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(128\,b^5d^3x^9+16\left(30\,b^5cd^2+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(128*b^5*d^3*x^
9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^5*c^2*d + 90*a*b^4*c*d^2
 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 -
a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 +
```

```
 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/1280*(15*(32*a^2*b^3*c^3 - 16*a^3
*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (128*b^5*d^3*x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^
5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^
2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^
2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] time = 45.9015, size = 665, normalized size = 2.44

$$\frac{3a^{\frac{9}{2}}d^3x}{256b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{9a^{\frac{7}{2}}cd^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{7}{2}}d^3x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{5}{2}}c^2dx}{16b\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{5}{2}}cd^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{5}{2}}d^3x^5}{640b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] 3*a**(9/2)*d**3*x/(256*b**3*sqrt(1 + b*x**2/a)) - 9*a**(7/2)*c*d**2*x/(128*
b**2*sqrt(1 + b*x**2/a)) + a**(7/2)*d**3*x**3/(256*b**2*sqrt(1 + b*x**2/a))
 + 3*a**(5/2)*c**2*d*x/(16*b*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d**2*x**3/(
128*b*sqrt(1 + b*x**2/a)) - a**(5/2)*d**3*x**5/(640*b*sqrt(1 + b*x**2/a)) +
 a**(3/2)*c**3*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**3*x/(8*sqrt(1 + b*x**2/
a)) + 17*a**(3/2)*c**2*d*x**3/(16*sqrt(1 + b*x**2/a)) + 39*a**(3/2)*c*d**2*
x**5/(64*sqrt(1 + b*x**2/a)) + 23*a**(3/2)*d**3*x**7/(160*sqrt(1 + b*x**2/a
)) + 3*sqrt(a)*b*c**3*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*c**2*d*x**
5/(8*sqrt(1 + b*x**2/a)) + 15*sqrt(a)*b*c*d**2*x**7/(16*sqrt(1 + b*x**2/a))
 + 19*sqrt(a)*b*d**3*x**9/(80*sqrt(1 + b*x**2/a)) - 3*a**5*d**3*asinh(sqrt(
b)*x/sqrt(a))/(256*b**(7/2)) + 9*a**4*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*
b**(5/2)) - 3*a**3*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*a**2*c
**3*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c**3*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*c**2*d*x**7/(2*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*b**2*c*
d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**3*x**11/(10*sqrt(a)*sqrt
(1 + b*x**2/a))
```

Giac [A] time = 1.11162, size = 351, normalized size = 1.29

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,bd^3x^2 + \frac{30\,b^9cd^2 + 11\,ab^8d^3}{b^8}\right)x^2 + \frac{80\,b^9c^2d + 90\,ab^8cd^2 + a^2b^7d^3}{b^8}\right)x^2 + \frac{5\left(32\,b^9c^3 + 112\,ab^8c^2d + 6\,a^2b^7c\right.}{b^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)*x^2 + (80
*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5*(32*b^9*c^3 + 112*a
*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b^8)*x^2 + 5*(160*a*b^8*c^3 + 4
8*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2 + 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a)*x
- 3/256*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.54 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=196

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh}{128b^{5/2}}$$

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*b^2) + ((4
8*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(192*b^2) + (d*(10
*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) + (d*x*(a + b*x^2)^(5/2)*(c + d
*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rubi [A] time = 0.115935, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```


```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*b^2) + ((4
8*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(192*b^2) + (d*(10
*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) + (d*x*(a + b*x^2)^(5/2)*(c + d
*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2 dx = \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b} + \frac{\int \left(a+bx^2\right)^{3/2}\left(c(8bc-ad)+d(10bc-3ad)x^2\right) dx}{8b}$$
$$= \frac{d(10bc-3ad)x\left(a+bx^2\right)^{5/2}}{48b^2} + \frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b} - \frac{(ad(10bc-3ad)-6bc(8bc-ad}{48b^2}$$
$$= \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x\left(a+bx^2\right)^{5/2}}{48b^2} + \frac{dx\left(a+bx^2\right)^{5/2}}{8b}$$
$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2} + \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^2} + \frac{d(}{}$$
$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2} + \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^2} + \frac{d(}{}$$
$$= \frac{a\left(48b^2c^2-16abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{128b^2} + \frac{\left(48b^2c^2-16abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{192b^2} + \frac{d(}{}$$

Mathematica [C] time = 2.50854, size = 157, normalized size = 0.8

$$\frac{x\sqrt{a+bx^2}\left(6bx^2\left(c+dx^2\right)^2 \text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)+12bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(-\frac{1}{2},\frac{3}{2};\frac{9}{2}\right.\right.}{105\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2F1
[-3/2, 1/2, 7/2, -((b*x^2)/a)] + 12*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hyp
ergeometric2F1[-1/2, 3/2, 9/2, -((b*x^2)/a)] + 6*b*x^2*(c + d*x^2)^2*Hyperg
eometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*Sqrt[1 + (b*x^2)
/a])
```

Maple [A] time = 0.007, size = 249, normalized size = 1.3

$$\frac{d^2x^3}{8\,b}\left(bx^2+a\right)^{\frac{5}{2}} - \frac{ad^2x}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{a^2d^2x}{64\,b^2}\left(bx^2+a\right)^{\frac{3}{2}} + \frac{3\,a^3d^2x}{128\,b^2}\sqrt{bx^2+a} + \frac{3\,d^2a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}} + \frac{cdx}{3\,b}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/8*d^2*x^3*(b*x^2+a)^(5/2)/b-1/16*d^2/b^2*a*x*(b*x^2+a)^(5/2)+1/64*d^2/b^2
*a^2*x*(b*x^2+a)^(3/2)+3/128*d^2/b^2*a^3*x*(b*x^2+a)^(1/2)+3/128*d^2/b^(5/2
)*a^4*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*c*d*x*(b*x^2+a)^(5/2)/b-1/12*c*d/b*
a*x*(b*x^2+a)^(3/2)-1/8*c*d/b*a^2*x*(b*x^2+a)^(1/2)-1/8*c*d/b^(3/2)*a^3*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*c^2*x*(b*x^2+a)^(3/2)+3/8*c^2*a*x*(b*x^2+a)^
(1/2)+3/8*c^2*a^2/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06499, size = 776, normalized size = 3.96

$$\Bigg[\frac{3\left(48a^2b^2c^2 - 16a^3bcd + 3a^4d^2\right)\sqrt{b}\log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{b}x - a\right) + 2\Big(48b^4d^2x^7 + 8\left(16b^4cd + 9ab^3d^2\right)x^5 + 2\Big(4}{768b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*
a*b^3*d^2)*x^5 + 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80
*a*b^3*c^2 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/384*
(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - (48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^5 + 2*(4
8*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80*a*b^3*c^2 + 16*a^2*b
^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] time = 26.2037, size = 440, normalized size = 2.24

$$-\frac{3a^{\frac{7}{2}}d^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cdx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}d^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}c^2x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}cdx^3}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}d^2x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}bc}{8\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(7/2)*d**2*x/(128*b**2*sqrt(1 + b*x**2/a)) + a**(5/2)*c*d*x/(8*b*sqrt
(1 + b*x**2/a)) - a**(5/2)*d**2*x**3/(128*b*sqrt(1 + b*x**2/a)) + a**(3/2)*
c**2*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**2*x/(8*sqrt(1 + b*x**2/a)) + 17*a
**(3/2)*c*d*x**3/(24*sqrt(1 + b*x**2/a)) + 13*a**(3/2)*d**2*x**5/(64*sqrt(1
 + b*x**2/a)) + 3*sqrt(a)*b*c**2*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b
*c*d*x**5/(12*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*d**2*x**7/(16*sqrt(1 + b*x*
*2/a)) + 3*a**4*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) - a**3*c*d*asi
nh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + 3*a**2*c**2*asinh(sqrt(b)*x/sqrt(a))/(
8*sqrt(b)) + b**2*c**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*c*d*x**7/
(3*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/
a))
```

Giac [A] time = 1.10363, size = 236, normalized size = 1.2

$$\frac{1}{384}\left(2\left(4\left(6\,bd^2x^2+\frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2+\frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2+\frac{3\left(80\,ab^6c^2+16\,a^2b^5cd-3\,a^3b^4d^2\right)}{b^6}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 + (48*b^7*c^
2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b^6*c^2 + 16*a^2*b^5*
c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(48*a^2*b^2*c^2 - 16*a^
3*b*c*d + 3*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.55 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \, dx$

Optimal. Leaf size=118

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a + b*x^2)^(
3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*b*c - a*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.0400311, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a + b*x^2)^(
3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*b*c - a*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)dx = \frac{dx\left(a+bx^2\right)^{5/2}}{6b} - \frac{(-6bc+ad)\int \left(a+bx^2\right)^{3/2}dx}{6b}$$
$$= \frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{dx\left(a+bx^2\right)^{5/2}}{6b} + \frac{(a(6bc-ad))\int \sqrt{a+bx^2}\,dx}{8b}$$
$$= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{dx\left(a+bx^2\right)^{5/2}}{6b} + \frac{\left(a^2(6bc-ad)\right)\int \frac{}{\sqrt{a\cdot}}}{16b}$$
$$= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{dx\left(a+bx^2\right)^{5/2}}{6b} + \frac{\left(a^2(6bc-ad)\right)\text{Subs}}{}$$
$$= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x\left(a+bx^2\right)^{3/2}}{24b} + \frac{dx\left(a+bx^2\right)^{5/2}}{6b} + \frac{a^2(6bc-ad)\tanh^{-1}}{16b^{3/2}}$$

Mathematica [A] time = 0.199388, size = 109, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(3a^2d+2ab\left(15c+7dx^2\right)+4b^2x^2\left(3c+2dx^2\right)\right)-\frac{3a^{3/2}(ad-6bc)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2*d + 4*b^2*x^2*(3*c + 2*d*x^2) + 2*a*b*(1
5*c + 7*d*x^2)) - (3*a^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

Maple [A] time = 0.003, size = 131, normalized size = 1.1

$$\frac{dx}{6b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{adx}{24b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{da^2x}{16b}\sqrt{bx^2+a}-\frac{da^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{cx}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,acx}{8}\sqrt{bx^2+a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c),x)
```

```
[Out] 1/6*d*x*(b*x^2+a)^(5/2)/b-1/24*d/b*a*x*(b*x^2+a)^(3/2)-1/16*d/b*a^2*x*(b*x^
2+a)^(1/2)-1/16*d/b^(3/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*c*x*(b*x^2+
a)^(3/2)+3/8*c*a*x*(b*x^2+a)^(1/2)+3/8*c*a^2/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72869, size = 483, normalized size = 4.09

$$\left[-\frac{3\left(6\,a^2bc-a^3d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,b^3dx^5+2\left(6\,b^3c+7\,ab^2d\right)x^3+3\left(10\,ab^2c+a^2bd\right)x\right)\sqrt{bx^2}}{96\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x^3 + 3*(10*a*b^2*c +
 a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)*ar
```

```
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x
^3 + 3*(10*a*b^2*c + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] time = 13.2377, size = 253, normalized size = 2.14

$$\frac{a^{\frac{5}{2}}dx}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}cx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}bcx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}bdx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3d\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{3a^2c\,\mathrm{asinh}}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] a**(5/2)*d*x/(16*b*sqrt(1 + b*x**2/a)) + a**(3/2)*c*x*sqrt(1 + b*x**2/a)/2
+ a**(3/2)*c*x/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*d*x**3/(48*sqrt(1 + b*x
**2/a)) + 3*sqrt(a)*b*c*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*d*x**5/(
24*sqrt(1 + b*x**2/a)) - a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*
a**2*c*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.10687, size = 139, normalized size = 1.18

$$\frac{1}{48}\left(2\left(4\,bdx^2+\frac{6\,b^5c+7\,ab^4d}{b^4}\right)x^2+\frac{3\left(10\,ab^4c+a^2b^3d\right)}{b^4}\right)\sqrt{bx^2+a}x-\frac{\left(6\,a^2bc-a^3d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4*c + a^2*b
^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

3.56 $\int \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0158949, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{3/2} dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\,dx \\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0933082, size = 65, normalized size = 1.

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+5ax+2bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/8
```

Maple [A] time = 0., size = 51, normalized size = 0.8

$$\frac{x}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,ax}{8}\sqrt{bx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a*x*(b*x^2+a)^(1/2)+3/8*a^2/b^(1/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63215, size = 294, normalized size = 4.52

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{16\,b},-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 2.97141, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Giac [A] time = 1.0819, size = 66, normalized size = 1.02

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x-\frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}-\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rubi [A] time = 0.108391, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 217, 206, 377, 208}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}-\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{c+dx^2}\,dx &= \frac{bx\sqrt{a+bx^2}}{2d}+\frac{\int \frac{-a(bc-2ad)-b(2bc-3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2d}\\
&= \frac{bx\sqrt{a+bx^2}}{2d}-\frac{(b(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^2}+\frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d^2}\\
&= \frac{bx\sqrt{a+bx^2}}{2d}-\frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\right.}{d^2}\\
&= \frac{bx\sqrt{a+bx^2}}{2d}-\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}
\end{aligned}$$

Mathematica [A] time = 0.194883, size = 110, normalized size = 0.97

$$\frac{\sqrt{b}(3ad-2bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}+bdx\sqrt{a+bx^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2] + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]
*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + Sqrt[b]*(-2*b*c + 3*a*d)*Log[b*x
+ Sqrt[b]*Sqrt[a + b*x^2]])/(2*d^2)
```

Maple [B] time = 0.014, size = 1845, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/d*b^(1/2)*ln((b*(-c*d)^(1/2)/d+b
*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a+1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^
2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a-1/2/(-c*d)^(
1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))
/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2))*c-1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/
d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^
(1/2)/d))*a^2+1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2
*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2
)/d))*a*b*c-1/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1
/2)/d))*b^2*c^2-1/6/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
```

```
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/d*b^(1/2)*ln((-b
*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a-1/2/(-c*d)^(1/2)*((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2)*a+1/2/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(
x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln
((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)
*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2))/(x+(-c*d)^(1/2)/d))*a^2-1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*
(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x+(-c*d)^(1/2)/d))*a*b*c+1/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((
2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*(
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2))/(x+(-c*d)^(1/2)/d))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.98249, size = 1582, normalized size = 14.

$$\left[\frac{2\sqrt{bx^2+a}\,bdx-(2\,bc-3\,ad)\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-(bc-ad)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,ab\right.}{}\right.}{4\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log(((8*b^2*
c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 -
4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d
^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x + 2*(2*b*c -
3*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt((b*c
- a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*s
qrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 +
a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
- a^2*d)*x)) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*d*x + (2*b*c - 3*a*d)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1
/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c -
 a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

Giac [A] time = 1.13534, size = 205, normalized size = 1.81

$$\frac{\sqrt{bx^2 + a}bx}{2\,d} + \frac{\left(2\,b^{\frac{3}{2}}c - 3\,a\sqrt{b}d\right)\log\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{4\,d^2} - \frac{\left(b^{\frac{5}{2}}c^2 - 2\,ab^{\frac{3}{2}}cd + a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 d + 2\,bc - ad}{2\,\sqrt{-b^2c^2 + abcd}}\right)}{\sqrt{-b^2c^2 + abcd}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x/d + 1/4*(2*b^(3/2)*c - 3*a*sqrt(b)*d)*log((sqrt(b)*
x - sqrt(b*x^2 + a))^2)/d^2 - (b^(5/2)*c^2 - 2*a*b^(3/2)*c*d + a^2*sqrt(b)*
d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2
*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*d^2)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2} dx$

Optimal. Leaf size=131

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) + (b^(3/2)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^2)
```

Rubi [A] time = 0.0903201, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 523, 217, 206, 377, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) + (b^(3/2)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^2}\,dx &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\int \frac{a(bc+ad)+2b^2cx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2cd}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^2}-\frac{((bc-ad)(2bc+ad))\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2cd^2}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{((bc-ad)(2bc+ad))\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\right.}{2cd^2}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}
\end{aligned}$$

Mathematica [A] time = 0.121052, size = 142, normalized size = 1.08

$$\frac{\frac{(a^2d^2+abcd-2b^2c^2)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{ad-bc}}+2b^{3/2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)-\frac{dx\sqrt{a+bx^2}(bc-ad)}{c(c+dx^2)}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] (-((d*(b*c - a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2))) + ((-2*b^2*c^2 + a*b*
c*d + a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c
^(3/2)*Sqrt[-(b*c) + a*d]) + 2*b^(3/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/
(2*d^2)
```

Maple [B] time = 0.016, size = 4621, normalized size = 35.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] -3/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a-1/12/(-c*d)^(1/2)/c*((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/4/d^2
*b^(3/2)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/12/(-
c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(3/2)-1/4/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d
))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2))+3/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((
2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*(
(x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2))/(x-(-c*d)^(1/2)/d))*a^2-3/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d
)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c
)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2+3/8/d*b^2/(a*d-b*c)*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+9/8/d
*b^(3/2)/(a*d-b*c)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
```

```
)*a+3/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(
1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/4*c/d^2*b^(5/2)/(a*d-b*c)*ln
((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-1/4/c*b/(a*d-b*c)*(
(x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
3/2)*x-3/8/c*b^(1/2)/(a*d-b*c)*a^2*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d
))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2))-3/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*
b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/4*c/d^2*b^(5/2
)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-1/4/
c*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(3/2)*x-3/8/c*b^(1/2)/(a*d-b*c)*a^2*ln((-b*(-c*d)^(1/2)/d+b*(
x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/8/c*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/8/c/d*b^(1/2)*ln((-b*
(-c*d)^(1/2)/d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a+1/4/(-c*d)^(1/2)/c/((
a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2
*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2+1/8/c*b/d*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/8/c
/d*b^(1/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/b^(1/2)+((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a-1/4/
(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2+3/8/d*
b^2/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)*x+9/8/d*b^(3/2)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+b*(x+(-
c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a-1/4/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-
2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a+1/4/(-c*d)^(1/2)
/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2)*b+1/4/(-c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a-1/4/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b+1/4/c/(a*d
-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)+1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)+1/4
/(-c*d)^(1/2)*c/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b^2-1
/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-3/8/c*b/(a*d-b*c)*a*((x-(-c*d)^(1/2)/
d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/c/d*b
```

```
*(-c*d)^(1/2)/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-3/8/c*b/(a*d-b*c)*a*((x+(-c*d)^(1/2)/d)^2*b-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/2/(-c*d)^(1/2)/
d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a*b-1/4/(-c*d)^(1/2)*c/
d^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b^2-1/2/(-c*d)^(1/2)/
d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a*b+3/4*c/d^3*b^3*(-c*d
)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-3/2/d^2
*b^2*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d
))*a+3/2/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)
/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-
c*d)^(1/2)/d))*a-3/4*c/d^3*b^3*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2
)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))+3/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)*((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)
```

Fricas [A] time = 2.7808, size = 1941, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(b)*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*c^2 + a*c*d + (2*b*c
*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)
*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*
c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2))
)/(c*d^3*x^2 + c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 8*(b*c
*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*
c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (
2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d
*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a
)*x - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan
(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c
 - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - 2*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b
*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 4*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-
(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(
b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/(c*d^3*x^2 + c^2*
d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)
```

Giac [B] time = 1.16482, size = 428, normalized size = 3.27

$$-\frac{b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^2}+\frac{\left(2\,b^{\frac{5}{2}}c^2-ab^{\frac{3}{2}}cd-a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}cd^2}-\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}c}{\left(\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 + 1/2*(2*b^(5/2)*c^2
- a*b^(3/2)*c*d - a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a)
)^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c*
d^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^(3/2)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*
d^2 + a^2*b^(3/2)*c*d - a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*
d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a*d + a^2*d)*c*d^2)
```

3.59 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=113

$$\frac{3a^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2])/(8*c^2*
(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0574365, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{3a^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2])/(8*c^2*
(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx &= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{(3a)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{4c}\\
&= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)}+\frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2}\\
&= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8c^2}\\
&= \frac{x\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{3ax\sqrt{a+bx^2}}{8c^2\left(c+dx^2\right)}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.670222, size = 163, normalized size = 1.44

$$\frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\left(5ac+3adx^2+2bcx^2\right)}{\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}}+\frac{3a\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{8c^3\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(5*a*c + 2*b*c*x
^2 + 3*a*d*x^2))/((c + d*x^2)*Sqrt[1 + (d*x^2)/c]) + (3*a*ArcSin[Sqrt[(-(b/
```

```
a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(8*
c^3*Sqrt[1 + (b*x^2)/a])
```

Maple [B] time = 0.022, size = 9059, normalized size = 80.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

Fricas [B] time = 2.51491, size = 1072, normalized size = 9.49

$$\left[\frac{3\left(a^2d^2x^4+2\,a^2cdx^2+a^2c^2\right)\sqrt{bc^2-acd}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)+4\left(\right.}{32\left(bc^6-ac^5d+\left(bc^4d^2-ac^3d^3\right)x^4+2\left(bc^5d-ac^4d^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log(((
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*
x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d
^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 +
 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2
- a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2), -1/16*(3*(a^2*d^2*x^4 + 2*
a^2*c*d*x^2 + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)
*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*
c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 + 5*(a*b*
c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*
d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**3, x)
```

Giac [B] time = 2.30446, size = 609, normalized size = 5.39

$$-\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}\,c^2}+\frac{8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}c^2d-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a^2\sqrt{b}d^3+16\left(\sqrt{b}x-\sqrt{bx^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/8*a^2*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*
d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^2) + 1/4*(8*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6
```

```
*a^2*sqrt(b)*d^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 8*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d - 18*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3
+ 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d + 16*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*s
qrt(b)*d^3 + 2*a^4*b^(3/2)*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b
*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^2)
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4} dx$

Optimal. Leaf size=199

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3\left(c+dx^2\right)(bc-ad)}-\frac{dx\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^(5/2))/(6*c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a*d)
*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + (a*(6*b*c - 5*a*
d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c + d*x^2)) + (a^2*(6*b*c - 5*a*
d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c
 - a*d)^(3/2))
```

Rubi [A] time = 0.11543, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {382, 378, 377, 208}

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3\left(c+dx^2\right)(bc-ad)}-\frac{dx\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```


```
[Out] -(d*x*(a + b*x^2)^(5/2))/(6*c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a*d)
*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + (a*(6*b*c - 5*a*
d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c + d*x^2)) + (a^2*(6*b*c - 5*a*
d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c
 - a*d)^(3/2))
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^4}\,dx = -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{(a(6bc-5ad))\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{\left(a^2(6bc-5ad)\right)\int}{16c^3(bc}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{\left(a^2(6bc-5ad)\right)\mathrm{Su}}{}$$

$$= -\frac{dx\left(a+bx^2\right)^{5/2}}{6c(bc-ad)\left(c+dx^2\right)^3}+\frac{(6bc-5ad)x\left(a+bx^2\right)^{3/2}}{24c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\left(c+dx^2\right)}+\frac{a^2(6bc-5ad)\tanh}{16c^{7/2}(bc-}$$

Mathematica [A] time = 0.775098, size = 247, normalized size = 1.24

$$\frac{ax\left(\frac{bx^2}{a}+1\right)\Big(c\big(a^2b\left(11c^2dx^2-30c^3+32cd^2x^4+15d^3x^6\right)+a^3d\left(33c^2+40cdx^2+15d^2x^4\right)-2ab^2cx^2\left(21c^2+13cdx^2+4\right.}{48c^4\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(-4*b^3*c^2*x^4*(3*c + d*x^2) - 2*a*b^2*c*x^2*(21*c
^2 + 13*c*d*x^2 + 4*d^2*x^4) + a^3*d*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4) + a
^2*b*(-30*c^3 + 11*c^2*d*x^2 + 32*c*d^2*x^4 + 15*d^3*x^6)) + (3*a^2*(-6*b*c
 + 5*a*d)*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/S
qrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(48*c^4*(-(b*c) + a*d)*(a + b*x^2)
^(3/2)*(c + d*x^2)^3)
```

Maple [B] time = 0.03, size = 13766, normalized size = 69.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

Fricas [B] time = 3.95139, size = 1994, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3*
(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^2
)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
- a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((4*b^3*c^4*d +
4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a
*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*
a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*c^8*d +
a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*
d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2
*c^6*d^3)*x^2), -1/96*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^
3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a
^3*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b
*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) - 2*((4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^
3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2
*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^
2 + a))/(b^2*c^9 - 2*a*b*c^8*d + a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4
 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3
*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2*c^6*d^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 20.881, size = 1241, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(6*a^2*b^(3/2)*c - 5*a^3*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^4 - a*c^3*d)*sq
rt(-b^2*c^2 + a*b*c*d)) - 1/24*(18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(
3/2)*c*d^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 - 96*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*a*b^(7/2)*c^3*d^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*
d^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 - 128*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 720*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 968*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 620*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*
b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 96*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d - 288*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^
(5/2)*c^2*d^3 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 48*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^4*b^(7/2)*c^3*d^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
5/2)*c^2*d^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 4*a^6*b^(5/2)*c^2*d^3 - 8*
a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/((b*c^4*d^2 - a*c^3*d^3)*((sqrt(b)*
x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

3.61 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx$

Optimal. Leaf size=300

$$\frac{x\sqrt{a+bx^2}\left(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{a^2\left(35a^2d^2-80ab\right.}{12}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c*d*(c + d*x^2)^4) + ((2*b*c + 7*a*d)*x
*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^2 + 24*a*b*c*d - 35*
a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c - a*d)*(c + d*x^2)^2) + ((16*b^
3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/
(384*c^4*d*(b*c - a*d)^2*(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*
a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/
2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.365153, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {413, 527, 12, 377, 208}

$$\frac{x\sqrt{a+bx^2}\left(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{a^2\left(35a^2d^2-80ab\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c*d*(c + d*x^2)^4) + ((2*b*c + 7*a*d)*x
*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^2 + 24*a*b*c*d - 35*
a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c - a*d)*(c + d*x^2)^2) + ((16*b^
3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/
(384*c^4*d*(b*c - a*d)^2*(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*
a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/
2)*(b*c - a*d)^(5/2))
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{\int \frac{a(bc+7ad)+2b(bc+3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^4}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\int \frac{a(bc-ad)(4bc+35ad)+4b(bc-ad)(2bc+7ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{48c^2d(bc-ad)}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2} + \frac{\int \frac{a(bc-ad)}{}}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2} + \frac{\left(16b^3c^3\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2} + \frac{\left(16b^3c^3\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2} + \frac{\left(16b^3c^3\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4} + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3} + \frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2} + \frac{\left(16b^3c^3\right.}{}$$

Mathematica [A] time = 1.32984, size = 362, normalized size = 1.21

$$\frac{ax\left(\frac{bx^2}{a}+1\right)\left(c\left(2a^2b^2c\left(-345c^2d^2x^4-160c^3dx^2+120c^4-294cd^3x^6-85d^4x^8\right)+a^3bd\left(-117c^2d^2x^4-563c^3dx^2-528c^4+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(16*b^4*c^3*x^4*(6*c^2 + 4*c*d*x^2 + d^2*x^4) + 8*a
*b^3*c^2*x^2*(42*c^3 + 34*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6) + a^4*d^2*(
```

```
279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6) + 2*a^2*b^2*c*(120*c
^4 - 160*c^3*d*x^2 - 345*c^2*d^2*x^4 - 294*c*d^3*x^6 - 85*d^4*x^8) + a^3*b*
d*(-528*c^4 - 563*c^3*d*x^2 - 117*c^2*d^2*x^4 + 215*c*d^3*x^6 + 105*d^4*x^8
)) + (3*a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*(c + d*x^2)^4*ArcTanh[Sq
rt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]]))/(384*c^5*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^4)
```

Maple [B] time = 0.038, size = 18791, normalized size = 62.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\left(dx^2 + c\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

Fricas [B] time = 15.271, size = 3330, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*
c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3
*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 +
 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*
d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 +
a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4
*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 - 105
*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3 +
1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*d -
 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(80*a
*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sqrt(
b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3 + (
b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(b^3
*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3*c^
10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c^11
*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2), -1/768*(3*(48*
a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*c^2*d^4 - 80*a^
3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35
*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4
)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*
b^4*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 -
105*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3
 + 1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*
d - 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(8
0*a*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sq
rt(b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3
+ (b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(
b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3
*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c
^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 6.29575, size = 2102, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/128*(48*a^2*b^(5/2)*c^2 - 80*a^3*b^(3/2)*c*d + 35*a^4*sqrt(b)*d^2)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1
/192*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c^2*d^5 - 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^14*a^3*b^(3/2)*c*d^6 + 105*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*a^4*sqrt(b)*d^7 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2
)*c^3*d^4 - 4368*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c^2*d^5 + 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6 - 735*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^
(13/2)*c^6*d + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c^5*d^2 + 7
936*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*c^4*d^3 - 26624*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3*d^4 + 26944*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^4*b^(5/2)*c^2*d^5 - 12320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b
^(3/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/2)*c^7 - 1024*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a*b^(13/2)*c^6*d + 27392*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(1
1/2)*c^5*d^2 - 65920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 +
81680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4 - 58840*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d^5 + 22750*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^6*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b
)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^6*d - 8192*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(11/2)*c^5*d^2 + 47104*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^4*b^(9/2)*c^4*d^3 - 74240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
^5*b^(7/2)*c^3*d^4 + 56416*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*
d^5 - 22400*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 + 3675*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*sqrt(b)*d^7 - 1536*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a^4*b^(11/2)*c^5*d^2 - 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/
2)*c^4*d^3 + 17696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 - 23
152*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(5/2)*c^2*d^5 + 11690*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^8*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^9*sqrt(b)*d^7 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^
3 - 512*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(7/2)*c^3*d^4 + 2896*(sqrt(b)
```

```
*x - sqrt(b*x^2 + a))^2*a^8*b^(5/2)*c^2*d^5 - 2800*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^9*b^(3/2)*c*d^6 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b
)*d^7 - 16*a^8*b^(7/2)*c^3*d^4 - 40*a^9*b^(5/2)*c^2*d^5 + 170*a^10*b^(3/2)*
c*d^6 - 105*a^11*sqrt(b)*d^7)/((b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*
((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

3.62 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=349

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right.}{}$$

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 -
 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) + ((320*b^3*c^3 - 120*a*b^2*c^2
*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*
b^2*c^2 - 68*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*
b*c - 5*a*d)*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^
(7/2)*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*
b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rubi [A] time = 0.247141, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 528, 388, 195, 217, 206}

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(36a^2bcd^2-5a^3d^3-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 -
 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) + ((320*b^3*c^3 - 120*a*b^2*c^2
*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*
b^2*c^2 - 68*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*
b*c - 5*a*d)*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^
(7/2)*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*
b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3 dx = \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} + \frac{\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(c(12bc-ad)+d(16bc-5ad)x^2\right)}{12b}$$

$$= \frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{120b^2} + \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} + \int \left(a+bx^2\right)^{5/2}\left(c\right.$$

$$= \frac{d\left(152b^2c^2-68abcd+15a^2d^2\right)x\left(a+bx^2\right)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{120b^2} +$$

$$= \frac{\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\left(a+bx^2\right)^{5/2}}{1920b^3} + \frac{d\left(152b^2c^2-68abcd+15a^2\right.}{960b^3}$$

$$= \frac{a\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\left(a+bx^2\right)^{3/2}}{1536b^3} + \frac{\left(320b^3c^3-120ab^2c^2d+3\right.}{1}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2c^2d+3\right.}{15}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2c^2d+3\right.}{15}$$

$$= \frac{a^2\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)x\sqrt{a+bx^2}}{1024b^3} + \frac{a\left(320b^3c^3-120ab^2c^2d+3\right.}{15}$$

Mathematica [A] time = 5.17283, size = 270, normalized size = 0.77

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(40a^3b^2d\left(45c^2+9cdx^2+d^2x^4\right)+48a^2b^3\left(295c^2dx^2+220c^3+186cd^2x^4+45d^3x^6\right)-10a^4bd^2\left(54c+5dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d*x^2) + 40
*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4*(20*c^3 + 45*c^2*d*
x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3*(220*c^3 + 295*c^2*d*x^2 + 18
6*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189*c*d
^2*x^4 + 50*d^3*x^6)) - 15*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^2*b*c
*d^2 + 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

Maple [A] time = 0.015, size = 476, normalized size = 1.4

$$\frac{d^3x^5}{12\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{ad^3x^3}{24\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{d^3a^2x}{64\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{a^3d^3x}{384\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,d^3a^4x}{1536\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,d^3a^5x}{1024\,b^3}\sqrt{bx^2+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/12*d^3*x^5*(b*x^2+a)^(7/2)/b-1/24*d^3/b^2*a*x^3*(b*x^2+a)^(7/2)+1/64*d^3/
b^3*a^2*x*(b*x^2+a)^(7/2)-1/384*d^3/b^3*a^3*x*(b*x^2+a)^(5/2)-5/1536*d^3/b^
3*a^4*x*(b*x^2+a)^(3/2)-5/1024*d^3/b^3*a^5*x*(b*x^2+a)^(1/2)-5/1024*d^3/b^(
7/2)*a^6*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/10*c*d^2*x^3*(b*x^2+a)^(7/2)/b-9/8
0*c*d^2/b^2*a*x*(b*x^2+a)^(7/2)+3/160*c*d^2/b^2*a^2*x*(b*x^2+a)^(5/2)+3/128
*c*d^2/b^2*a^3*x*(b*x^2+a)^(3/2)+9/256*c*d^2/b^2*a^4*x*(b*x^2+a)^(1/2)+9/25
6*c*d^2/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/8*c^2*d*x*(b*x^2+a)^(7/
2)/b-1/16*c^2*d/b*a*x*(b*x^2+a)^(5/2)-5/64*c^2*d/b*a^2*x*(b*x^2+a)^(3/2)-15
/128*c^2*d/b*a^3*x*(b*x^2+a)^(1/2)-15/128*c^2*d/b^(3/2)*a^4*ln(x*b^(1/2)+(b
*x^2+a)^(1/2))+1/6*c^3*x*(b*x^2+a)^(5/2)+5/24*c^3*a*x*(b*x^2+a)^(3/2)+5/16*
c^3*a^2*x*(b*x^2+a)^(1/2)+5/16*c^3*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.51186, size = 1385, normalized size = 3.97

$$\left[-\frac{15\left(320\,a^3b^3c^3-120\,a^4b^2c^2d+36\,a^5bcd^2-5\,a^6d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(1280\,b^6d^3x^{11}+128\left(36\,b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6
*d^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*b^6
*d^3*x^11 + 128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*
a*b^5*c*d^2 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 111
6*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2
*d + 36*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*
b^3*c^2*d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/153
60*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*x^11 + 128*(36*b
^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*a*b^5*c*d^2 + 15*a^2*
b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 1116*a^2*b^4*c*d^2 + 5*a
^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2*d + 36*a^3*b^3*c*d^2
 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*b^3*c^2*d - 36*a^4*b^
2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] time = 87.8307, size = 796, normalized size = 2.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(11/2)*d**3*x/(1024*b**3*sqrt(1 + b*x**2/a)) - 9*a**(9/2)*c*d**2*x/(25
6*b**2*sqrt(1 + b*x**2/a)) + 5*a**(9/2)*d**3*x**3/(3072*b**2*sqrt(1 + b*x**
2/a)) + 15*a**(7/2)*c**2*d*x/(128*b*sqrt(1 + b*x**2/a)) - 3*a**(7/2)*c*d**2
*x**3/(256*b*sqrt(1 + b*x**2/a)) - a**(7/2)*d**3*x**5/(1536*b*sqrt(1 + b*x*
*2/a)) + a**(5/2)*c**3*x*sqrt(1 + b*x**2/a)/2 + 3*a**(5/2)*c**3*x/(16*sqrt(
1 + b*x**2/a)) + 133*a**(5/2)*c**2*d*x**3/(128*sqrt(1 + b*x**2/a)) + 387*a*
*(5/2)*c*d**2*x**5/(640*sqrt(1 + b*x**2/a)) + 55*a**(5/2)*d**3*x**7/(384*sq
rt(1 + b*x**2/a)) + 35*a**(3/2)*b*c**3*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a
**(3/2)*b*c**2*d*x**5/(64*sqrt(1 + b*x**2/a)) + 219*a**(3/2)*b*c*d**2*x**7/
(160*sqrt(1 + b*x**2/a)) + 67*a**(3/2)*b*d**3*x**9/(192*sqrt(1 + b*x**2/a))
 + 17*sqrt(a)*b**2*c**3*x**5/(24*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*c**2
*d*x**7/(16*sqrt(1 + b*x**2/a)) + 87*sqrt(a)*b**2*c*d**2*x**9/(80*sqrt(1 +
b*x**2/a)) + 7*sqrt(a)*b**2*d**3*x**11/(24*sqrt(1 + b*x**2/a)) - 5*a**6*d**
3*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(7/2)) + 9*a**5*c*d**2*asinh(sqrt(b)*x/
sqrt(a))/(256*b**(5/2)) - 15*a**4*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
3/2)) + 5*a**3*c**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + b**3*c**3*x**7/
(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*b**3*c**2*d*x**9/(8*sqrt(a)*sqrt(1 + b*x
```

```
**2/a)) + 3*b**3*c*d**2*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d**3*x
**13/(12*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.18821, size = 433, normalized size = 1.24

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2d^3x^2+\frac{36\,b^{12}cd^2+25\,ab^{11}d^3}{b^{10}}\right)x^2+\frac{9\left(40\,b^{12}c^2d+84\,ab^{11}cd^2+15\,a^2b^{10}d^3\right)}{b^{10}}\right)x^2+\frac{320\,b^{12}c^3+204}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*d^3)/b^10)
*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10*d^3)/b^10)*x^2 + (3
20*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^10*c*d^2 + 5*a^3*b^9*d^3)/b^10
)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*a^4*
b^8*d^3)/b^10)*x^2 + 15*(704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4*b^8*
c*d^2 + 5*a^5*b^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*c^3 -
120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(7/2)
```

3.63 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=241

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd\right.}{256b^2}$$

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(256*b^2) + (
a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(384*b^2) + ((
80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(5/2))/(480*b^2) + (3*d*
(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d
*x^2))/(10*b) + (a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rubi [A] time = 0.14837, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd\right.}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```


```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(256*b^2) + (
a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(384*b^2) + ((
80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(5/2))/(480*b^2) + (3*d*
(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d
*x^2))/(10*b) + (a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2 dx = \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}+\frac{\int \left(a+bx^2\right)^{5/2}\left(c(10bc-ad)+3d(4bc-ad)x^2\right) dx}{10b}$$
$$=\frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}-\frac{(3ad(4bc-ad)-8bc(10bc-a}{80b^2}$$
$$=\frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{480b^2}+\frac{3d(4bc-ad)x\left(a+bx^2\right)^{7/2}}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/}}{10b}$$
$$=\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}+\frac{\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{5/2}}{480b^2}$$
$$=\frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}$$
$$=\frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}$$
$$=\frac{a^2\left(80b^2c^2-20abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{256b^2}+\frac{a\left(80b^2c^2-20abcd+3a^2d^2\right)x\left(a+bx^2\right)^{3/2}}{384b^2}$$

Mathematica [C] time = 2.62425, size = 158, normalized size = 0.66

$$\frac{ax\sqrt{a+bx^2}\left(10bx^2\left(c+dx^2\right)^2 \text{HypergeometricPFQ}\left(\left\{-\frac{3}{2},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)+20bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(-\frac{3}{2}\right.\right.}{105\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*x*Sqrt[a + b*x^2]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometric2
F1[-5/2, 1/2, 7/2, -((b*x^2)/a)] + 20*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*H
ypergeometric2F1[-3/2, 3/2, 9/2, -((b*x^2)/a)] + 10*b*x^2*(c + d*x^2)^2*Hyp
ergeometricPFQ[{-3/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*Sqrt[1 + (b*x
^2)/a])
```

Maple [A] time = 0.007, size = 308, normalized size = 1.3

$$\frac{d^2x^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3\,ad^2x}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2d^2x}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{d^2a^3x}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,d^2a^4x}{256\,b^2}\sqrt{bx^2+a}+\frac{3\,d^2a^5}{256}\ln\left(x\sqrt{b}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/10*d^2*x^3*(b*x^2+a)^(7/2)/b-3/80*d^2/b^2*a*x*(b*x^2+a)^(7/2)+1/160*d^2/b
^2*a^2*x*(b*x^2+a)^(5/2)+1/128*d^2/b^2*a^3*x*(b*x^2+a)^(3/2)+3/256*d^2/b^2*
a^4*x*(b*x^2+a)^(1/2)+3/256*d^2/b^(5/2)*a^5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1
/4*c*d*x*(b*x^2+a)^(7/2)/b-1/24*c*d/b*a*x*(b*x^2+a)^(5/2)-5/96*c*d/b*a^2*x*
(b*x^2+a)^(3/2)-5/64*c*d/b*a^3*x*(b*x^2+a)^(1/2)-5/64*c*d/b^(3/2)*a^4*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))+1/6*c^2*x*(b*x^2+a)^(5/2)+5/24*c^2*a*x*(b*x^2+a)^(
3/2)+5/16*c^2*a^2*x*(b*x^2+a)^(1/2)+5/16*c^2*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9962, size = 963, normalized size = 4.

$$\left[\frac{15\left(80\,a^{3}b^{2}c^{2} - 20\,a^{4}bcd + 3\,a^{5}d^{2}\right)\sqrt{b}\log\left(-2\,bx^{2} - 2\,\sqrt{bx^{2} + a}\sqrt{b}x - a\right) + 2\left(384\,b^{5}d^{2}x^{9} + 48\left(20\,b^{5}cd + 21\,ab^{4}d^{2}\right)x^{7} + 8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d^2*x^9 + 48*(20*b^5*c*d
+ 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 +
 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3
*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(
80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
```

```
b*x^2 + a)) - (384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7 + 8*(80
*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2 + 236*a^
2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*c*d - 3*a
^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] time = 50.966, size = 537, normalized size = 2.23

$$-\frac{3a^{\frac{9}{2}}d^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{7}{2}}cdx}{64b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}d^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}c^2x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}cdx^3}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{129a^{\frac{5}{2}}d^2x^5}{640\sqrt{1+\frac{bx^2}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(9/2)*d**2*x/(256*b**2*sqrt(1 + b*x**2/a)) + 5*a**(7/2)*c*d*x/(64*b*s
qrt(1 + b*x**2/a)) - a**(7/2)*d**2*x**3/(256*b*sqrt(1 + b*x**2/a)) + a**(5/
2)*c**2*x*sqrt(1 + b*x**2/a)/2 + 3*a**(5/2)*c**2*x/(16*sqrt(1 + b*x**2/a))
+ 133*a**(5/2)*c*d*x**3/(192*sqrt(1 + b*x**2/a)) + 129*a**(5/2)*d**2*x**5/(
640*sqrt(1 + b*x**2/a)) + 35*a**(3/2)*b*c**2*x**3/(48*sqrt(1 + b*x**2/a)) +
 127*a**(3/2)*b*c*d*x**5/(96*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*d**2*x**7/
(160*sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 23*sqrt(a)*b**2*c*d*x**7/(24*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*d**2
*x**9/(80*sqrt(1 + b*x**2/a)) + 3*a**5*d**2*asinh(sqrt(b)*x/sqrt(a))/(256*b
**(5/2)) - 5*a**4*c*d*asinh(sqrt(b)*x/sqrt(a))/(64*b**(3/2)) + 5*a**3*c**2*
asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + b**3*c**2*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a)) + b**3*c*d*x**9/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d**2*x**11
/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.17603, size = 298, normalized size = 1.24

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{20\,b^{10}cd+21\,ab^9d^2}{b^8}\right)x^2+\frac{80\,b^{10}c^2+340\,ab^9cd+93\,a^2b^8d^2}{b^8}\right)x^2+\frac{5\left(208\,ab^9c^2+236\,a^2b^8cd\right.}{b^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8)*x^2 + (8
0*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 + 5*(208*a*b^9*c^2 +
236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 15*(176*a^2*b^8*c^2 + 20*a^3*b^
7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 - 20*
a^4*b*c*d + 3*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.64 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) dx$

Optimal. Leaf size=149

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}+\cdots$$

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a*d)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(5/2))/(48*b) + (d*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.0522127, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {388, 195, 217, 206}

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a*d)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(5/2))/(48*b) + (d*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)dx = \frac{dx\left(a+bx^2\right)^{7/2}}{8b}-\frac{(-8bc+ad)\int \left(a+bx^2\right)^{5/2}dx}{8b}$$
$$= \frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}+\frac{(5a(8bc-ad))\int \left(a+bx^2\right)^{3/2}dx}{48b}$$
$$= \frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b}+\frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}+\frac{\left(5a^2(8bc-ad)\right)}{64}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b}+\frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b}+\frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b}+\frac{dx\left(a\right.}{}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b}+\frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b}+\frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b}+\frac{dx\left(a\right.}{}$$
$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b}+\frac{5a(8bc-ad)x\left(a+bx^2\right)^{3/2}}{192b}+\frac{(8bc-ad)x\left(a+bx^2\right)^{5/2}}{48b}+\frac{dx\left(a\right.}{}$$

Mathematica [A] time = 0.221856, size = 130, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(2a^2b\left(132c+59dx^2\right)+15a^3d+8ab^2x^2\left(26c+17dx^2\right)+16b^3x^4\left(4c+3dx^2\right)\right)-\frac{15a^{5/2}(ad-8bc)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*d + 16*b^3*x^4*(4*c + 3*d*x^2) + 8*a*b^
2*x^2*(26*c + 17*d*x^2) + 2*a^2*b*(132*c + 59*d*x^2)) - (15*a^(5/2)*(-8*b*c
 + a*d)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(3/2))
```

Maple [A] time = 0.003, size = 166, normalized size = 1.1

$$\frac{dx}{8b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{adx}{48b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5da^2x}{192b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5da^3x}{128b}\sqrt{bx^2+a}-\frac{5da^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{cx}{6}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c),x)
```

```
[Out] 1/8*d*x*(b*x^2+a)^(7/2)/b-1/48*d/b*a*x*(b*x^2+a)^(5/2)-5/192*d/b*a^2*x*(b*x
^2+a)^(3/2)-5/128*d/b*a^3*x*(b*x^2+a)^(1/2)-5/128*d/b^(3/2)*a^4*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))+1/6*c*x*(b*x^2+a)^(5/2)+5/24*c*a*x*(b*x^2+a)^(3/2)+5/16*
c*a^2*x*(b*x^2+a)^(1/2)+5/16*c*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.24254, size = 605, normalized size = 4.06

$$\left[-\frac{15\left(8a^3bc-a^4d\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48b^4dx^7+8\left(8b^4c+17ab^3d\right)x^5+2\left(104ab^3c+59a^2b^2\right.\right.}{768b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*a^3*b*c - a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^
3*c + 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/
b^2, -1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^3*c + 59*a
^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] time = 26.2698, size = 316, normalized size = 2.12

$$\frac{5a^{\frac{7}{2}}dx}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}cx}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}dx^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bcx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bdx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{17\sqrt{a}b^2cx^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}b^2d}{48\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] 5*a**(7/2)*d*x/(128*b*sqrt(1 + b*x**2/a)) + a**(5/2)*c*x*sqrt(1 + b*x**2/a)
/2 + 3*a**(5/2)*c*x/(16*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*d*x**3/(384*sqrt
(1 + b*x**2/a)) + 35*a**(3/2)*b*c*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a**(3/
2)*b*d*x**5/(192*sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c*x**5/(24*sqrt(1 +
b*x**2/a)) + 23*sqrt(a)*b**2*d*x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*d*asin
h(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + 5*a**3*c*asinh(sqrt(b)*x/sqrt(a))/(16
*sqrt(b)) + b**3*c*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.15324, size = 182, normalized size = 1.22

$$\frac{1}{384}\left(2\left(4\left(6\,b^2dx^2+\frac{8\,b^8c+17\,ab^7d}{b^6}\right)x^2+\frac{104\,ab^7c+59\,a^2b^6d}{b^6}\right)x^2+\frac{3\left(88\,a^2b^6c+5\,a^3b^5d\right)}{b^6}\right)\sqrt{bx^2+ax}-\frac{5\left(8\,a^3bc-a^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104*a*b^7*c +
 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d)/b^6)*sqrt(b*x^2 +
a)*x - 5/128*(8*a^3*b*c - a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

3.65 $\int \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] time = 0.023137, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{5/2}\,dx &= \frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2}\,dx\\ &= \frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\,dx\\ &= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+b}}\right.\\ &= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.10306, size = 76, normalized size = 0.9

$$\frac{1}{48}\sqrt{a+bx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+33a^2x+26abx^3+8b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/48
```

Maple [A] time = 0.001, size = 66, normalized size = 0.8

$$\frac{x}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,ax}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2x}{16}\sqrt{bx^2+a}+\frac{5\,a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/24*a*x*(b*x^2+a)^(3/2)+5/16*a^2*x*(b*x^2+a)^(1/2)+5
/16*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63455, size = 347, normalized size = 4.13

$$\left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx\right)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 4.18772, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

Giac [A] time = 1.14066, size = 85, normalized size = 1.01

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

3.66 $\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{8d^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}+\frac{bx\,(a+bx}{4d}$$

```
[Out] -(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(8*d^2) + (b*x*(a + b*x^2)^(3/2))/(4
*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(S
qrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^3)
```

Rubi [A] time = 0.198957, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {416, 528, 523, 217, 206, 377, 208}

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{8d^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}+\frac{bx\,(a+bx}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] -(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(8*d^2) + (b*x*(a + b*x^2)^(3/2))/(4
*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(S
qrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^3)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx = \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\int \frac{\sqrt{a+bx^2}\left(-a(bc-4ad)-b(4bc-7ad)x^2\right)}{c+dx^2}\,dx}{4d}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\int \frac{a\left(4b^2c^2-9abcd+8a^2d^2\right)+b\left(8b^2c^2-20abcd+15a^2d^2\right)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8d^2}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d^3} + \frac{\left(b\left(8b^2c^2-20abcd+1\right.\right.}{8d^3}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} - \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d^3} + \frac{\left(b\left(8b^2\right.\right.}{}$$

$$= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)^{3/2}}{4d} + \frac{\sqrt{b}\left(8b^2c^2-20abcd+15a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3} - \frac{(bc}{}$$

Mathematica [A] time = 0.12079, size = 140, normalized size = 0.89

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+bdx\sqrt{a+bx^2}\left(9ad-4bc+2bdx^2\right)+\frac{8(ad-bc)^{5/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) + (8*(-(b*c) + a*d)^(5/
2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + Sqrt
[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
])/(8*d^3)
```

Maple [B] time = 0.014, size = 3053, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(3/2)*a+1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a^2-1/6/(-c*d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*
a-1/2/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2)*a^2-3/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x-(-c*d)^(1/2)/d))*a*b^2*c^2-3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2
)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2*b*c+3/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/
2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^
(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a*b^2*c^2-1/10/(-c*d)^(1/2)*((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)+1/10/(-c
*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(5/2)-1/4/d^2*b^2*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*c-5/4/d^2*b^(3/2)*ln((-b*(-c*d)^(1/2)/d
+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c*a-1/2/(-c*d)^(1/2)/d^2*((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b^2*c^
2+7/16*b/d*a*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2)*x+7/16*b/d*a*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/6/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)
/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*b*c-1/4/d^
2*b^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)*x*c-5/4/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))/
b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))*c*a+1/2/(-c*d)^(1/2)/d^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b^2*c^2-1/2/(-c*d)^(1/2)/d^3/
((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b^3*c^3+1/6/(-c*d)^(1/2)/
d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(3/2)*b*c+3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2
*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2
)/d))*a^2*b*c-1/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a*b*c+1/2/(-c*d)^(1/2)/d^3/((a*d-b*c)/
d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*
c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b^3*c^3+1/(-c*d)^(1/2)/d*((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a*b*c
+1/8*b/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
```

```
-b*c)/d)^(3/2)*x+15/16/d*b^(1/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/d))
/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2))*a^2+1/2/d^3*b^(5/2)*ln((b*(-c*d)^(1/2)/d+b*(x-(-c*d)^(1/2)/
d))/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))*c^2-1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)
/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(
1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-
c*d)^(1/2)/d))*a^3+1/8*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+15/16/d*b^(1/2)*ln((-b*(-c*d)^(1/2)/d+b*
(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a^2+1/2/d^3*b^(5/2)*ln((-b*(-c*d)^(1/2)/
d+b*(x+(-c*d)^(1/2)/d))/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c^2+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.08999, size = 2022, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a
*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
- 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt
((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*b^2*c
```

```
*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15
*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*
d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, 1/16*(8*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a
*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 +
(a*b*c - a^2*d)*x)) + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d -
9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*
d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 4*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) -
(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

Giac [A] time = 1.1661, size = 290, normalized size = 1.85

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,b^2x^2}{d}-\frac{4\,b^4cd^4-9\,ab^3d^5}{b^2d^6}\right)x-\frac{\left(8\,b^{\frac{5}{2}}c^2-20\,ab^{\frac{3}{2}}cd+15\,a^2\sqrt{b}d^2\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,d^3}+\frac{\left(b^{\frac{7}{2}}c^3-3\,ab^{\frac{5}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*b^2*x^2/d - (4*b^4*c*d^4 - 9*a*b^3*d^5)/(b^2*d^6))*x
 - 1/16*(8*b^(5/2)*c^2 - 20*a*b^(3/2)*c*d + 15*a^2*sqrt(b)*d^2)*log((sqrt(b
```

```
)*x - sqrt(b*x^2 + a))^2)/d^3 + (b^(7/2)*c^3 - 3*a*b^(5/2)*c^2*d + 3*a^2*b^
(3/2)*c*d^2 - a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*
d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*d^3)
```

3.67 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=175

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}(2bc-ad)}{2cd^2}-\frac{x\left(a+bx^2\right)^{3/2}}{2cd\left(c+\right.}$$

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a + b*x^2)^
(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^3)
```

Rubi [A] time = 0.225999, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 528, 523, 217, 206, 377, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}(2bc-ad)}{2cd^2}-\frac{x\left(a+bx^2\right)^{3/2}}{2cd\left(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a + b*x^2)^
(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^3)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+ad)+2b(2bc-ad)x^2\right)}{c+dx^2}\,dx}{2cd}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{-2a\left(2b^2c^2-2abcd-a^2d^2\right)-2b^2c(4bc-5ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{4cd^2}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{\left(b^2(4bc-5ad)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^3} + \frac{\left((bc-ad)^2(4bc-\right.}{}$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{\left(b^2(4bc-5ad)\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2d^3} +$$

$$= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{2cd\left(c+dx^2\right)} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(bc-ad)^{3/2}(}{}$$

Mathematica [A] time = 0.160223, size = 144, normalized size = 0.82

$$\frac{dx\sqrt{a+bx^2}\left(\frac{(bc-ad)^2}{c\left(c+dx^2\right)}+b^2\right)+b^{3/2}(-(4bc-5ad))\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{(ad-bc)^{3/2}(ad+4bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(b^2 + (b*c - a*d)^2/(c*(c + d*x^2))) + ((-(b*c) + a*d
)^(3/2)*(4*b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2
])])/c^(3/2) - b^(3/2)*(4*b*c - 5*a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/
(2*d^3)
```

Maple [B] time = 0.02, size = 7345, normalized size = 42.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

Fricas [A] time = 6.33458, size = 2584, normalized size = 14.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*b^2*c^3 - 3*a*b*c^2*
d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)
/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3
*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b
*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/8*(
4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*
b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(
a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*b^2*c^3 - 3*a*b
*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-(b*c
```

```
- a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c -
 a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/4*(2*(4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2
 - a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)
*x)) + 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x
^2 + a))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

Giac [B] time = 1.23954, size = 547, normalized size = 3.13

$$\frac{\sqrt{bx^2+a}\,ab^2x}{2\,d^2}+\frac{\left(4\,b^{\frac{5}{2}}c-5\,ab^{\frac{3}{2}}d\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,d^3}-\frac{\left(4\,b^{\frac{7}{2}}c^3-7\,ab^{\frac{5}{2}}c^2d+2\,a^2b^{\frac{3}{2}}cd^2+a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{}\right.}{2}\right.}{2\sqrt{-b^2c^2+abcd}\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x/d^2 + 1/4*(4*b^(5/2)*c - 5*a*b^(3/2)*d)*log((sqrt
(b)*x - sqrt(b*x^2 + a))^2)/d^3 - 1/2*(4*b^(7/2)*c^3 - 7*a*b^(5/2)*c^2*d +
2*a^2*b^(3/2)*c*d^2 + a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
```

```
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*c*d^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2)*c^3 - 5*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*b^(5/2)*c^2*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*
b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^3 + a^2*b^(5/
2)*c^2*d - 2*a^3*b^(3/2)*c*d^2 + a^4*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)*c*d^3)
```

3.68 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-\frac{x\left(a\right.}{}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(3/2))/(4*c*d*(c + d*x^2)^2) - ((b*c - a*d)*(4*
b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2)) + (b^(5/2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c - a*d]*(8*b^2*c^2 + 4*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^
(5/2)*d^3)
```

Rubi [A] time = 0.194419, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 526, 523, 217, 206, 377, 208}

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-\frac{x\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(3/2))/(4*c*d*(c + d*x^2)^2) - ((b*c - a*d)*(4*
b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2)) + (b^(5/2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c - a*d]*(8*b^2*c^2 + 4*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^
(5/2)*d^3)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+3ad)+4b^2cx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}-\frac{\int \frac{-a\left(4b^2c^2+ad(bc+3ad)\right)-8b^3c^2x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^3}-\frac{\left((bc-ad)\left(8b^2c^2+\right.\right.}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{\left(\left(\right.\right.}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{\sqrt{bc-ad}\left(8b\right.}{}$$

Mathematica [A] time = 0.193342, size = 184, normalized size = 0.95

$$\frac{\frac{\left(a^2bcd^2+3a^3d^3+4ab^2c^2d-8b^3c^3\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}\sqrt{ad-bc}}+8b^{5/2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{dx\sqrt{a+bx^2}(ad-bc)\left(ad\left(5c+3dx^2\right)+2bc\left(2c+3dx^2\right)\right)}{c^2\left(c+dx^2\right)^2}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] ((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + 3*d*x^2) + a*d*(5*c + 3*
d*x^2)))/(c^2*(c + d*x^2)^2) + ((-8*b^3*c^3 + 4*a*b^2*c^2*d + a^2*b*c*d^2 +
 3*a^3*d^3)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5
/2)*Sqrt[-(b*c) + a*d]) + 8*b^(5/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*
d^3)
```

Maple [B] time = 0.026, size = 14133, normalized size = 72.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

Fricas [B] time = 4.26576, size = 3125, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2
*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a
*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x +
 (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c
*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^
3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d
^4*x^2 + c^4*d^3), -1/32*(32*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^2*c^4 + 4*a*b*c^3*d + 3*
a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*
d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*
```

```
c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^
4 + 2*c*d*x^2 + c^2)) + 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4
*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 +
2*c^3*d^4*x^2 + c^4*d^3), 1/16*((8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 +
(8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*
d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
+ a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c -
 a^2*d)*x)) + 8*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^
3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2
+ a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), -1/16*(16*(b^2*c^2*d^2*x^4 +
 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*
a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*
c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c
 - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + 2*(3*(2*b^2*c^2*d^2
 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*
sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\left(c+dx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**3, x)
```

Giac [B] time = 1.23433, size = 890, normalized size = 4.59

$$-\frac{b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^3}+\frac{\left(8\,b^{\frac{7}{2}}c^3-4\,ab^{\frac{5}{2}}c^2d-a^2b^{\frac{3}{2}}cd^2-3\,a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\sqrt{-b^2c^2+abcd}\,c^2d^3}-\frac{16\left(\sqrt{b}x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + 1/8*(8*b^(7/2)*c^3
- 4*a*b^(5/2)*c^2*d - a^2*b^(3/2)*c*d^2 - 3*a^3*sqrt(b)*d^3)*arctan(1/2*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(s
qrt(-b^2*c^2 + a*b*c*d)*c^2*d^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
b^(7/2)*c^3*d - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2 + (sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^3*sqrt(b)*d^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(9/2)*c^4 - 7
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c^3*d + 18*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2)*c^2*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*d^4 + 32*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c^3*d - 28*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^3*b^(5/2)*c^2*d^2 - 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*
c*d^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*d^4 + 6*a^4*b^(5/2)*c
^2*d^2 - 3*a^5*b^(3/2)*c*d^3 - 3*a^6*sqrt(b)*d^4)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)^2*c^2*d^3)
```

3.69 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4} dx$

Optimal. Leaf size=144

$$\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5a^3\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3/2))/(24*c
^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(c + d*x^2)) + (5*a^3
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*Sqrt[b
*c - a*d])
```

Rubi [A] time = 0.0732243, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 208}

$$\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5a^3\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3/2))/(24*c
^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(c + d*x^2)) + (5*a^3
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*Sqrt[b
*c - a*d])
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4}\,dx = \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}+\frac{(5a)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{6c}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{\left(5a^2\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx}{8c^2}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{16c^3}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16c^3}$$

$$= \frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.777666, size = 201, normalized size = 1.4

$$\frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\left(a^2\left(33c^2+40cdx^2+15d^2x^4\right)+2abcx^2\left(13c+5dx^2\right)+8b^2c^2x^4\right)}{\left(c+dx^2\right)^2\sqrt{\frac{dx^2}{c}+1}}+\frac{15a^2\sin^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{48c^4\sqrt{\frac{bx^2}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(8*b^2*c^2*x^4 +
 2*a*b*c*x^2*(13*c + 5*d*x^2) + a^2*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4)))/((
c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]) + (15*a^2*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]
/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(48*c^4*Sqrt[1 +
(b*x^2)/a])
```

Maple [B] time = 0.033, size = 21220, normalized size = 147.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

Fricas [B] time = 3.20487, size = 1438, normalized size = 9.99

$$\left[\frac{15\left(a^3d^3x^6+3\,a^3cd^2x^4+3\,a^3c^2dx^2+a^3c^3\right)\sqrt{bc^2-acd}\,\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left(\left(2\,bc-ad\right)x^3+acx\right)\sqrt{bc^2-}}{d^2x^4+2\,cdx^2+c^2}\right.}{192\left(bc^8-ac^7d+\left(bc^5d^3-ac^4d^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt
(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4 + 2*a*b^2*
c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3
*d - 20*a^3*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(
b*c^8 - a*c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x
^4 + 3*(b*c^7*d - a*c^6*d^2)*x^2), -1/96*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4
 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2
 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3*d - 20*a^3*c^2*d^2)*x^
3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(b*c^8 - a*c^7*d + (b*c^
5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x^4 + 3*(b*c^7*d - a*c^6
*d^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 22.0103, size = 1142, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -5/16*a^3*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a
*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^3) + 1/24*(48*(sq
```

rt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 288*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 - 420*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 + 8*a^6*b^(5/2)*c^2*d^3 + 10*a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3*c^3*d^3)

3.70 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx$

Optimal. Leaf size=249

$$\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4\left(c+dx^2\right)(bc-ad)}+\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-7ad)}{48c^2\left(c+dx^2\right)^3(bc-ad)}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-7a}{192c^3\left(c+dx^2\right)^2(bc-ad}$$

```
[Out] -(d*x*(a + b*x^2)^(7/2))/(8*c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a*d)
*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3) + (5*a*(8*b*c - 7*
a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d)*(c + d*x^2)^2) + (5*a^2*(8*b
*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3*(
8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128
*c^(9/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.137348, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {382, 378, 377, 208}

$$\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4\left(c+dx^2\right)(bc-ad)}+\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-7ad)}{48c^2\left(c+dx^2\right)^3(bc-ad)}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-7a}{192c^3\left(c+dx^2\right)^2(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] -(d*x*(a + b*x^2)^(7/2))/(8*c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a*d)
*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3) + (5*a*(8*b*c - 7*
a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d)*(c + d*x^2)^2) + (5*a^2*(8*b
*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3*(
8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128
*c^(9/2)*(b*c - a*d)^(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5}\,dx = -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4}\,dx}{8c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{(5a(8bc-7ad))\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3}\,dx}{48c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{\left(5a^2(8bc-7a\right.}{64c^3(b}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2(8bc-7ad}{128c^4(bc-ad}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2(8bc-7ad}{128c^4(bc-ad}$$

$$= -\frac{dx\left(a+bx^2\right)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x\left(a+bx^2\right)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x\left(a+bx^2\right)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{5a^2(8bc-7ad}{128c^4(bc-ad}$$

Mathematica [A] time = 1.00828, size = 306, normalized size = 1.23

$$\frac{x\left(\frac{15a^3\left(c+dx^2\right)^4(7ad-8bc)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}} - c\left(2a^2b^2cx^2\left(173c^2dx^2+236c^3+106cd^2x^4+25d^3x^6\right)+a^3b\left(-323c^2d^2x^4-21c^3d\right.\right.\right.}{384c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] (x*(-(c*(16*b^4*c^3*x^6*(4*c + d*x^2) + 8*a*b^3*c^2*x^4*(34*c^2 + 13*c*d*x^
2 + 3*d^2*x^4) + 2*a^2*b^2*c*x^2*(236*c^3 + 173*c^2*d*x^2 + 106*c*d^2*x^4 +
 25*d^3*x^6) - a^4*d*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6
) + a^3*b*(264*c^4 - 21*c^3*d*x^2 - 323*c^2*d^2*x^4 - 335*c*d^3*x^6 - 105*d
^4*x^8))) + (15*a^3*(-8*b*c + 7*a*d)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d
```

```
)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^
5*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^4)
```

Maple [B] time = 0.048, size = 28625, normalized size = 115.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

Fricas [B] time = 8.06962, size = 2612, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 +
4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)
*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
```

```
 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c
^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^
5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208
*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 +
3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d + 93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(
b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2 + (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*
c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c
^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 +
 a^2*c^8*d^3)*x^2), -1/768*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4
- 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d
^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(-b*c^
2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b
*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^4*c
^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c
*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*
c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^
3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 + 3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d +
93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2
+ (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*
c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*
x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 + a^2*c^8*d^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.8351, size = 1955, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -5/128*(8*a^3*b^(3/2)*c - 7*a^4*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^5 - a*c^4*d)*s
qrt(-b^2*c^2 + a*b*c*d)) - 1/192*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*
b^(3/2)*c*d^6 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d^7 - 768*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(11/2)*c^5*d^2 + 768*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2)*c^4*d^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3
*b^(5/2)*c^2*d^5 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6
 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*b^(13/2)*c^6*d + 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a^2*b^(9/2)*c^4*d^3 + 8320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3
*d^4 - 15600*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^5 + 9800*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/
2)*c^7 + 1024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*d - 4864*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^2 + 21888*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 - 38000*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^
4*b^(7/2)*c^3*d^4 + 37400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d
^5 - 18550*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*d^6 + 3675*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a^2*b^(13/2)*c^6*d - 9472*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(9/2)*
c^4*d^3 + 32896*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2)*c^3*d^4 - 35376
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*d^5 + 18200*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^8*sqrt(b)*d^7 - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(11/2)*c^5*d^2
 - 1536*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2)*c^4*d^3 - 2944*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 + 12528*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^7*b^(5/2)*c^2*d^5 - 9170*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^8*b^(3
/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*sqrt(b)*d^7 - 256*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^3 - 256*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^7*b^(7/2)*c^3*d^4 - 608*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^8*b^
(5/2)*c^2*d^5 + 1960*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b^(3/2)*c*d^6 - 73
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b)*d^7 - 16*a^8*b^(7/2)*c^3*d^4
 - 24*a^9*b^(5/2)*c^2*d^5 - 50*a^10*b^(3/2)*c*d^6 + 105*a^11*sqrt(b)*d^7)/(
(b*c^5*d^3 - a*c^4*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx$

Optimal. Leaf size=30

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0143253, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {402, 216, 377, 203}

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(1 + x^2),x]
```

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx &= 2\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx-\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &=-\sin^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\ &=-\sin^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0276604, size = 30, normalized size = 1.

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(1 + x^2),x]
```

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Maple [A] time = 0.011, size = 33, normalized size = 1.1

$$-\arcsin(x)-\sqrt{2}\arctan\left(\frac{x\sqrt{2}}{x^2-1}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2+1),x)
```

```
[Out] -arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

Fricas [A] time = 1.51973, size = 111, normalized size = 3.7

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,x}\right)+2\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) + 2*arctan((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

Giac [B] time = 1.21887, size = 128, normalized size = 4.27

$$-\frac{1}{2}\pi\operatorname{sgn}(x)+\frac{1}{2}\sqrt{2}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x
^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arctan(-1/2*x*((sqrt(-x^2
+ 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

```
[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0139642, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {402, 215, 377, 207}

$$\sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx &= 2\int \frac{1}{\left(-1+x^2\right)\sqrt{1+x^2}}\,dx+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \sinh^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right)\\ &= \sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0246694, size = 64, normalized size = 2.37

$$\frac{\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)-\log\left(\sqrt{2}\sqrt{x^2+1}+x+1\right)+\log(1-x)-\log(x+1)}{\sqrt{2}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] + (Log[1 - x] - Log[1 + x] + Log[1 - x + Sqrt[2]*Sqrt[1 + x^2]]
- Log[1 + x + Sqrt[2]*Sqrt[1 + x^2]])/Sqrt[2]
```

Maple [B] time = 0.008, size = 84, normalized size = 3.1

$$-\frac{1}{2}\sqrt{(1+x)^2-2\,x}+\text{Arcsinh}\,(x)+\frac{\sqrt{2}}{2}\text{Artanh}\left(\frac{(-2\,x+2)\sqrt{2}}{4}\frac{1}{\sqrt{(1+x)^2-2\,x}}\right)+\frac{1}{2}\sqrt{(-1+x)^2+2\,x}-\frac{\sqrt{2}}{2}\text{Artanh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^2-1),x)
```

```
[Out] -1/2*((1+x)^2-2*x)^(1/2)+arcsinh(x)+1/2*2^(1/2)*arctanh(1/4*(-2*x+2)*2^(1/2
)/((1+x)^2-2*x)^(1/2))+1/2*((-1+x)^2+2*x)^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2+
2*x)*2^(1/2)/((-1+x)^2+2*x)^(1/2))
```

Maxima [B] time = 1.4827, size = 80, normalized size = 2.96

$$-\frac{1}{2}\sqrt{2}\operatorname{arsinh}\left(\frac{2x}{|2x+2|}-\frac{2}{|2x+2|}\right)-\frac{1}{2}\sqrt{2}\operatorname{arsinh}\left(\frac{2x}{|2x-2|}+\frac{2}{|2x-2|}\right)+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqrt(2)*arcsi
nh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**2-1),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

Giac [B] time = 1.19632, size = 95, normalized size = 3.52

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|2\left(x-\sqrt{x^2+1}\right)^2-4\sqrt{2}-6\right|}{\left|2\left(x-\sqrt{x^2+1}\right)^2+4\sqrt{2}-6\right|}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(2*(x - sq
rt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - log(-x + sqrt(x^2 + 1))
```

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rubi [A] time = 0.0128449, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {402, 216, 377, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\left(-1+2x^2\right)}\,dx\\ &= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\ &= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0101558, size = 25, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Maple [B] time = 0.029, size = 187, normalized size = 7.5

$$-\frac{\sqrt{2}}{2}\left(\frac{1}{4}\sqrt{-4\,\left(x+1/2\,\sqrt{2}\right)^{2}+4\,\left(x+1/2\,\sqrt{2}\right)\sqrt{2}+2}+\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}}{4}\operatorname{Artanh}\left(\sqrt{2}\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\frac{1}{\sqrt{-4\,(x+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(2*x^2-1),x)
```

```
[Out] -1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)+
1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/
2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))+1/2*2^(1/2)*(
```

```
1/4*(-4*(x-1/2*2^(1/2))^2-4*2^(1/2)*(x-1/2*2^(1/2))+2)^(1/2)-1/4*2^(1/2)*ar
csin(x)-1/4*2^(1/2)*arctanh((-2^(1/2)*(x-1/2*2^(1/2))+1)*2^(1/2)/(-4*(x-1/2
*2^(1/2))^2-4*2^(1/2)*(x-1/2*2^(1/2))+2)^(1/2)))
```

Maxima [B] time = 1.51138, size = 149, normalized size = 5.96

$$-\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin\left(x\right)-\sqrt{2}\log\left(\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{\left|4x+2\sqrt{2}\right|}+\frac{1}{\left|4x+2\sqrt{2}\right|}\right)+\sqrt{2}\log\left(-\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^2+1}}{\left|4x-2\sqrt{2}\right|}+\frac{1}{\left|4x-2\sqrt{2}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqrt(2)*sqrt(
-x^2 + 1)/abs(4*x + 2*sqrt(2)) + 1/abs(4*x + 2*sqrt(2))) + sqrt(2)*log(-1/4
*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x - 2*sqrt(2)) + 1/abs(4*x - 2*sqrt
(2))))
```

Fricas [B] time = 1.56759, size = 192, normalized size = 7.68

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) - x
 - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)
```

Giac [B] time = 1.14134, size = 159, normalized size = 6.36

$$-\frac{1}{4}\,\pi\mathrm{sgn}\left(x\right)-\frac{1}{2}\,\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)-\frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)
/x + 2)) + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2
))
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=169

$$\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2}$$

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*b^3) + (5*
d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rubi [A] time = 0.144695, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 528, 388, 217, 206}

$$\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```



```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*b^3) + (5*
d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b} + \frac{\int \frac{\left(c+dx^2\right)\left(c(6bc-ad)+5d(2bc-ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b} + \frac{\int \frac{c\left(24b^2c^2-14abcd+5a^2d^2\right)+d\left(44b^2c^2-44abcd+1\right.}{\sqrt{a+bx^2}}}{24b^2}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}\left(c+dx\right.}{6b}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}\left(c+dx\right.}{6b}$$

$$= \frac{d\left(44b^2c^2-44abcd+15a^2d^2\right)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24b^2} + \frac{dx\sqrt{a+bx^2}\left(c+dx\right.}{6b}$$

Mathematica [A] time = 5.09434, size = 140, normalized size = 0.83

$$\frac{\sqrt{b}dx\sqrt{a+bx^2}\left(15a^2d^2-2abd\left(27c+5dx^2\right)+4b^2\left(18c^2+9cdx^2+2d^2x^4\right)\right)+3\left(18a^2bcd^2-5a^3d^3-24ab^2c^2d+16b^3c^3\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^2) + 4*b^2
*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + 3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Maple [A] time = 0.012, size = 228, normalized size = 1.4

$$\frac{d^3x^5}{6b}\sqrt{bx^2+a}-\frac{5ad^3x^3}{24b^2}\sqrt{bx^2+a}+\frac{5d^3a^2x}{16b^3}\sqrt{bx^2+a}-\frac{5a^3d^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{3cd^2x^3}{4b}\sqrt{bx^2+a}-\frac{9cd^2ax}{8b^2}\sqrt{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*d^3*x^5/b*(b*x^2+a)^(1/2)-5/24*d^3/b^2*a*x^3*(b*x^2+a)^(1/2)+5/16*d^3/b
^3*a^2*x*(b*x^2+a)^(1/2)-5/16*d^3/b^(7/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
+3/4*c*d^2*x^3/b*(b*x^2+a)^(1/2)-9/8*c*d^2/b^2*a*x*(b*x^2+a)^(1/2)+9/8*c*d^
2/b^(5/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/2*c^2*d*x/b*(b*x^2+a)^(1/2)-3
/2*c^2*d*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c^3*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8594, size = 675, normalized size = 3.99

$$\left[-\frac{3\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,b^3d^3x^5+2\left(18\,b^3cd^2-5\,ab^2\right.\right.}{96\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b
)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*d^3*x^5 + 2*(1
8*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2*c*d^2 + 5*a^2*b
*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a
^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^
3*d^3*x^5 + 2*(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2
*c*d^2 + 5*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] time = 11.9673, size = 400, normalized size = 2.37

$$\frac{5a^{\frac{5}{2}}d^3x}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{9a^{\frac{3}{2}}cd^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}d^3x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}c^2dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{3\sqrt{a}cd^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}d^3x^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{16b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*d**3*x/(16*b**3*sqrt(1 + b*x**2/a)) - 9*a**(3/2)*c*d**2*x/(8*b**
2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d**3*x**3/(48*b**2*sqrt(1 + b*x**2/a)) +
 3*sqrt(a)*c**2*d*x*sqrt(1 + b*x**2/a)/(2*b) - 3*sqrt(a)*c*d**2*x**3/(8*b*s
qrt(1 + b*x**2/a)) - sqrt(a)*d**3*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*d
**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 9*a**2*c*d**2*asinh(sqrt(b)*x/
sqrt(a))/(8*b**(5/2)) - 3*a*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) +
c**3*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
```

```
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + 3*c*d**2*x**5/(4*sqrt(a)*sq
rt(1 + b*x**2/a)) + d**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.16584, size = 203, normalized size = 1.2

$$\frac{1}{48}\left(2\left(\frac{4\,d^3x^2}{b}+\frac{18\,b^4cd^2-5\,ab^3d^3}{b^5}\right)x^2+\frac{3\left(24\,b^4c^2d-18\,ab^3cd^2+5\,a^2b^2d^3\right)}{b^5}\right)\sqrt{bx^2+ax}-\frac{\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*(24*b^4*c^
2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3
*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(7/2)
```

3.75 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=108

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}$$

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x^2]*(c + d
*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.0556046, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 217, 206}

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x^2]*(c + d
*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{\sqrt{a+bx^2}}\,dx &= \frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}+\frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x^2}{\sqrt{a+bx^2}}\,dx}{4b}\\ &= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}-\frac{(3ad(2bc-ad)-2bc(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}\\ &= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}-\frac{(3ad(2bc-ad)-2bc(4bc-ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{8b^2}\\ &= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [C] time = 2.35633, size = 160, normalized size = 1.48

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\left(-2bx^2\left(c+dx^2\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)-4bx^2\left(2c^2+3cdx^2+d^2x^4\right){}_2F_1\left(\frac{3}{2},\frac{3}{2};\frac{9}{2};-\right.\right.}{105a\sqrt{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[1 + (b*x^2)/a]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometri
c2F1[1/2, 1/2, 7/2, -((b*x^2)/a)] - 4*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*H
ypergeometric2F1[3/2, 3/2, 9/2, -((b*x^2)/a)] - 2*b*x^2*(c + d*x^2)^2*Hyper
```

```
geometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a*Sqrt[a + b*x^2
])
```

Maple [A] time = 0.006, size = 131, normalized size = 1.2

$$\frac{d^2x^3}{4b}\sqrt{bx^2+a}-\frac{3\,ad^2x}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2d^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{b}\sqrt{bx^2+a}-cda\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*d^2*x^3/b*(b*x^2+a)^(1/2)-3/8*d^2/b^2*a*x*(b*x^2+a)^(1/2)+3/8*d^2/b^(5/
2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c*d*x/b*(b*x^2+a)^(1/2)-c*d*a/b^(3/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66686, size = 440, normalized size = 4.07

$$\left[\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2d^2x^3+\left(8\,b^2cd-3\,abd^2\right)x\right)\sqrt{bx^2+a}}{16\,b^3},-\frac{\left(8\,b^2c^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)*sqr
t(b*x^2 + a))/b^3, -1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)
*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 6.26663, size = 238, normalized size = 2.2

$$-\frac{3a^{\frac{3}{2}}d^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cdx\sqrt{1+\frac{bx^2}{a}}}{b}-\frac{\sqrt{a}d^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{acd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+c^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a \\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a \\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*d**2*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*d*x*sqrt(1 + b*x
**2/a)/b - sqrt(a)*d**2*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*d**2*asinh(s
qrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*c*d*asinh(sqrt(b)*x/sqrt(a))/b**(3/2) +
c**2*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)),
(sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acos
h(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + d**2*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a))
```

Giac [A] time = 1.12174, size = 122, normalized size = 1.13

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^2x^2}{b}+\frac{8\,b^2cd-3\,abd^2}{b^3}\right)x-\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x - 1/8*(8*
b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
```

5/2)

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.0171933, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 217, 206}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.0203176, size = 57, normalized size = 0.98

$$\frac{dx\sqrt{a+bx^2}}{2b} - \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) - ((-2*b*c + a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.004, size = 62, normalized size = 1.1

$$\frac{dx}{2\,b}\sqrt{bx^2+a} - \frac{ad}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}} + c\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*d*x*(b*x^2+a)^(1/2)/b-1/2*d*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60352, size = 275, normalized size = 4.74

$$\left[\frac{2\sqrt{bx^2+a}bdx - (2bc-ad)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4b^2}, \frac{\sqrt{bx^2+a}bdx - (2bc-ad)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] time = 2.50644, size = 126, normalized size = 2.17

$$\frac{\sqrt{a}dx\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0 \wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*sqrt(1 + b*x**2/a)/(2*b) - a*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
```

Giac [A] time = 1.12063, size = 66, normalized size = 1.14

$$\frac{\sqrt{bx^2+a}dx}{2b}-\frac{(2bc-ad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0062912, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0046142, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.002, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57856, size = 153, normalized size = 6.12

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 0.992651, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A] time = 1.16954, size = 31, normalized size = 1.24

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

Rubi [A] time = 0.0215087, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0183035, size = 49, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c - a*d])
```

Maple [B] time = 0.013, size = 300, normalized size = 6.1

$$-\frac{1}{2}\ln\left(\left(2\frac{ad-bc}{d}+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}\right)\left(x-\frac{1}{d}\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] -1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.9915, size = 513, normalized size = 10.47

$$\left[\frac{\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\sqrt{bc^2-acd}}, -\frac{\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2\,bc-ad)x^2\right.}{2\left(\left(b^2c^2-abcd\right)x^3+\left(abc\right.\right.}\right.}{2\left(bc^2-acd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), -1/2*sqrt(-b*c^2
 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^2 - a*c*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

Giac [A] time = 1.14349, size = 95, normalized size = 1.94

$$-\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(
-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=101

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*ArcTa
nh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(
3/2))
```

Rubi [A] time = 0.0489142, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*ArcTa
nh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(
3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.286529, size = 126, normalized size = 1.25

$$
\frac{x\left(\frac{\left(c+dx^2\right)(2bc-ad)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{c\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}-d\left(a+bx^2\right)\right)}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] (x*(-(d*(a + b*x^2)) + ((2*b*c - a*d)*(c + d*x^2)*ArcTanh[Sqrt[((b*c - a*d)
*x^2)/(c*(a + b*x^2))]])/(c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))])))/(2*c
*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2))
```

Maple [B] time = 0.015, size = 809, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/(x+(-c*d)^
(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)+1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(
a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
))/(x+(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)
^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-
(-c*d)^(1/2)/d))+1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)
^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

Fricas [B] time = 2.7679, size = 957, normalized size = 9.48

$$\left[-\frac{4\left(bc^2d-acd^2\right)\sqrt{bx^2+a}x-\left(2\,bc^2-acd+\left(2\,bcd-ad^2\right)x^2\right)\sqrt{bc^2-acd}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4}{d^2x^4+2\,cdx^2+c^2}\right.}{8\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 - a*c*d + (2*b*c*
d - a*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b^2*
c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)
*x^2), -1/4*(2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x + (2*b*c^2 - a*c*d + (
2*b*c*d - a*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*
((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c
^2 - a^2*c*d)*x)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.18589, size = 327, normalized size = 3.24

$$\frac{1}{2} b^{\frac{3}{2}} \Bigg(\frac{(2\,bc - ad)\arctan\Big(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\Big)}{\left(b^2c^2 - abcd\right)\sqrt{-b^2c^2 + abcd}} - \frac{2\Big(2\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 bc - \left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 ad \cdots}{\Big(\Big(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 d + 4\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 bc - 2\left(\sqrt{b}x - \sqrt{bx^2 + a} \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*((2*b*c - a*d)*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d +
 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b*c*d)*sqrt(-b^2*c^2
+ a*b*c*d)) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - (sqrt(b)*x - sqrt(
```

```
b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d
)*(b^2*c^2 - a*b*c*d)))
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*b*c - a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) + ((8*b^2*c^2 - 8*a*b
*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(
8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.119151, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*b*c - a*d)
*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) + ((8*b^2*c^2 - 8*a*b
*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(
8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx = -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-2bdx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-8abcd+3a^2d^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right.}}{8c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\text{Subst}\left(\int \frac{}{c-(b}\right.}{8c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{bc-}}{\sqrt{c}\sqrt{a}}\right.}{8c^{5/2}(bc-ad)^{5/2}}$$

Mathematica [A] time = 0.622278, size = 192, normalized size = 1.18

$$\frac{x\left(\frac{\left(c+dx^2\right)^2\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}}-cd\left(a^2(-d)\left(5c+3dx^2\right)+ab\left(8c^2+cdx^2-3d^2x^4\right)+2b^2cx^2\left(4c+3dx^2\right)\right)\right)}{8c^3\sqrt{a+bx^2}\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] (x*(-(c*d*(2*b^2*c*x^2*(4*c + 3*d*x^2) - a^2*d*(5*c + 3*d*x^2) + a*b*(8*c^2
 + c*d*x^2 - 3*d^2*x^4))) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*(c + d*x^2
)^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))]))/(8*c^3*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^2)
```

Maple [B] time = 0.02, size = 1815, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c^2/d*b*(-c*d)^(1/2)/(a*d-
b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/16/(-c*d)^(1/2)/c/
(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d
)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2)+3/16/(-c*d)^(1/2)*b^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+
(-c*d)^(1/2)/d))+1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*
(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x+(-c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/c^
2/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b
-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/
d))-3/16/(-c*d)^(1/2)/c^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))
+3/16/(-c*d)^(1/2)/c^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+1/
16/(-c*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2*((x-(-c*d)^(1/2)/d)^2*b+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c*b/(a*d-b*c)^2
/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2)*b^2/(a*d-b*c)^2/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-b*
c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d
-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

Fricas [B] time = 4.63851, size = 1750, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^3*d^2 -
 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c^3*d^2 + 5*a^2*c^2
*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*
c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x
^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x^2),
-1/16*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*
c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(
3*(2*b^2*c^3*d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c
^3*d^2 + 5*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^
2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^
4 - a^3*c^3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a
^3*c^4*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 3.54621, size = 726, normalized size = 4.45

$$-\frac{1}{8}b^{\frac{5}{2}}\left(\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{2\left(8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^2c^2d-8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sqrt(b)*x -
sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^4 - 2
*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(8*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d
^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*b^3*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 42*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d - 40*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*d
^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2
)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*
c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

3.81 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}-\frac{dx\sqrt{a+bx^2}\left(290a^2bcd^2-105a^3d^3-248ab^2c^2d+48b^3c^3\right)}{48ab^4}+\frac{d\,(12}{}$$

```
[Out] -(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*Sqrt[a
 + b*x^2])/(48*a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*(c
+ d*x^2)^2)/(6*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2])
 + (d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rubi [A] time = 0.259272, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {413, 528, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}-\frac{dx\sqrt{a+bx^2}\left(290a^2bcd^2-105a^3d^3-248ab^2c^2d+48b^3c^3\right)}{48ab^4}+\frac{d\,(12}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*Sqrt[a
 + b*x^2])/(48*a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*(c
+ d*x^2)^2)/(6*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2])
 + (d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)^2\left(acd-d(6bc-7ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(6bc-7ad)x\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)\left(acd(12bc-7ad)-d\left(24b^2c^2-64abcd+3\right.\right.}{\sqrt{a+bx^2}}}{6ab^2}$$

$$= -\frac{d\left(24b^2c^2-64abcd+35a^2d^2\right)x\sqrt{a+bx^2}\left(c+dx^2\right)}{24ab^3} - \frac{d(6bc-7ad)x\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6ab^2} + \frac{(bc-a}{ab}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a^2d^2\right)x\sqrt{a+}}{24ab^3}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a^2d^2\right)x\sqrt{a+}}{24ab^3}$$

$$= -\frac{d\left(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3\right)x\sqrt{a+bx^2}}{48ab^4} - \frac{d\left(24b^2c^2-64abcd+35a^2d^2\right)x\sqrt{a+}}{24ab^3}$$

Mathematica [A] time = 5.19608, size = 172, normalized size = 0.67

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(3d^2\left(19a^2d^2-56abcd+48b^2c^2\right)+2bd^3x^2(24bc-11ad)+\frac{48(bc-ad)^4}{a(a+bx^2)}+8b^2d^4x^4\right)+3d\left(120a^2bcd^2-35a^3d^3\right.}{48b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(3*d^2*(48*b^2*c^2 - 56*a*b*c*d + 19*a^2*d^2) +
2*b*d^3*(24*b*c - 11*a*d)*x^2 + 8*b^2*d^4*x^4 + (48*(b*c - a*d)^4)/(a*(a +
b*x^2))) + 3*d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3
)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(9/2))
```

Maple [A] time = 0.017, size = 340, normalized size = 1.3

$$\frac{d^4x^7}{6\,b}\frac{1}{\sqrt{bx^2+a}} - \frac{7\,ad^4x^5}{24\,b^2}\frac{1}{\sqrt{bx^2+a}} + \frac{35\,d^4a^2x^3}{48\,b^3}\frac{1}{\sqrt{bx^2+a}} + \frac{35\,a^3d^4x}{16\,b^4}\frac{1}{\sqrt{bx^2+a}} - \frac{35\,a^3d^4}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}} + \frac{cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*d^4*x^7/b/(b*x^2+a)^(1/2)-7/24*d^4/b^2*a*x^5/(b*x^2+a)^(1/2)+35/48*d^4/
b^3*a^2*x^3/(b*x^2+a)^(1/2)+35/16*d^4/b^4*a^3*x/(b*x^2+a)^(1/2)-35/16*d^4/b
^(9/2)*a^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c*d^3*x^5/b/(b*x^2+a)^(1/2)-5/2*c*
d^3/b^2*a*x^3/(b*x^2+a)^(1/2)-15/2*c*d^3/b^3*a^2*x/(b*x^2+a)^(1/2)+15/2*c*d
^3/b^(7/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3*c^2*d^2*x^3/b/(b*x^2+a)^(1/2
)+9*c^2*d^2/b^2*a*x/(b*x^2+a)^(1/2)-9*c^2*d^2/b^(5/2)*a*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))-4*c^3*d*x/b/(b*x^2+a)^(1/2)+4*c^3*d/b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))+c^4*x/a/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32665, size = 1272, normalized size = 4.95

$$\left[-\frac{3\left(64\,a^2b^3c^3d-144\,a^3b^2c^2d^2+120\,a^4bcd^3-35\,a^5d^4+\left(64\,ab^4c^3d-144\,a^2b^3c^2d^2+120\,a^3b^2cd^3-35\,a^4bd^4\right)x^2\right)\sqrt{b}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^
5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*
b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*
a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a*b^4*c^2*d^2
 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64*a*b^4*c^3*d
 + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*sqrt(b*x^2 +
```

```
a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2
 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 1
20*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a
*b^4*c^2*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64
*a*b^4*c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*s
qrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

Giac [A] time = 1.15544, size = 317, normalized size = 1.23

$$\frac{\left(\left(2\left(\frac{4d^4x^2}{b}+\frac{24ab^6cd^3-7a^2b^5d^4}{ab^7}\right)x^2+\frac{144ab^6c^2d^2-120a^2b^5cd^3+35a^3b^4d^4}{ab^7}\right)x^2+\frac{3\left(16b^7c^4-64ab^6c^3d+144a^2b^5c^2d^2-120a^3b^4cd^3+35a^4b^3d^4\right)}{ab^7}\right)x}{48\sqrt{bx^2+a}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))*x^2 + (14
4*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(a*b^7))*x^2 + 3*(16*
b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^2 - 120*a^3*b^4*c*d^3 + 35*a^4
*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c^2*d
^2 + 120*a^2*b*c*d^3 - 35*a^3*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

3.82 $\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=169

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)(4bc-5ad)}{4ab^2}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}$$

```
[Out] -(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*b^3) - (d*(4*b*
c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + ((b*c - a*d)*x*(c + d
*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] time = 0.1972, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {413, 528, 388, 217, 206}

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)(4bc-5ad)}{4ab^2}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*b^3) - (d*(4*b*
c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + ((b*c - a*d)*x*(c + d
*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{\left(c+dx^2\right)\left(acd-d(4bc-5ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= -\frac{d(4bc-5ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd(8bc-5ad)-d(2bc-5ad)(4bc-3ad)x^2}{\sqrt{a+bx^2}}\,dx}{4ab^2}$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} +$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} +$$

$$= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}\left(c+dx^2\right)}{4ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{ab\sqrt{a+bx^2}} +$$

Mathematica [A] time = 5.09691, size = 122, normalized size = 0.72

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(d^2(12bc-7ad)+\frac{8(bc-ad)^3}{a\left(a+bx^2\right)}+2bd^3x^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(d^2*(12*b*c - 7*a*d) + 2*b*d^3*x^2 + (8*(b*c - a*d)^3)/
(a*(a + b*x^2))))/(8*b^3) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[b
*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Maple [A] time = 0.008, size = 219, normalized size = 1.3

$$\frac{d^3x^5}{4\,b}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,ad^3x^3}{8\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,a^2d^3x}{8\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,a^2d^3}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{3\,cd^2x^3}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{9\,ac}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*d^3*x^5/b/(b*x^2+a)^(1/2)-5/8*d^3/b^2*a*x^3/(b*x^2+a)^(1/2)-15/8*d^3/b^
3*a^2*x/(b*x^2+a)^(1/2)+15/8*d^3/b^(7/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+
3/2*c*d^2*x^3/b/(b*x^2+a)^(1/2)+9/2*c*d^2/b^2*a*x/(b*x^2+a)^(1/2)-9/2*c*d^2
/b^(5/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-3*c^2*d*x/b/(b*x^2+a)^(1/2)+3*c^2*
d/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c^3*x/a/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81024, size = 888, normalized size = 5.25

$$\left[\frac{3\left(8a^2b^2c^2d-12a^3bcd^2+5a^4d^3+\left(8ab^3c^2d-12a^2b^2cd^2+5a^3bd^3\right)x^2\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2ab^3\right.}{16\left(ab^5x^2+a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^
3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15*a^3*b*d^3)*x)*sqrt(
b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/8*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^
2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2
 - 5*a^2*b^2*d^3)*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15
*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

Giac [A] time = 1.16144, size = 212, normalized size = 1.25

$$\frac{\left(\left(\frac{2d^3x^2}{b}+\frac{12ab^4cd^2-5a^2b^3d^3}{ab^5}\right)x^2+\frac{8b^5c^3-24ab^4c^2d+36a^2b^3cd^2-15a^3b^2d^3}{ab^5}\right)x}{8\sqrt{bx^2+a}}-\frac{3\left(8b^2c^2d-12abcd^2+5a^2d^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+\right.\right.}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2 + (8*b^5*
c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)/(a*b^5))*x/sqrt(b
*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*log(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/b^(7/2)
```

3.83 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{x(bc-ad)^2}{ab^2\sqrt{a+bx^2}}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{d^2x\sqrt{a+bx^2}}{2b^2}$$

```
[Out] ((b*c - a*d)^2*x)/(a*b^2*Sqrt[a + b*x^2]) + (d^2*x*Sqrt[a + b*x^2])/(2*b^2)
 + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] time = 0.0620589, antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}(2bc-3ad)}{2ab^2}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^
2))/(a*b*Sqrt[a + b*x^2]) + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(2*b^(5/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{\int \frac{acd-d(2bc-3ad)x^2}{\sqrt{a+bx^2}}\,dx}{ab}\\ &= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{(d(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\ &= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{(d(4bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\ &= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^2\right)}{ab\sqrt{a+bx^2}}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [C] time = 2.36876, size = 160, normalized size = 1.78

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\left(-6bx^2\left(c+dx^2\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,\frac{5}{2}\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)-12bx^2\left(2c^2+3cdx^2+d^2x^4\right)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{9}{2}\right.\right.}{105a^2\sqrt{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^2)/a]*(7*a*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*Hypergeometri
c2F1[1/2, 3/2, 7/2, -((b*x^2)/a)] - 12*b*x^2*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*
```

```
Hypergeometric2F1[3/2, 5/2, 9/2, -((b*x^2)/a)] - 6*b*x^2*(c + d*x^2)^2*Hype
rgeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, -((b*x^2)/a)]))/(105*a^2*Sqrt[a + b*
x^2])
```

Maple [A] time = 0.006, size = 123, normalized size = 1.4

$$\frac{d^2x^3}{2b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,ad^2x}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,ad^2}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-2\,\frac{cdx}{b\sqrt{bx^2+a}}+2\,\frac{cd\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{3/2}}+\frac{c^2x}{a}\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*d^2*x^3/b/(b*x^2+a)^(1/2)+3/2*d^2/b^2*a*x/(b*x^2+a)^(1/2)-3/2*d^2/b^(5/
2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-2*c*d*x/b/(b*x^2+a)^(1/2)+2*c*d/b^(3/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c^2*x/a/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63845, size = 597, normalized size = 6.63

$$\left[-\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(ab^2d^2x^3+\left(2\,b^3c^2-4\,ab^2cd+3\,a\right.\right.}{4\left(ab^4x^2+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^2*x^3 + (2*b^3
*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
, -1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^2*x^3 + (2*b^3*c^2 - 4*a*b^
2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

Giac [A] time = 1.11688, size = 124, normalized size = 1.38

$$\frac{\left(\frac{d^2x^2}{b}+\frac{2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}}-\frac{\left(4\,bcd-3\,ad^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))*x/sqrt(b*
x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(5/2)
```

3.84 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rubi [A] time = 0.0172689, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 217, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\ &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\ &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0575813, size = 70, normalized size = 1.3

$$\frac{a^{3/2}d\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x(bc-ad)}{ab^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(b*c - a*d)*x + a^(3/2)*d*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(a*b^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.004, size = 54, normalized size = 1.

$$-\frac{dx}{b}\frac{1}{\sqrt{bx^2+a}}+d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}+\frac{cx}{a}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(3/2),x)
```

```
[Out] -d*x/b/(b*x^2+a)^(1/2)+d/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+c*x/a/(b*x^2
+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.545, size = 367, normalized size = 6.8

$$\left[\frac{2\left(b^2c - abd\right)\sqrt{bx^2 + a}x + \left(abdx^2 + a^2d\right)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right)}{2\left(ab^3x^2 + a^2b^2\right)}, \frac{\left(b^2c - abd\right)\sqrt{bx^2 + ax} - \left(abdx^2 + a^2d\right)\sqrt{}}{ab^3x^2 + a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - a*b*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), ((b^2*
c - a*b*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] time = 3.70417, size = 60, normalized size = 1.11

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1 + \frac{bx^2}{a}}}\right) + \frac{cx}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.1755, size = 68, normalized size = 1.26

$$-\frac{d\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(bc-ad)x}{\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/(sqrt(b*x
^2 + a)*a*b)
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0020305, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0034885, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$\frac{x}{a}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Maxima [A] time = 0.975591, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A] time = 1.49059, size = 47, normalized size = 2.94

$$\frac{\sqrt{bx^2+a}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] time = 0.533954, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A] time = 1.14702, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

3.86 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx$

Optimal. Leaf size=79

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0386197, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 208}

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.682355, size = 309, normalized size = 3.91

$$\frac{x\left(2dx^2\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+2c\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)-10dx^2\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}-15c\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}+\right.}{5c^2\left(a+bx^2\right)^{3/2}\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (x*(-15*c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 10*d*x^2*Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))] + 15*c*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]] + 10*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 2*c*((
(b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*
c - a*d)*x^2)/(c*(a + b*x^2))] + 2*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2))
)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))
/(5*c^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.013, size = 618, normalized size = 7.8

$$\frac{d}{2\,ad-2\,bc}\frac{1}{\sqrt{-cd}}\frac{1}{\sqrt{\left(x-\frac{1}{d}\sqrt{-cd}\right)^2b+2\,\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}}-\frac{bx}{(2\,ad-2\,bc)\,a}\frac{1}{\sqrt{\left(x-\frac{1}{d}\sqrt{-cd}\right)^2b+2\,\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x-1/2/(-c*d)^(1/
2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2/(-c*d)
^(1/2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x+1/2/(-c*d)^(1/2)/(a*d-b*
c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36161, size = 911, normalized size = 11.53

$$\left[\frac{4\left(b^2c^2-abcd\right)\sqrt{bx^2+ax}-\left(abdx^2+a^2d\right)\sqrt{bc^2-acd}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-}}{d^2x^4+2\,cdx^2+c^2}\right.}{4\left(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+\left(ab^3c^3-2\,a^2b^2c^2d+a^3bcd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(b*
c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*
b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*
sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(a^2*b^2*c^3 - 2*a^3*b*c^2*d
 + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2), 1/2*(2*(b^
2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(-b*c^2 + a*c*
d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a
)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^3 - 2*a^3*
b*c^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

Giac [A] time = 1.14144, size = 144, normalized size = 1.82

$$-\frac{\sqrt{b}d\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bc-ad)}+\frac{bx}{\left(abc-a^2d\right)\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sq
rt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c - a*d)) + b*x/((a*b*
c - a^2*d)*sqrt(b*x^2 + a))
```

3.87 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=143

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.109018, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```


```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(5/2))
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-2bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)}-\frac{\int \frac{ad(4bc-ad)}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2ac(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)}-\frac{(d(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,d}{2c(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)}-\frac{(d(4bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-a}\right.}{2c(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)}-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}$$

Mathematica [C] time = 2.52783, size = 758, normalized size = 5.3

$$x\left(\frac{24d^2x^4\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}\text{HypergeometricPFQ}\left(\left\{2,2,\frac{5}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{x^2(bc-ad)}{c(a+bx^2)}\right)}{c^2}+\frac{48dx^2\left(\frac{x^2(bc-ad)}{c(a+bx^2)}\right)^{7/2}\text{HypergeometricPFQ}\left(\left\{2,2,\frac{5}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{x^2(bc-ad)}{c(a+bx^2)}\right)}{c}+24\left(\frac{x^2}{c(}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*(-2625*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (5250*d*x^2*Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))])/c - (2310*d^2*x^4*Sqrt[((b*c - a*d)*x^2)/(c*(
a + b*x^2))])/c^2 + 70*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (560*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (280*d^2*x^4*(((b*c - a*d
)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + 2625*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c
*(a + b*x^2))]] + (5250*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c + (2310*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c^
2 - (945*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/
(c*(a + b*x^2)) + (2310*d*(-(b*c) + a*d)*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (1050*d^2*(-(b*c) + a*d)*x^6*ArcTan
h[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)) + 24*(((b*c -
 a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2},
((b*c - a*d)*x^2)/(c*(a + b*x^2))] + (48*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b
*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))])/c + (24*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*H
ypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])
/c^2))/(210*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*(a + b*x^2)^(3/2)*(
c + d*x^2))
```

Maple [B] time = 0.017, size = 1439, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/(
```

```
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)+3/4*b^2/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((a*d-b
*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*
d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/a/((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x+1/4/
c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
+3/4*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((a*d-b*c)/
d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*
c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x+1/4/c/(-
c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((x+(-c*d
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4
/c/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-
2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

Fricas [B] time = 4.48744, size = 1706, normalized size = 11.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4
*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - 4*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
(2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a))
/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c^
5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*c
^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2), 1/4*((4*a^2*b*c
^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*
d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 2*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
 (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a)
)/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c
^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*
c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 5.45968, size = 429, normalized size = 3.

$$\frac{b^2x}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{bx^2+a}}+\frac{\left(4\,b^{\frac{3}{2}}cd-a\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-b^2c^2+abcd}}+\frac{2\left(\sqrt{b}x-\right.}{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(\left(\sqrt{b}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2
)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*
c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqr
t(-b^2*c^2 + a*b*c*d)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d - (
sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2 + a^2*sqrt(b)*d^2)/((b^2*c^3 -
 2*a*b*c^2*d + a^2*c*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

3.88 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^2}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*c + a*d)*x
)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*(2*b
*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*d*(
8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[
a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.243398, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*c + a*d)*x
)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*(2*b
*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*d*(
8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[
a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(7/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{\int \frac{4bc-3ad-4bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\left(c+dx^2\right)} - \frac{\int \frac{ad(8bc-3ad)-2bd}{\sqrt{a+bx^2}(c+}}{4ac(bc-}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\left(c+dx^2\right)} + \frac{d(4bc-ad)(2bc}{8ac^2(bc-a}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\left(c+dx^2\right)} + \frac{d(4bc-ad)(2bc}{8ac^2(bc-a}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\left(c+dx^2\right)} + \frac{d(4bc-ad)(2bc}{8ac^2(bc-a}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\left(c+dx^2\right)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\left(c+dx^2\right)} + \frac{d(4bc-ad)(2bc}{8ac^2(bc-a}$$

Mathematica [C] time = 4.77052, size = 1392, normalized size = 6.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*(-108045*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (324135*d*x^2*Sqrt[((
b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (324135*d^2*x^4*Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))])/c^2 - (103320*d^3*x^6*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))])/c^3 + 42735*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (128205*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (139545*d^2*x^4*(((b*c -
a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + (46200*d^3*x^6*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(3/2))/c^3 - 3864*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)
 - (4032*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c - (4032*d^2*x^4
*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^2 - (1344*d^3*x^6*(((b*c - a*
d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^3 + 108045*ArcTanh[Sqrt[((b*c - a*d)*x^2)
```

```
/(c*(a + b*x^2))]] + (324135*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b
*x^2))]])/c + (324135*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c^2 + (103320*d^3*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]
)/c^3 + (8505*(b*c - a*d)^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]])/(c^2*(a + b*x^2)^2) + (17955*d*(b*c - a*d)^2*x^6*ArcTanh[Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)^2) + (21735*d^2*(b*c - a*d)
^2*x^8*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2
) + (7560*d^3*(b*c - a*d)^2*x^10*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]])/(c^5*(a + b*x^2)^2) - (78750*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (236250*d*(-(b*c) + a*d)*x^4*
ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (2475
90*d^2*(-(b*c) + a*d)*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])
/(c^3*(a + b*x^2)) + (80640*d^3*(-(b*c) + a*d)*x^8*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)) + 64*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(9/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d
)*x^2)/(c*(a + b*x^2))] + (192*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9
/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a
 + b*x^2))])/c + (192*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hyp
ergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^
2))])/c^2 + (64*d^3*x^6*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hypergeom
etricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/
c^3))/(2520*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*(a + b*x^2)^(3/2)*(
c + d*x^2)^2)
```

Maple [B] time = 0.02, size = 2919, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/c^2/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x-15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((a*d-b
*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*
d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c*d*b/(a*d-b*c
)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2)+3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-
b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(
x+(-c*d)^(1/2)/d))-1/4/c*b^2/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
```

d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/16/(-c*d)^(1/2)/c*d*b/
(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2)+9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2
)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2
b(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x-3/16/(-c*d)^(1/
2)/c^2/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+15/16/
(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2
*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2
)/d))-1/4/c*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-3/16/(-c*d)^(1/2)/c*d*b/(a
*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/
2)/c*d*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
d)^(1/2)/d(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/16
/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x
+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x
-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2)+3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/c^2/(a*d-b*c)/(x+(-c*d)
^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(1/2)-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2/((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-5/
16/c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1
/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-15/16/(-c*d)^(1/2)*d*b^2/(a*d-b
*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)-15/16*b^3/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*
c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2)+3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/16/(-c*d)^(1/2)/c/(a*d
-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-5/16/c*b/(a*d-b*c)^2/(x-(-c*d)^(1/2)/d)/((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2)+15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-15/16*b^3/(a*d-b*c)^3/a/((x-

```
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

Fricas [B] time = 9.68627, size = 2967, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x
)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*((8
*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a^3*b*c*d^5)*x^5 +
(16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d^4 + 3*a
^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c^4*d^2 - 17*a^3*b*
c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9 - 4*a^3*b^3*c^8*d
 + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a*b^5*c^7*d^2 - 4*a
^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 + a^5*b*c^3*d^6)*x^6
 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3 - 2*a^4*b^2*c^5*d
^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*a^2*b^4*c^8*d - 2*
a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2*a^6*c^4*d^5)*x^2)
, 1/16*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
```

```
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
 - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) + 2*((8*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a
^3*b*c*d^5)*x^5 + (16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a
^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c
^4*d^2 - 17*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9
 - 4*a^3*b^3*c^8*d + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a
*b^5*c^7*d^2 - 4*a^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 +
a^5*b*c^3*d^6)*x^6 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3
 - 2*a^4*b^2*c^5*d^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*
a^2*b^4*c^8*d - 2*a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2
*a^6*c^4*d^5)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 14.8726, size = 868, normalized size = 3.86

$$\frac{b^3 x}{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\sqrt{bx^2 + a}} + \frac{3\left(8\,b^{\frac{5}{2}}c^2d - 4\,ab^{\frac{3}{2}}cd^2 + a^2\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 d + 2\,bc - ad}{2\,\sqrt{-b^2c^2 + abcd}}\right)}{8\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3\right)\sqrt{-b^2c^2 + abcd}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(b*x^2 +
 a)) + 3/8*(8*b^(5/2)*c^2*d - 4*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan(1
/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
```

```
d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(-b^2*c^
2 + a*b*c*d)) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d^2 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*a^2*sqrt(b)*d^4 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3
*d - 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2 + 54*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
a^3*sqrt(b)*d^4 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d^2 -
52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3 + 9*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*sqrt(b)*d^4 + 10*a^4*b^(3/2)*c*d^3 - 3*a^5*sqrt(b)*d^4)/(
(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*((sqrt(b)*x - sqr
t(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abcd+8b^2c^2\right)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\left(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3\right)}{24a^2b^4}+\frac{d^2}{}$$

```
[Out] -(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(24*a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c - a*d)*(2*b*c + 7*a*d)*x*(c + d*
x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*
(a + b*x^2)^(3/2)) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] time = 0.244889, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 526, 528, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abcd+8b^2c^2\right)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\left(-170a^2bcd^2+105a^3d^3+40ab^2c^2d+16b^3c^3\right)}{24a^2b^4}+\frac{d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(24*a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a
+ b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c - a*d)*(2*b*c + 7*a*d)*x*(c + d*
x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*
(a + b*x^2)^(3/2)) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^3}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{\left(c+dx^2\right)^2\left(c(2bc+ad)-d(4bc-7ad)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+7ad)x\left(c+dx^2\right)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^3}{3ab\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{\left(c+dx^2\right)\left(acd(4bc-7ad)+d\left(8b^2c^2+24abcd-35a^2d\right.\right.}{\sqrt{a+bx^2}}}{3a^2b^2}$$

$$= -\frac{d\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}\left(c+dx^2\right)}{12a^2b^3} + \frac{(bc-ad)(2bc+7ad)x\left(c+dx^2\right)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad}{3ab\,(a}$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+b}}{12a^2b^3}$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+b}}{12a^2b^3}$$

$$= -\frac{d\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{24a^2b^4} - \frac{d\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+b}}{12a^2b^3}$$

Mathematica [A] time = 5.16222, size = 157, normalized size = 0.62

$$\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{9/2}} + \frac{x\sqrt{a+bx^2}\left(\frac{16(bc-ad)^3(5ad+bc)}{a^2\left(a+bx^2\right)} + 3d^3(16bc-11ad) + \frac{8(bc-ad)^4}{a\left(a+bx^2\right)^2} + \right.}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(3*d^3*(16*b*c - 11*a*d) + 6*b*d^4*x^2 + (8*(b*c - a*d)^
4)/(a*(a + b*x^2)^2) + (16*(b*c - a*d)^3*(b*c + 5*a*d))/(a^2*(a + b*x^2))))
/(24*b^4) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Log[b*x + Sqrt[b]*S
qrt[a + b*x^2]])/(8*b^(9/2))
```

Maple [A] time = 0.02, size = 351, normalized size = 1.4

$$\frac{d^4x^7}{4b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{7ad^4x^5}{8b^2}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{35a^2d^4x^3}{24b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{35a^2d^4x}{8b^4}\frac{1}{\sqrt{bx^2+a}}+\frac{35a^2d^4}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/4*d^4*x^7/b/(b*x^2+a)^(3/2)-7/8*d^4/b^2*a*x^5/(b*x^2+a)^(3/2)-35/24*d^4/b
^3*a^2*x^3/(b*x^2+a)^(3/2)-35/8*d^4/b^4*a^2*x/(b*x^2+a)^(1/2)+35/8*d^4/b^(9
/2)*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+2*c*d^3*x^5/b/(b*x^2+a)^(3/2)+10/3*c*
d^3/b^2*a*x^3/(b*x^2+a)^(3/2)+10*c*d^3/b^3*a*x/(b*x^2+a)^(1/2)-10*c*d^3/b^(
7/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-2*c^2*d^2*x^3/b/(b*x^2+a)^(3/2)-6*c^2*
d^2/b^2*x/(b*x^2+a)^(1/2)+6*c^2*d^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-4
/3*c^3*d/b*x/(b*x^2+a)^(3/2)+4/3*c^3*d/b/a*x/(b*x^2+a)^(1/2)+1/3*c^4*x/a/(b
*x^2+a)^(3/2)+2/3*c^4/a^2*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.78682, size = 1430, normalized size = 5.61

$$\left[\frac{3\left(48a^4b^2c^2d^2-80a^5bcd^3+35a^6d^4+\left(48a^2b^4c^2d^2-80a^3b^3cd^3+35a^4b^2d^4\right)x^4+2\left(48a^3b^3c^2d^2-80a^4b^2cd^3+35a^5b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4*c^
2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^2*d^2 - 80
*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d
^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*
d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^
2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 +
a^4*b^5), -1/24*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*
a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^
2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*
x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*d^3
- 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^2*c*
d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

Giac [A] time = 1.18895, size = 320, normalized size = 1.25

$$\frac{\left(\left(3\left(\frac{2\,d^4x^2}{b}+\frac{16\,a^2b^6cd^3-7\,a^3b^5d^4}{a^2b^7}\right)x^2+\frac{4\left(4\,b^8c^4+8\,ab^7c^3d-48\,a^2b^6c^2d^2+80\,a^3b^5cd^3-35\,a^4b^4d^4\right)}{a^2b^7}\right)x^2+\frac{3\left(8\,ab^7c^4-48\,a^3b^5c^2d^2+80\,a^4b^4cd^3-35\,a^5b^3d^4\right)}{a^2b^7}\right)}{24\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b^7))*x^2 +
 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80*a^3*b^5*c*d^3 - 35*
a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4 - 48*a^3*b^5*c^2*d^2 + 80*a^4*
b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^2*c^
2*d^2 - 80*a*b*c*d^3 + 35*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

3.90 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c+\right.}{3ab}$$

```
[Out] -(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(6*a^2*b^3) + (
(b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((
b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a*d
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] time = 0.156522, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {413, 526, 388, 217, 206}

$$-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c+\right.}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```


```
[Out] -(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(6*a^2*b^3) + (
(b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((
b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a*d
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{\left(c+dx^2\right)\left(c(2bc+ad)-d(2bc-5ad)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{acd(2bc-5ad)+d\left(4b^2c^2+8abcd-15a^2d^2\right)x^2}{\sqrt{a+bx^2}}\,dx}{3a^2b^2}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x\left(c+dx^2\right)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x\left(c+dx^2\right)^2}{3ab\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 5.09688, size = 125, normalized size = 0.73

$$\frac{x\left(3a^2d^3\left(a+bx^2\right)^2+2\left(a+bx^2\right)(bc-ad)^2(7ad+2bc)+2a(bc-ad)^3\right)}{6a^2b^3\left(a+bx^2\right)^{3/2}} + \frac{d^2(6bc-5ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*(b*c - a*d)^3 + 2*(b*c - a*d)^2*(2*b*c + 7*a*d)*(a + b*x^2) + 3*a^2
*d^3*(a + b*x^2)^2))/(6*a^2*b^3*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a*d)*L
og[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Maple [A] time = 0.007, size = 228, normalized size = 1.3

$$\frac{d^3x^5}{2b}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{5ad^3x^3}{6b^2}\left(bx^2+a\right)^{-\frac{3}{2}} + \frac{5ad^3x}{2b^3}\frac{1}{\sqrt{bx^2+a}} - \frac{5ad^3}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}} - \frac{cd^2x^3}{b}\left(bx^2+a\right)^{-\frac{3}{2}} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*d^3*x^5/b/(b*x^2+a)^(3/2)+5/6*d^3/b^2*a*x^3/(b*x^2+a)^(3/2)+5/2*d^3/b^3
*a*x/(b*x^2+a)^(1/2)-5/2*d^3/b^(7/2)*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-c*d^2*
x^3/b/(b*x^2+a)^(3/2)-3*c*d^2/b^2*x/(b*x^2+a)^(1/2)+3*c*d^2/b^(5/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))-c^2*d/b*x/(b*x^2+a)^(3/2)+c^2*d/b/a*x/(b*x^2+a)^(1/2
)+1/3*c^3*x/a/(b*x^2+a)^(3/2)+2/3*c^3/a^2*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96421, size = 1008, normalized size = 5.86

$$\left[-\frac{3\left(6\,a^4bcd^2 - 5\,a^5d^3 + \left(6\,a^2b^3cd^2 - 5\,a^3b^2d^3\right)x^4 + 2\left(6\,a^3b^2cd^2 - 5\,a^4bd^3\right)x^2\right)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) - 2\,(}{12\left(a^2b^6x^4 + 2\,a^3b^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2
*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*
d^2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b
^4), -1/6*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)
*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d - 12*a^2
*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*d^2 + 5*a^4
*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

Giac [A] time = 1.16924, size = 213, normalized size = 1.24

$$\frac{\left(\left(\frac{3\,d^3x^2}{b}+\frac{2\left(2\,b^6c^3+3\,ab^5c^2d-12\,a^2b^4cd^2+10\,a^3b^3d^3\right)}{a^2b^5}\right)x^2+\frac{3\left(2\,ab^5c^3-6\,a^3b^3cd^2+5\,a^4b^2d^3\right)}{a^2b^5}\right)x}{6\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\left(6\,bcd^2-5\,ad^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a^
3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*d^
3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*c*d^2 - 5*a*d^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.91 $\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*
x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.0503894, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 385, 217, 206}

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*
x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(5/2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{c(2bc+ad)+3ad^2x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\ &= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\ &= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\ &= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x\left(c+dx^2\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 4.13494, size = 214, normalized size = 2.04

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(-16b^3x^6\left(c+dx^2\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,\frac{7}{2}\right\},\left\{1,\frac{9}{2}\right\},-\frac{bx^2}{a}\right)+\frac{7a^2\left(15c^2+10cdx^2+3d^2x^4\right)\left(\sqrt{-\frac{bx^2\left(a+bx^2\right)}{a^2}}\left(2bx^2-3a\right.\right.}{\sqrt{-\frac{bx^2}{a}}}\right.}{168a^3b^2x^3\sqrt{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 + (b*x^2)/a]*((7*a^2*(15*c^2 + 10*c*d*x^2 + 3*d^2*x^4)*(Sqrt[-((b*x
^2*(a + b*x^2))/a^2)]*(-3*a + 2*b*x^2) + 3*a*ArcSin[Sqrt[-((b*x^2)/a)]]))/S
qrt[-((b*x^2)/a)] - 32*b^3*x^6*(2*c^2 + 3*c*d*x^2 + d^2*x^4)*Hypergeometric
2F1[3/2, 7/2, 9/2, -((b*x^2)/a)] - 16*b^3*x^6*(c + d*x^2)^2*HypergeometricP
FQ[{3/2, 2, 7/2}, {1, 9/2}, -((b*x^2)/a)]))/(168*a^3*b^2*x^3*Sqrt[a + b*x^2
])
```

Maple [A] time = 0.006, size = 136, normalized size = 1.3

$$-\frac{d^2x^3}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{d^2x}{b^2}\frac{1}{\sqrt{bx^2+a}}+d^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-\frac{2\,cdx}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,cdx}{3\,ab}\frac{1}{\sqrt{bx^2+a}}+\frac{c^2x}{3\,a}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*d^2*x^3/b/(b*x^2+a)^(3/2)-d^2/b^2*x/(b*x^2+a)^(1/2)+d^2/b^(5/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))-2/3*c*d/b*x/(b*x^2+a)^(3/2)+2/3*c*d/b/a*x/(b*x^2+a)
^(1/2)+1/3*c^2*x/a/(b*x^2+a)^(3/2)+2/3*c^2/a^2*x/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65094, size = 655, normalized size = 6.24

$$\left[\frac{3\left(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\left(b^4c^2+ab^3cd-2\,a^2b^2d^2\right)x^3+3\left(ab^3c^2-a^3\right.\right.}{6\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a
^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*(b^4*c^2 + a*b^3*c*d - 2
*a^2*b^2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*
x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^2}{\left(a + bx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

Giac [A] time = 1.16602, size = 139, normalized size = 1.32

$$\frac{x\left(\frac{2\left(b^4c^2+ab^3cd-2\,a^2b^2d^2\right)x^2}{a^2b^3}+\frac{3\left(ab^3c^2-a^3bd^2\right)}{a^2b^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{d^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*(a*b^3*c^2
 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

3.92 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}} dx$

Optimal. Leaf size=47

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.0096644, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x\left(c+dx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}} + \frac{(2c)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a} = \frac{2cx}{3a^2\sqrt{a+bx^2}} + \frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}}$$

Mathematica [A] time = 0.0160048, size = 37, normalized size = 0.79

$$\frac{x(3ac+adx^2+2bcx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$\frac{x(adx^2+2\,bcx^2+3\,ac)}{3\,a^2}(bx^2+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(a*d*x^2+2*b*c*x^2+3*a*c)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 0.959582, size = 92, normalized size = 1.96

$$\frac{2\,cx}{3\,\sqrt{bx^2+a}a^2} + \frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{dx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{dx}{3\,\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*x/((b
*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A] time = 1.54471, size = 115, normalized size = 2.45

$$\frac{\left((2\,bc+ad)x^3+3\,acx\right)\sqrt{bx^2+a}}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

Sympy [B] time = 10.7922, size = 144, normalized size = 3.06

$$c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

Giac [A] time = 1.11949, size = 54, normalized size = 1.15

$$\frac{x\left(\frac{3\,c}{a}+\frac{\left(2\,b^2c+abd\right)x^2}{a^2b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.93 $\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0057249, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{x}{3a(a+bx^2)^{3/2}} + \frac{2\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a}$$
$$= \frac{x}{3a(a+bx^2)^{3/2}} + \frac{2x}{3a^2\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0063237, size = 29, normalized size = 0.74

$$\frac{x(3a+2bx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$\frac{x(2\,bx^2+3\,a)}{3\,a^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 0.959649, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{bx^2+a}a^2} + \frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 1.54074, size = 99, normalized size = 2.54

$$\frac{\left(2\,bx^3+3\,ax\right)\sqrt{bx^2+a}}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] time = 0.782578, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

Giac [A] time = 1.13864, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

3.94 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=122

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)/(3*a^2*(b
*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sq
rt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.102706, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)/(3*a^2*(b
*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sq
rt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2bc+3ad-2bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\int \frac{3a^2d^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{3a^2(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}$$

Mathematica [C] time = 2.62801, size = 775, normalized size = 6.35

$$x\left(12c^2\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{7/2}\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)+12d^2x^4\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{7/2}\text{HypergeometricPFQ}\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (x*(-315*c^2*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 420*c*d*x^2*Sqrt[((b
*c - a*d)*x^2)/(c*(a + b*x^2))] - 168*d^2*x^4*Sqrt[((b*c - a*d)*x^2)/(c*(a
+ b*x^2))] - 105*c^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) - 140*c*d*x^
2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) - 56*d^2*x^4*(((b*c - a*d)*x^2)
/(c*(a + b*x^2)))^(3/2) + 315*c^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))]] + 420*c*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16
8*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 48*c^2*(((b*c
- a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, ((b*c - a
*d)*x^2)/(c*(a + b*x^2))] + 84*c*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^
(7/2)*Hypergeometric2F1[2, 7/2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 3
6*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Hypergeometric2F1[2, 7/
```

```
2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 12*c^2*(((b*c - a*d)*x^2)/(c*(
a + b*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^
2)/(c*(a + b*x^2))] + 24*c*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*
HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]
 + 12*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*HypergeometricPFQ[{
2, 2, 7/2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(63*c^3*(((b*c -
 a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*(a + b*x^2)^(5/2))
```

Maple [B] time = 0.014, size = 1070, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*
b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-1/3*b/(a*d-b*c
)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)*x+1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2*d/(a*d-b*c)^2/a/((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2)*b*x-1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c
)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^
(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(
-c*d)^(1/2)/d))-1/6/(-c*d)^(1/2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*
x-1/3*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2*d/
(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)*b*x+1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2
)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(
1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.24145, size = 1530, normalized size = 12.54

$$\left[\frac{3\left(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2\right)\sqrt{bc^2-acd}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2+4\left((2\,bc-ad)x^3+acx\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)+}{12\left(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+\left(a^2b^5c^4-3\,a^3b^4c^3d+3\,a^4b^3c^2d^2-a^5b^2cd^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b*c^2 - a*c*d)*
log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2
*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^4*c^3 - 7*a*b^3*c^2*d + 5*a^2*b^
2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x)*sqrt(b*x^
2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c*d^3 + (a^2
*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*x^4 + 2*(a^
3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x^2), -1/6*(
3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-b*c^2 + a*c*d)*arctan
(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c
^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^4*c^3 - 7*a*b^3*c^2*d
 + 5*a^2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x
)*sqrt(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c
*d^3 + (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*
x^4 + 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

Giac [B] time = 1.17883, size = 432, normalized size = 3.54

$$-\frac{\sqrt{b}d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{\left(\frac{\left(2\,b^6c^3-9\,ab^5c^2d+12\,a^2b^4cd^2-5\,a^3b^3d^3\right)x^2}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}+\frac{3\left(ab^5c^3-4\,a^2b^4c^2d+5\,a^3b^3cd^2-2\,a^4b^2d^3\right)}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c^2 + a
*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d + 12*a^2*b^4*c*d^2 - 5*a^3*b^3*d
^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4) + 3*(a*b^5*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*a^4*b
^2*d^3)/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

3.95 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3ad+2}{6ac\left(a+bx^2\right)^{3/2}}$$

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (b*(4*b^2*c
^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(7/2))
```

Rubi [A] time = 0.228392, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3ad+2}{6ac\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (b*(4*b^2*c
^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(7/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-4bdx^2}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}dx}{2c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} - \frac{\int \frac{-4b^2c^2+12abcd-3a^2d^2-2bd(}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right.}}{6ac(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}(c+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}(c+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}(c+}$$

$$= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}(c+}$$

Mathematica [A] time = 5.48197, size = 170, normalized size = 0.84

$$\frac{1}{6}\left(x\sqrt{a+bx^2}\left(\frac{4b^2(4ad-bc)}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{2b^2}{a\left(a+bx^2\right)^2(bc-ad)^2} - \frac{3d^3}{c\left(c+dx^2\right)(bc-ad)^3}\right) + \frac{3d^2(ad-6bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx}}\right.}{c^{3/2}(ad-bc)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((2*b^2)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (4*b^2*(-(b*c
) + 4*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (3*d^3)/(c*(b*c - a*d)^3*(
c + d*x^2))) + (3*d^2*(-6*b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]
*Sqrt[a + b*x^2])]])/(c^(3/2)*(-(b*c) + a*d)^(7/2)))/6
```

Maple [B] time = 0.02, size = 2371, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] 5/4*d*b^2/(a*d-b*c)^3/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/c*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+1/2/c*b/(a*d-b*
c)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2)*x-1/4/c*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x-1/4/c*d/(a*d-b*c)^2/a/((x-(-
c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*
b*x+5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/
d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c
*d)^(1/2)/d))-5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*
(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x
-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))+1/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1
/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)
^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((a*d-
b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a
*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*c
)^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2)+5/4*d*b^2/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/c*b/(a*d-b*c)/a/((x+(-c*d)^(1
/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+1/2/c
*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)*x+5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+5/12/c*b*
(-c*d)^(1/2)/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+5/12*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2
*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+5/6*b^2/(a*d-
b*c)^2/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2)*x+1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4/(-c*d)^(1/2)/c
*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)-1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^
2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/4/(-c*d)^(1/
```

```
2)/c*d^2/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2)-5/12/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+5/12*b^2
/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(3/2)*x+5/6*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/c/(a*d-b*c)/(x+(
-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(3/2)+1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

Fricas [B] time = 9.3097, size = 2849, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6
 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c
^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2
- 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((
2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2
*c*d*x^2 + c^2)) + 4*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3
+ 3*a^3*b^2*c*d^4)*x^5 + 2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 +
 6*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d
+ 6*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b
^4*c^7 - 4*a^5*b^3*c^6*d + 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^
```

```
4 + (a^2*b^6*c^6*d - 4*a^3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*
d^4 + a^6*b^2*c^2*d^5)*x^6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5
*d^2 + 8*a^5*b^3*c^4*d^3 - 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^
3*b^5*c^7 - 7*a^4*b^4*c^6*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7
*b*c^3*d^4 + a^8*c^2*d^5)*x^2), -1/12*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*
a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt
(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((4*
b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 + 3*a^3*b^2*c*d^4)*x^5 +
2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3 + 3*a^
4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - a^4
*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b^4*c^7 - 4*a^5*b^3*c^6*d
+ 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^4 + (a^2*b^6*c^6*d - 4*a^
3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*d^4 + a^6*b^2*c^2*d^5)*x^
6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5*d^2 + 8*a^5*b^3*c^4*d^3
- 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^3*b^5*c^7 - 7*a^4*b^4*c^6
*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7*b*c^3*d^4 + a^8*c^2*d^5)
*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 7.72122, size = 837, normalized size = 4.14

$$\frac{\left(\frac{2\left(b^{8}c^{4}-7\,ab^{7}c^{3}d+15\,a^{2}b^{6}c^{2}d^{2}-13\,a^{3}b^{5}cd^{3}+4\,a^{4}b^{4}d^{4}\right)x^{2}}{a^{2}b^{7}c^{6}-6\,a^{3}b^{6}c^{5}d+15\,a^{4}b^{5}c^{4}d^{2}-20\,a^{5}b^{4}c^{3}d^{3}+15\,a^{6}b^{3}c^{2}d^{4}-6\,a^{7}b^{2}cd^{5}+a^{8}bd^{6}}+\frac{3\left(ab^{7}c^{4}-6\,a^{2}b^{6}c^{3}d+12\,a^{3}b^{5}c^{2}d^{2}-10\,a^{4}b^{4}cd^{3}+3\,a^{5}b^{3}d^{4}\right)}{a^{2}b^{7}c^{6}-6\,a^{3}b^{6}c^{5}d+15\,a^{4}b^{5}c^{4}d^{2}-20\,a^{5}b^{4}c^{3}d^{3}+15\,a^{6}b^{3}c^{2}d^{4}-6\,a^{7}b^{2}cd^{5}+a^{8}bd^{6}}\right)}{3\left(bx^{2}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^8*c^4 - 7*a*b^7*c^3*d + 15*a^2*b^6*c^2*d^2 - 13*a^3*b^5*c*d^3 + 4
*a^4*b^4*d^4)*x^2/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*
a^5*b^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6) + 3*(a*
b^7*c^4 - 6*a^2*b^6*c^3*d + 12*a^3*b^5*c^2*d^2 - 10*a^4*b^4*c*d^3 + 3*a^5*b
^3*d^4)/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*a^5*b^4*c^
3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6))*x/(b*x^2 + a)^(3
/2) + 1/2*(6*b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(-1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^3*c^4 - 3*a*b^
2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*
a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^
2 - a^3*c*d^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

3.96 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{dx\sqrt{a+bx^2}\left(-42a^2bcd^2+9a^3d^3-88ab^2c^2d+16b^3c^3\right)}{24a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+4\right.}{8c^{5/2}(b}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*b*c + 3*a*
d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)) + (b*(8*b^2*c^2
- 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c + d
*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sq
rt[a + b*x^2])/(24*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2 -
16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2
])])/(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rubi [A] time = 0.399588, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 208}

$$\frac{dx\sqrt{a+bx^2}\left(-42a^2bcd^2+9a^3d^3-88ab^2c^2d+16b^3c^3\right)}{24a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+4\right.}{8c^{5/2}(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*b*c + 3*a*
d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)) + (b*(8*b^2*c^2
- 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c + d
*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sq
rt[a + b*x^2])/(24*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2 -
16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2
])])/(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{\int \frac{4bc-3ad-6bdx^2}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} - \frac{\int \frac{-8b^2c^2+ \cdots}{\cdots}}{\cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{b\left(8b^2 \cdots\right.}{12a^2c(bc \cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{b\left(8b^2 \cdots\right.}{12a^2c(bc \cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{b\left(8b^2 \cdots\right.}{12a^2c(bc \cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{b\left(8b^2 \cdots\right.}{12a^2c(bc \cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{b\left(8b^2 \cdots\right.}{12a^2c(bc \cdots}$$

Mathematica [A] time = 5.65287, size = 221, normalized size = 0.71

$$\frac{1}{24}\left(x\sqrt{a+bx^2}\left(\frac{8b^3(2bc-11ad)}{a^2\left(a+bx^2\right)(bc-ad)^4} - \frac{8b^3}{a\left(a+bx^2\right)^2(ad-bc)^3} + \frac{3d^3(3ad-14bc)}{c^2\left(c+dx^2\right)(bc-ad)^4} - \frac{6d^3}{c\left(c+dx^2\right)^2(bc-ad)^3}\right) + \cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((-8*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)^2) + (8*b^3*(2
*b*c - 11*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^2)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^2)^2) + (3*d^3*(-14*b*c + 3*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^2)))
```

```
 + (3*d^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5/2)*(-(b*c) + a*d)^(9/2)))/24
```

Maple [B] time = 0.024, size = 4495, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] -3/16/c^2*d/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x-3/16/c^2*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/
2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b*x+5/8/
c*d*b^2/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b
*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*
d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+5/8/c*d*b^2/(a*d-b*c)^3/a/((x-(-
c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*
x-15/16/c^2*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/c^2*b/(a*d-b*c)/a/((x+(
-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)
*x+3/8/c^2*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/8/c*b^2/(a*d-b*c)^2/a/((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3
/2)*x-3/4/c*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-5/48/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2
/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(3/2)-7/16/c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/16/(-c*d)^(1/2)/c^2/
(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(3/2)-3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/16/(-c*d)^(
1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-7/16/c*b/(a*d-b*c)^2/(x-(-c*d)^
(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(3/2)+35/48/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-35/48*b^3/(a*d-b*
```

```
c)^3/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(3/2)*x-35/24*b^3/(a*d-b*c)^3/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(1/2)*d^2*b^2
/(a*d-b*c)^4/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2)+1/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*
b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+3/16/(-c*d)^(1/2
)/c^2*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)-5/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-35/24
*b^3/(a*d-b*c)^3/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2)*x-35/16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/((x+(-c
*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+5
/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-35/48/(-c*d)^(1/2)*d*b^2/(a*d-b*c)
^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(3/2)+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+3/16/c^2/(a*d-b*c)/(x-
(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(3/2)-5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+15/16/c^2*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-15/16/c^2*d*b*(-c*d)^(1/2)
/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-35/48*b^3/(a
*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(3/2)*x-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2/((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2
)-35/16*d*b^3/(a*d-b*c)^4/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/16/c^2*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)
/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+3/8/c^2*
b/(a*d-b*c)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2)*x-3/8/c*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-3/4/c*b^2/(a*d-b*c)
^2/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2)*x+15/16/c^2*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2
*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/(-c*d)^(1/
2)/c^2*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+5/
16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*
```

```
c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+5/48/(-c*d)^(1/2)/c*
d*b/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(3/2)-35/16*d*b^3/(a*d-b*c)^4/a/((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-35/16/(-c*d)^(1/2)*
d^2*b^2/(a*d-b*c)^4/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-5/16/
(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

Fricas [B] time = 35.9545, size = 4520, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b
^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3
+ 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^
4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^
6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*
a^6*c*d^5)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((
```

```
16*b^6*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^
5 - 9*a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^
4*d^3 + 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16
*b^6*c^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a
^4*b^2*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*
a^2*b^4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4
- 5*a^6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a
^6*b^3*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*
b^7*c^8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 +
 5*a^6*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^
8*d^2 + 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c
^3*d^7)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^
4*c^7*d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^
3*d^7)*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*
b^2*c^6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2), -1/48*(3*(48*a^4*b^2*c^4
*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b^4*c^2*d^4 - 16*a^3*b^3*
c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 1
3*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*
d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^3*c
^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*x^2)*sqrt(-b*
c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt
(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^6
*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^5 - 9*
a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^4*d^3
+ 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16*b^6*c
^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a^4*b^2
*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*a^2*b^
4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4 - 5*a^
6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a^6*b^3
*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*b^7*c^
8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 + 5*a^6
*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^8*d^2
+ 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c^3*d^7
)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^4*c^7*
d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^3*d^7)
*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*b^2*c^
6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.70934, size = 1364, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/3*((2*b^10*c^5 - 19*a*b^9*c^4*d + 56*a^2*b^8*c^3*d^2 - 74*a^3*b^7*c^2*d^3
 + 46*a^4*b^6*c*d^4 - 11*a^5*b^5*d^5)*x^2/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d +
28*a^4*b^7*c^6*d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c
^3*d^5 + 28*a^8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8) + 3*(a*b^9*c^5
- 8*a^2*b^8*c^4*d + 22*a^3*b^7*c^3*d^2 - 28*a^4*b^6*c^2*d^3 + 17*a^5*b^5*c*
d^4 - 4*a^6*b^4*d^5)/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d + 28*a^4*b^7*c^6*d^2 -
56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c^3*d^5 + 28*a^8*b^3*c
^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^(5/
2)*c^2*d^2 - 16*a*b^(3/2)*c*d^3 + 3*a^2*sqrt(b)*d^4)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^6
- 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-
b^2*c^2 + a*b*c*d)) - 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d
^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4 + 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*d^5 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7
/2)*c^3*d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^3 + 66*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^4 - 9*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^3*sqrt(b)*d^5 + 88*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c
^2*d^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^4 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^5 + 14*a^4*b^(3/2)*c*d^4 - 3*a^5*sqrt(
b)*d^5)/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a
^4*c^2*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

3.97 $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx$

Optimal. Leaf size=224

$$\frac{8a^2x(a+bx^2)(9bc-8ad)}{315c^4(c+dx^2)^{3/2}(bc-ad)}+\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}(bc-ad)}+\frac{x(a+bx^2)^3(9bc-8ad)}{63c^2(c+dx^2)^{7/2}(bc-ad)}+\frac{2ax(a+bx^2)^2(9bc-8ad)}{105c^3(c+dx^2)^{5/2}(bc-ad)}-\frac{}{9c}$$

```
[Out] -(d*x*(a + b*x^2)^4)/(9*c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a*d)
*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)) + (2*a*(9*b*c - 8*
a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c + d*x^2)^(5/2)) + (8*a^2*(9*b
*c - 8*a*d)*x*(a + b*x^2))/(315*c^4*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*a^
3*(9*b*c - 8*a*d)*x)/(315*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.101163, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{8a^2x(a+bx^2)(9bc-8ad)}{315c^4(c+dx^2)^{3/2}(bc-ad)}+\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}(bc-ad)}+\frac{x(a+bx^2)^3(9bc-8ad)}{63c^2(c+dx^2)^{7/2}(bc-ad)}+\frac{2ax(a+bx^2)^2(9bc-8ad)}{105c^3(c+dx^2)^{5/2}(bc-ad)}-\frac{}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```



```
[Out] -(d*x*(a + b*x^2)^4)/(9*c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a*d)
*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)) + (2*a*(9*b*c - 8*
a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c + d*x^2)^(5/2)) + (8*a^2*(9*b
*c - 8*a*d)*x*(a + b*x^2))/(315*c^4*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*a^
3*(9*b*c - 8*a*d)*x)/(315*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{11/2}}\,dx = -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^{9/2}}\,dx}{9c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}}+\frac{(2a(9bc-8ad))\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{7/2}}\,dx}{21c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}}+\frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}}+\frac{\left(8a^2(9bc-8\right.}{105}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}}+\frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}}+\frac{8a^2(9bc-8}{315c^4(bc-a}$$

$$= -\frac{dx\left(a+bx^2\right)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}}+\frac{(9bc-8ad)x\left(a+bx^2\right)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}}+\frac{2a(9bc-8ad)x\left(a+bx^2\right)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}}+\frac{8a^2(9bc-8}{315c^4(bc-a}$$

Mathematica [A] time = 0.10089, size = 163, normalized size = 0.73

$$\frac{3a^2bcx^3\left(126c^2dx^2+105c^3+72cd^2x^4+16d^3x^6\right)+a^3\left(1008c^2d^2x^5+840c^3dx^3+315c^4x+576cd^3x^7+128d^4x^9\right)+3ab^2c^2}{315c^5\left(c+dx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d*x^2 + 8*d
^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c*d^2*x^4 + 16*d^3*x^
6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^2*d^2*x^5 + 576*c*d^3*x^7 + 12
8*d^4*x^9))/(315*c^5*(c + d*x^2)^(9/2))
```

Maple [A] time = 0.007, size = 190, normalized size = 0.9

$$\frac{x\left(128\,a^3d^4x^8+48\,a^2bcd^3x^8+24\,ab^2c^2d^2x^8+10\,b^3c^3dx^8+576\,a^3cd^3x^6+216\,a^2bc^2d^2x^6+108\,ab^2c^3dx^6+45\,b^3c^4x^6+\cdots\right.}{315\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)^(11/2),x)
```

```
[Out] 1/315*x*(128*a^3*d^4*x^8+48*a^2*b*c*d^3*x^8+24*a*b^2*c^2*d^2*x^8+10*b^3*c^3
*d*x^8+576*a^3*c*d^3*x^6+216*a^2*b*c^2*d^2*x^6+108*a*b^2*c^3*d*x^6+45*b^3*c
^4*x^6+1008*a^3*c^2*d^2*x^4+378*a^2*b*c^3*d*x^4+189*a*b^2*c^4*x^4+840*a^3*c
^3*d*x^2+315*a^2*b*c^4*x^2+315*a^3*c^4)/(d*x^2+c)^(9/2)/c^5
```

Maxima [B] time = 1.05908, size = 628, normalized size = 2.8

$$-\frac{b^3x^5}{4\left(dx^2+c\right)^{\frac{9}{2}}d}-\frac{5\,b^3cx^3}{24\left(dx^2+c\right)^{\frac{9}{2}}d^2}-\frac{ab^2x^3}{2\left(dx^2+c\right)^{\frac{9}{2}}d}+\frac{128\,a^3x}{315\,\sqrt{dx^2+c}c^5}+\frac{64\,a^3x}{315\left(dx^2+c\right)^{\frac{3}{2}}c^4}+\frac{16\,a^3x}{105\left(dx^2+c\right)^{\frac{5}{2}}c^3}+\frac{8\cdots}{63\left(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^(9/2)*d^2)
 - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/(sqrt(d*x^2 + c)*c^5
) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16/105*a^3*x/((d*x^2 + c)^(5/2)*
c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2)*c)
 + 1/84*b^3*x/((d*x^2 + c)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^2*d^3
) + 1/63*b^3*x/((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)^(7/2)
```

```
*d^3) - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^2 + c)^
(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105*a*b^2*x/((d*x^
2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^(5/2)*c*d^2) - 1/6*a*b^2*
c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*x/((d*x^2 + c)^(9/2)*d) + 16/105*a^
2*b*x/(sqrt(d*x^2 + c)*c^4*d) + 8/105*a^2*b*x/((d*x^2 + c)^(3/2)*c^3*d) + 2
/35*a^2*b*x/((d*x^2 + c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 + c)^(7/2)*c*d
)
```

Fricas [A] time = 3.34891, size = 487, normalized size = 2.17

$$\frac{\left(2\left(5b^3c^3d+12ab^2c^2d^2+24a^2bcd^3+64a^3d^4\right)x^9+315a^3c^4x+9\left(5b^3c^4+12ab^2c^3d+24a^2bc^2d^2+64a^3cd^3\right)x^7+63\left(3\right.\right.}{315\left(c^5d^5x^{10}+5c^6d^4x^8+10c^7d^3x^6+10c^8d^2x^4+5c^9dx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^3*d^4)*x^9
 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a^2*b*c^2*d^2 + 64*a^
3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*d + 16*a^3*c^2*d^2)*x^5 + 105*
(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sqrt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*d^4*
x^8 + 10*c^7*d^3*x^6 + 10*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17216, size = 294, normalized size = 1.31

$$\frac{\left(\left(\left(\left(x^2\left(\frac{2\left(5b^3c^3d^5+12ab^2c^2d^6+24a^2bcd^7+64a^3d^8\right)x^2}{c^5d^4}+\frac{9\left(5b^3c^4d^4+12ab^2c^3d^5+24a^2bc^2d^6+64a^3cd^7\right)}{c^5d^4}\right)+\frac{63\left(3ab^2c^4d^4+6a^2bc^3d^5+16a^3c^2d^6\right)}{c^5d^4}\right)x^2+\frac{105}{}\right.\right.\right.\right.}{315\left(dx^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^7 + 64*a^3
*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d^5 + 24*a^2*b*c^2*d^
6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + 16*a
^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5*d^4
))*x^2 + 315*a^3/c)*x/(d*x^2 + c)^(9/2)
```

3.98 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}} dx$

Optimal. Leaf size=174

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}(bc-ad)}+\frac{x(a+bx^2)^2(7bc-6ad)}{35c^2(c+dx^2)^{5/2}(bc-ad)}+\frac{4ax(a+bx^2)(7bc-6ad)}{105c^3(c+dx^2)^{3/2}(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^3)/(7*c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a*d)
*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)) + (4*a*(7*b*c - 6*
a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (8*a^2*(7*b*c
 - 6*a*d)*x)/(105*c^4*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0707433, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}(bc-ad)}+\frac{x(a+bx^2)^2(7bc-6ad)}{35c^2(c+dx^2)^{5/2}(bc-ad)}+\frac{4ax(a+bx^2)(7bc-6ad)}{105c^3(c+dx^2)^{3/2}(bc-ad)}-\frac{dx(a+bx^2)^3}{7c(c+dx^2)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```


```
[Out] -(d*x*(a + b*x^2)^3)/(7*c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a*d)
*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)) + (4*a*(7*b*c - 6*
a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (8*a^2*(7*b*c
 - 6*a*d)*x)/(105*c^4*(b*c - a*d)*Sqrt[c + d*x^2])
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx = -\frac{dx\left(a+bx^2\right)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{7/2}}\,dx}{7c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x\left(a+bx^2\right)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{(4a(7bc-6ad))\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}}\,dx}{35c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x\left(a+bx^2\right)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{4a(7bc-6ad)x\left(a+bx^2\right)}{105c^3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{\left(8a^2(7bc-\right.}{105}$$

$$= -\frac{dx\left(a+bx^2\right)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x\left(a+bx^2\right)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{4a(7bc-6ad)x\left(a+bx^2\right)}{105c^3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{8a^2(7b}{105c^4(bc-}$$

Mathematica [A] time = 0.063847, size = 107, normalized size = 0.61

$$\frac{3a^2\left(70c^2dx^3+35c^3x+56cd^2x^5+16d^3x^7\right)+2abcx^3\left(35c^2+28cdx^2+8d^2x^4\right)+3b^2c^2x^5\left(7c+2dx^2\right)}{105c^4\left(c+dx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] (3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2 + 8*d^2*x
^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 16*d^3*x^7))/(105*c^4
```

```
*(c + d*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 115, normalized size = 0.7

$$\frac{x\left(48\,a^{2}d^{3}x^{6}+16\,abcd^{2}x^{6}+6\,b^{2}c^{2}dx^{6}+168\,a^{2}cd^{2}x^{4}+56\,abc^{2}dx^{4}+21\,b^{2}c^{3}x^{4}+210\,a^{2}c^{2}dx^{2}+70\,abc^{3}x^{2}+105\,a^{2}c^{3}\right)}{105\,c^{4}}\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(9/2),x)
```

```
[Out] 1/105*x*(48*a^2*d^3*x^6+16*a*b*c*d^2*x^6+6*b^2*c^2*d*x^6+168*a^2*c*d^2*x^4+
56*a*b*c^2*d*x^4+21*b^2*c^3*x^4+210*a^2*c^2*d*x^2+70*a*b*c^3*x^2+105*a^2*c^
3)/(d*x^2+c)^(7/2)/c^4
```

Maxima [A] time = 1.01433, size = 336, normalized size = 1.93

$$-\frac{b^{2}x^{3}}{4\left(dx^{2}+c\right)^{\frac{7}{2}}d}+\frac{16\,a^{2}x}{35\,\sqrt{dx^{2}+c}c^{4}}+\frac{8\,a^{2}x}{35\left(dx^{2}+c\right)^{\frac{3}{2}}c^{3}}+\frac{6\,a^{2}x}{35\left(dx^{2}+c\right)^{\frac{5}{2}}c^{2}}+\frac{a^{2}x}{7\left(dx^{2}+c\right)^{\frac{7}{2}}c}+\frac{3\,b^{2}x}{140\left(dx^{2}+c\right)^{\frac{5}{2}}d^{2}}+\frac{2\,b^{2}x}{35\,\sqrt{dx^{2}+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)*c^4) + 8/
35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 + c)^(5/2)*c^2) + 1/7
*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((d*x^2 + c)^(5/2)*d^2) + 2/35*b
^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) - 3/2
8*b^2*c*x/((d*x^2 + c)^(7/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) + 16/10
5*a*b*x/(sqrt(d*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*d) + 2
/35*a*b*x/((d*x^2 + c)^(5/2)*c*d)
```

Fricas [A] time = 1.94563, size = 319, normalized size = 1.83

$$\frac{\left(2\left(3\,b^{2}c^{2}d+8\,abcd^{2}+24\,a^{2}d^{3}\right)x^{7}+105\,a^{2}c^{3}x+7\left(3\,b^{2}c^{3}+8\,abc^{2}d+24\,a^{2}cd^{2}\right)x^{5}+70\left(abc^{3}+3\,a^{2}c^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}}{105\left(c^{4}d^{4}x^{8}+4\,c^{5}d^{3}x^{6}+6\,c^{6}d^{2}x^{4}+4\,c^{7}dx^{2}+c^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c^3*x + 7*(
3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c^3 + 3*a^2*c^2*d)*x^
3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x^6 + 6*c^6*d^2*x^4 + 4*c^7*d*x
^2 + c^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(9/2), x)
```

Giac [A] time = 1.25454, size = 186, normalized size = 1.07

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,b^2c^2d^4+8\,abcd^5+24\,a^2d^6\right)x^2}{c^4d^3}+\frac{7\left(3\,b^2c^3d^3+8\,abc^2d^4+24\,a^2cd^5\right)}{c^4d^3}\right)+\frac{70\left(abc^3d^3+3\,a^2c^2d^4\right)}{c^4d^3}\right)x^2+\frac{105\,a^2}{c}\right)x}{105\left(dx^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^4*d^3) + 7
*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^3)) + 70*(a*b*c^3*d^
3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c)*x/(d*x^2 + c)^(7/2)
```

3.99 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

```
[Out] -((b*c - a*d)*x)/(5*c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(15*c^2*d*(c
 + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0288756, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```


```
[Out] -((b*c - a*d)*x)/(5*c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(15*c^2*d*(c
 + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c + d*x^2])
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx &= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx}{5cd}\\ &= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{(2(bc+4ad))\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{15c^2d}\\ &= -\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0224532, size = 59, normalized size = 0.65

$$\frac{a\left(15c^2x+20cdx^3+8d^2x^5\right)+bcx^3\left(5c+2dx^2\right)}{15c^3\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] (b*c*x^3*(5*c + 2*d*x^2) + a*(15*c^2*x + 20*c*d*x^3 + 8*d^2*x^5))/(15*c^3*(
c + d*x^2)^(5/2))
```

Maple [A] time = 0.003, size = 57, normalized size = 0.6

$$\frac{x\left(8\,ad^2x^4+2\,bcdx^4+20\,acdx^2+5\,bc^2x^2+15\,ac^2\right)}{15\,c^3}\left(dx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*d^2*x^4+2*b*c*d*x^4+20*a*c*d*x^2+5*b*c^2*x^2+15*a*c^2)/(d*x^2+c
)^(5/2)/c^3
```

Maxima [A] time = 0.970178, size = 139, normalized size = 1.53

$$\frac{8\,ax}{15\,\sqrt{dx^2+c}c^3}+\frac{4\,ax}{15\left(dx^2+c\right)^{\frac{3}{2}}c^2}+\frac{ax}{5\left(dx^2+c\right)^{\frac{5}{2}}c}-\frac{bx}{5\left(dx^2+c\right)^{\frac{5}{2}}d}+\frac{2\,bx}{15\,\sqrt{dx^2+c}c^2d}+\frac{bx}{15\left(dx^2+c\right)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2) + 1/5*a*x
/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d) + 2/15*b*x/(sqrt(d*x
^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*c*d)
```

Fricas [A] time = 1.55856, size = 185, normalized size = 2.03

$$\frac{\left(2\left(bcd+4\,ad^2\right)x^5+15\,ac^2x+5\left(bc^2+4\,acd\right)x^3\right)\sqrt{dx^2+c}}{15\left(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*x^3)*sqrt(
d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 + c^6)
```

Sympy [B] time = 36.6034, size = 566, normalized size = 6.22

$$a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx}{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1
+ d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3
*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(15*c**(17/2)*sqrt(1 + d*x**2/c)
 + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 +
 d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2*x**5
/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c)
+ 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1
 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(
15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c)
 + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c))) + b*(5*c*x**3/(15*c**(9/2)*s
qrt(1 + d*x**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**
2*x**4*sqrt(1 + d*x**2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*
c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x**2/
c)))
```

Giac [A] time = 1.1769, size = 97, normalized size = 1.07

$$\frac{\left(x^2\left(\frac{2\left(bcd^3+4\,ad^4\right)x^2}{c^3d^2}+\frac{5\left(bc^2d^2+4\,acd^3\right)}{c^3d^2}\right)+\frac{15\,a}{c}\right)x}{15\left(dx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4*a*c*d^3)/
(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

3.100 $\int \frac{1}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}$$

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0056034, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(-5/2),x]
```

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{x}{3c(c+dx^2)^{3/2}} + \frac{2\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{3c}$$
$$= \frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0093256, size = 29, normalized size = 0.74

$$\frac{x(3c+2dx^2)}{3c^2(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(-5/2),x]
```

```
[Out] (x*(3*c + 2*d*x^2))/(3*c^2*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 26, normalized size = 0.7

$$\frac{x(2\,dx^2+3\,c)}{3\,c^2}\left(dx^2+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*x*(2*d*x^2+3*c)/(d*x^2+c)^(3/2)/c^2
```

Maxima [A] time = 0.952149, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{dx^2+c}c^2} + \frac{x}{3\left(dx^2+c\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

Fricas [A] time = 1.49205, size = 99, normalized size = 2.54

$$\frac{\left(2\,dx^{3}+3\,cx\right)\sqrt{dx^{2}+c}}{3\left(c^{2}d^{2}x^{4}+2\,c^{3}dx^{2}+c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)
```

Sympy [B] time = 0.792314, size = 95, normalized size = 2.44

$$\frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^{2}}{c}}+3c^{\frac{5}{2}}dx^{2}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{2dx^{3}}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^{2}}{c}}+3c^{\frac{5}{2}}dx^{2}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**2+c)**(5/2),x)
```

```
[Out] 3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*x**2/c)
) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*
x**2/c))
```

Giac [A] time = 1.16096, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,dx^{2}}{c^{2}}+\frac{3}{c}\right)}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="giac")

[Out] 1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.046164, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Mathematica [C] time = 2.58988, size = 236, normalized size = 2.99

$$-\frac{\frac{15c\left(3c+2dx^2\right)\left(c\left(a+bx^2\right)\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}-a\left(c+dx^2\right)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)\right)}{\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}}+\frac{4x^4\left(c+dx^2\right)(bc-ad)^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}}{15c^3x\left(a+bx^2\right)\sqrt{c+dx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((15*c*(3*c + 2*d*x^2)*(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))
/(c^2*(a + b*x^2)^2)] - a*(c + d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a +
 b*x^2))]]))/Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)] + (4
*(b*c - a*d)^2*x^4*(c + d*x^2)*Hypergeometric2F1[2, 2, 7/2, ((b*c - a*d)*x^
2)/(c*(a + b*x^2))])/(a + b*x^2))/(15*c^3*(-(b*c) + a*d)*x*(a + b*x^2)*Sqrt
[c + d*x^2])
```

Maple [B] time = 0.033, size = 628, normalized size = 8.

$$\frac{b}{2ad-2bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}+\frac{dx}{(2ad-2bc)c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^{2}d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-
a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*
d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))-1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)*x
*d+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1
/b*(-a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30654, size = 914, normalized size = 11.57

$$\left[-\frac{4\left(abcd-a^{2}d^{2}\right)\sqrt{dx^{2}+cx}-\left(bcdx^{2}+bc^{2}\right)\sqrt{-abc+a^{2}d}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}+4\left((bc-2\,ad)x^{3}-acx\right)\sqrt{-a}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right.}{4\left(ab^{2}c^{4}-2\,a^{2}bc^{3}d+a^{3}c^{2}d^{2}+\left(ab^{2}c^{3}d-2\,a^{2}bc^{2}d^{2}+a^{3}cd^{3}\right)x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A] time = 1.1703, size = 144, normalized size = 1.82

$$\frac{b\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}-\frac{dx}{\left(bc^2-acd\right)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

3.102 $\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rubi [A] time = 0.0547293, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}\\ &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}\\ &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.757111, size = 405, normalized size = 4.05

$$\frac{x\sqrt{c+dx^2}\left(-30dx^2\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}-45c\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}+16dx^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+16c}{30c^2\left(a+bx^2\right)^2\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*(-45*c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*
x^2)^2)] - 30*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2
)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 30*d*x^2*ArcSin
[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16*c*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2,
3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 16*d*x^2*(((b*c - a*d)*x^2)/(c
*(a + b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(30*c^2*(((b*c - a*d)*x^2
)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^2*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))
```

```
])
```

Maple [B] time = 0.024, size = 823, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(a
*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] time = 2.61771, size = 954, normalized size = 9.54

$$\left[\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}x-\left(abc-2a^2d+\left(b^2c-2abd\right)x^2\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4}{b^2x^4+2abx^2+a^2}\right.}{8\left(a^3b^2c^2-2a^4bcd+a^5d^2+\left(a^2b^3c^2-2a^3b^2cd+a^4bd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.18637, size = 304, normalized size = 3.04

$$-\frac{1}{2}d^{\frac{3}{2}}\left(\frac{(bc-2ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2-}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

3.103 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{c+dx^2}(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2)) + (b*x*
(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*(b*c - a*d)^
(3/2))
```

Rubi [A] time = 0.0794741, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {382, 378, 377, 205}

$$\frac{x\sqrt{c+dx^2}(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2)) + (b*x*
(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*(b*c - a*d)^
(3/2))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^3}\,dx &= \frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{(3bc-4ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx}{4a(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{(c(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8a^2(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{8a^2(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\left(a+bx^2\right)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 5.17001, size = 130, normalized size = 0.87

$$\frac{\frac{\sqrt{a}x\sqrt{c+dx^2}\left(-4a^2d+ab\left(5c-2dx^2\right)+3b^2cx^2\right)}{\left(a+bx^2\right)^2(bc-ad)}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*x*Sqrt[c + d*x^2]*(-4*a^2*d + 3*b^2*c*x^2 + a*b*(5*c - 2*d*x^2)))
/((b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(8*a^(5/2))
```

Maple [B] time = 0.028, size = 5177, normalized size = 34.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

Fricas [B] time = 2.89617, size = 1439, normalized size = 9.66

$$\left[-\frac{\left(3\,a^2bc^2 - 4\,a^3cd + \left(3\,b^3c^2 - 4\,ab^2cd\right)x^4 + 2\left(3\,ab^2c^2 - 4\,a^2bcd\right)x^2\right)\sqrt{-abc + a^2d}\,\log\left(\frac{\left(b^2c^2 - 8\,abcd + 8\,a^2d^2\right)x^4 + a^2c^2 - 2\left(3\,abc^2 - 4\right.}{b^2x^4}\right.}{32\left(a^5b^2c^2 - 2\,a^6bcd + a^7d^2 + \left(a^3b^4c^2 - 2\,a^4b^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b
^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^
2 - 9*a^3*b*c*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d
 + a^7*d^2 + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c
^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x^2), 1/16*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b
^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c -
 a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*a*b^3*c^2
- 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^2 - 9*a^3*b*c*d + 4*a^4*d
^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (a^3*b^4*c^2
 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*
b*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{\left(a + bx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**3, x)
```

Giac [B] time = 3.77492, size = 657, normalized size = 4.41

$$-\frac{\left(3\,bc^2\sqrt{d} - 4\,acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{8\left(a^2bc - a^3d\right)\sqrt{abcd - a^2d^2}} - \frac{3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^6 b^3c^2\sqrt{d} - 4\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^6 ab^2cd^{\frac{3}{2}} - 9\left(\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b*c^2*sqrt(d) - 4*a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a
*b*c*d - a^2*d^2)) - 1/4*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^3*c^2*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^2*c*d^(3/2) - 9*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b^3*c^3*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c^
2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) + 16*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a^3*d^(7/2) + 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^
3*c^4*sqrt(d) - 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2) + 16*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c^2*d^(5/2) - 3*b^3*c^5*sqrt(d) + 2*a*
b^2*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^2*(a^2*b
^2*c - a^3*b*d))
```

3.104 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx$

Optimal. Leaf size=199

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{cx\sqrt{c+dx^2}(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*x^2)) + ((
5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)*(a + b*x^2)^2) + (b
*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + (c^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c -
 a*d)^(3/2))
```

Rubi [A] time = 0.113364, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {382, 378, 377, 205}

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{cx\sqrt{c+dx^2}(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```


```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*x^2)) + ((
5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)*(a + b*x^2)^2) + (b
*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + (c^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c -
 a*d)^(3/2))
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^4}\,dx = \frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3}+\frac{(5bc-6ad)\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^3}\,dx}{6a(bc-ad)}$$

$$= \frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3}+\frac{(c(5bc-6ad))\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx}{8a^2(bc-ad)}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)}+\frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3}+\frac{\left(c^2(5bc-6ad)\right)\int}{16a^3(bc}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)}+\frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3}+\frac{\left(c^2(5bc-6ad)\right)\mathrm{Su}}{}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\left(a+bx^2\right)}+\frac{(5bc-6ad)x\left(c+dx^2\right)^{3/2}}{24a^2(bc-ad)\left(a+bx^2\right)^2}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a(bc-ad)\left(a+bx^2\right)^3}+\frac{c^2(5bc-6ad)\tan}{16a^{7/2}(bc-}$$

Mathematica [A] time = 5.26762, size = 179, normalized size = 0.9

$$\frac{\frac{3c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}-\frac{\sqrt{a}x\sqrt{c+dx^2}\left(a^2b\left(33c^2-22cdx^2-4d^2x^4\right)-6a^3d\left(5c+2dx^2\right)+8ab^2cx^2\left(5c-dx^2\right)+15b^3c^2x^4\right)}{\left(a+bx^2\right)^3(ad-bc)}}{48a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*c - d*x^2)
 - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2 - 4*d^2*x^4)))/((-(
b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(48*a^(7/2))
```

Maple [B] time = 0.044, size = 13964, normalized size = 70.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)
```

Fricas [B] time = 4.26016, size = 1994, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 + 3
*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^
2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^
2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*a*b^4*c^
3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2*b^3*c
^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b^2*c^3
- 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 - 2*a^8*b
*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 + 3*(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^7*b^2*c*d
+ a^8*b*d^2)*x^2), 1/96*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^
3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6
*a^3*b*c^2*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^5 + 2*(20*a^2*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5
*d^3)*x^3 + 3*(11*a^3*b^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^
2 + c))/(a^7*b^2*c^2 - 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d
 + a^6*b^3*d^2)*x^6 + 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3
*(a^6*b^3*c^2 - 2*a^7*b^2*c*d + a^8*b*d^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 23.1522, size = 1241, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -1/16*(5*b*c^3*sqrt(d) - 6*a*c^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x
^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c - a^4*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/24*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^5*c^3*s
qrt(d) - 18*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b^4*c^2*d^(3/2) - 75*(sqrt(d
)*x - sqrt(d*x^2 + c))^8*b^5*c^4*sqrt(d) + 240*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a*b^4*c^3*d^(3/2) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*b^3*c^2*d^(
5/2) - 96*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^3*b^2*c*d^(7/2) + 96*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^4*b*d^(9/2) + 150*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^5*c^5*sqrt(d) - 620*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^4*c^4*d^(3/2) + 96
8*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*b^3*c^3*d^(5/2) - 720*(sqrt(d)*x - sq
rt(d*x^2 + c))^6*a^3*b^2*c^2*d^(7/2) + 64*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a
^4*b*c*d^(9/2) + 128*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^5*d^(11/2) - 150*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b^5*c^6*sqrt(d) + 600*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a*b^4*c^5*d^(3/2) - 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b^3*c^4
*d^(5/2) + 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b^2*c^3*d^(7/2) + 96*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^4*b*c^2*d^(9/2) + 75*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b^5*c^7*sqrt(d) - 210*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^4*c^6*d^
(3/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^3*c^5*d^(5/2) + 48*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a^3*b^2*c^4*d^(7/2) - 15*b^5*c^8*sqrt(d) + 8*a*b^4
*c^7*d^(3/2) + 4*a^2*b^3*c^6*d^(5/2))/((a^3*b^3*c - a^4*b^2*d)*((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^3)
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=20

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0046504, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {21, 191}

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{d\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{b}$$
$$= \frac{dx}{bc\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0071051, size = 20, normalized size = 1.

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Maple [A] time = 0.003, size = 19, normalized size = 1.

$$\frac{dx}{bc}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x)
```

```
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50166, size = 55, normalized size = 2.75

$$\frac{\sqrt{dx^2 + c}dx}{bcdx^2 + bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\int \frac{1}{c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
```

```
[Out] d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

Giac [A] time = 1.11679, size = 24, normalized size = 1.2

$$\frac{dx}{\sqrt{dx^2 + c}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] d*x/(sqrt(d*x^2 + c)*b*c)
```

3.106 $\int \frac{1}{\sqrt{1-x^2}(1+x^2)} dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rubi [A] time = 0.0076408, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0050939, size = 25, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Maple [A] time = 0.003, size = 28, normalized size = 1.1

$$-\frac{\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{x^2-1}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+1\right)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 1.50801, size = 69, normalized size = 2.76

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

Giac [B] time = 1.14642, size = 69, normalized size = 2.76

$$\frac{1}{4}\sqrt{2}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^
2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.0204205, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0136459, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Maple [B] time = 0.009, size = 306, normalized size = 6.2

$$\frac{1}{2}\ln\left(\left(-2\frac{ad-bc}{b}-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-2\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x+\frac{1}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99852, size = 513, normalized size = 10.47

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(abc-a^2d\right)}, \frac{\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-a\right.}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2\right.\right.}\right.}{2\sqrt{abc-a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A] time = 1.10731, size = 95, normalized size = 1.94

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.108 $\int \frac{-1+x^2}{\left(1+x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=15

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi [A] time = 0.0040774, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {385, 215}

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)^{3/2}}\,dx = -\frac{2x}{\sqrt{1+x^2}} + \int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{2x}{\sqrt{1+x^2}} + \sinh^{-1}(x)$$

Mathematica [A] time = 0.0119162, size = 15, normalized size = 1.

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\mathrm{Arcsinh}\,(x) - 2\,\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)^(3/2),x)
```

```
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

Maxima [A] time = 1.45842, size = 18, normalized size = 1.2

$$-\frac{2\,x}{\sqrt{x^2+1}} + \mathrm{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x/sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [B] time = 1.51439, size = 108, normalized size = 7.2

$$-\frac{2\,x^2+\left(x^2+1\right)\log\left(-x+\sqrt{x^2+1}\right)+2\,\sqrt{x^2+1}x+2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^2 + 1)*log(-x + sqrt(x^2 + 1)) + 2*sqrt(x^2 + 1)*x + 2)/(x^2 +
 1)
```

Sympy [B] time = 3.45313, size = 31, normalized size = 2.07

$$\frac{x^2\operatorname{asinh}(x)}{x^2+1}-\frac{2x}{\sqrt{x^2+1}}+\frac{\operatorname{asinh}(x)}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 + 1)
```

Giac [A] time = 1.08485, size = 34, normalized size = 2.27

$$-\frac{2\,x}{\sqrt{x^2+1}}-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*x/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

3.109 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx$

Optimal. Leaf size=648

$$\frac{24192\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\cdots}{1235\left(\left(1\cdots\right.\right.}$$

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(5/3))/6175
 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x*(a - b*x^2)^(5/3)*(
3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36288*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24192*Sqrt[2]*3^(3/4)*
a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)])
```

Rubi [A] time = 0.597054, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {416, 528, 388, 195, 235, 304, 219, 1879}

$$-\frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}+\frac{24192\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(5/3))/6175
 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x*(a - b*x^2)^(5/3)*(
3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36288*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24192*Sqrt[2]*3^(3/4)*
a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
```

```
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = -\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2 - \frac{3\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\left(-78a^2b-42ab^2x^2\right) dx}{25b}$$

$$= -\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2 + \frac{9\int \left(a-bx^2\right)^{2/3}\left(1608a\right.}{475}$$

$$= -\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

$$= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235} - \frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(\right.$$

$$= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235} - \frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(\right.$$

$$= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235} - \frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(\right.$$

$$= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235} - \frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(\right.$$

Mathematica [C] time = 5.04722, size = 99, normalized size = 0.15

$$-\frac{3\left(8992a^2b^2x^5 - 40320a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right) + 3390a^3bx^3 - 15255a^4x + 2626ab^3x^7 + 247b^4x^9\right)}{6175\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (-3*(-15255*a^4*x + 3390*a^3*b*x^3 + 8992*a^2*b^2*x^5 + 2626*a*b^3*x^7 + 24
7*b^4*x^9 - 40320*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3
/2, (b*x^2)/a]))/(6175*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{2}{3}}\left(bx^2+3a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^3 \left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3x^6+9\,ab^2x^4+27\,a^2bx^2+27\,a^3\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
, x)
```

Sympy [A] time = 4.05953, size = 136, normalized size = 0.21

$$27a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
+ a**(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)
/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + 3a\right)^3 \left(-bx^2 + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

3.110 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx$

Optimal. Leaf size=617

$$\frac{10368\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\cdots}{1729\left(\left(1-\cdots\right.\right.}$$

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(1729*((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (103
68*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.425737, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {416, 388, 195, 235, 304, 219, 1879}

$$-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{10368\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\cdots}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(1729*((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (103
68*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx = -\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3\int \left(a-bx^2\right)^{2/3}\left(-60a^2b-28ab^2x^2\right)dx}{19b}$$

$$= -\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{1}{247}\left(2592a^2\right)\int \left(a-bx^2\right)^{2/3}dx$$

$$= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{\left(10368a^3\right)\int}{172}$$

$$= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{\left(15552a^3\sqrt{-}\right.}{}$$

$$= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{\left(15552a^3\sqrt{-}\right.}{}$$

$$= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{1729\left(\left(1-\sqrt{}\right.\right.}$$

Mathematica [C] time = 3.22512, size = 176, normalized size = 0.29

$$\frac{x\left(a-bx^2\right)^{2/3}\left(4b\text{Gamma}\left(\frac{1}{3}\right)\left(3ax+bx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{1}{3},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},\frac{bx^2}{a}\right)+8bx^2\text{Gamma}\left(\frac{1}{3}\right)\left(18a^2+9\right.\right.}{105a\text{Gamma}\left(-\frac{2}{3}\right)\left(1-\frac{bx^2}{a}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-2/3]*Hype
rgeometric2F1[-2/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[1/3]*Hypergeometric2F1[1/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x
+ b*x^3)^2*Gamma[1/3]*HypergeometricPFQ[{1/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]
))/(105*a*(1 - (b*x^2)/a)^(2/3)*Gamma[-2/3])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{2}{3}}\left(bx^2+3\,a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2x^4+6\,abx^2+9\,a^2\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 2.9094, size = 99, normalized size = 0.16

$$9a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

3.111 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\,dx$

Optimal. Leaf size=588

$$\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{72}{13\left(\left(1-\sqrt{3}\right)\right.}$$

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*a^2*x)/(13
*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3
]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1
3*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.369367, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {388, 195, 235, 304, 219, 1879}

$$-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*a^2*x)/(13
*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3
```

```
]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1
3*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)dx = -\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}(42a)\int \left(a-bx^2\right)^{2/3}dx$$
$$= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}\left(24a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}dx$$
$$= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a-bx^2}\right)}{13bx}$$
$$= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a-b}\right.}{13bx}$$
$$= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a}{}$$

Mathematica [C] time = 0.0720905, size = 62, normalized size = 0.11

$$\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(\frac{14a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}-a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (3*x*(a - b*x^2)^(2/3)*(-a + b*x^2 + (14*a*Hypergeometric2F1[-2/3, 1/2, 3/2
, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/13
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(-bx^2 + a\right)^{\frac{2}{3}} \left(bx^2 + 3\,a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + 3\,a\right)\left(-bx^2 + a\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + 3\,a\right)\left(-bx^2 + a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 2.02609, size = 63, normalized size = 0.11

$$3a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(5/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

3.112 $\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=740

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}}{}$$

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1/6)*ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
n[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sq
rt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b])
 + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*
x^2)^(1/3)))])/Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.320949, antiderivative size = 740, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {396, 235, 304, 219, 1879, 393}

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1/6)*ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
n[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sq
rt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b])
 + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*
x^2)^(1/3)))])/Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^2)^(2/3)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[1/(a + b*x^2)^(1/3), x], x] - Dist[(b*c - a*d)/d, Int[1/((a + b*x^2)
^(1/3)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& EqQ[b*c + 3*a*d, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{2/3}}{3a+bx^2}\,dx = (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx - \int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-}}{}$$

$$= \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-}}{}$$

$$= \frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\,\mathrm{ta}}{3}$$

Mathematica [C] time = 0.150068, size = 162, normalized size = 0.22

$$\frac{9ax\left(a-bx^2\right)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(9aF_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-2bx^2\left(F_1\left(\frac{3}{2};-\frac{2}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (9*a*x*(a - b*x^2)^(2/3)*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3
*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(
3*a)] - 2*b*x^2*(AppellF1[3/2, -2/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2
*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+3\,a}\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{2}{3}}}{3a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2 + a\right)^{\frac{2}{3}}}{bx^2 + 3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

3.113 $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=584

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\cdots}{\cdots}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(4*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.374212, antiderivative size = 584, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {412, 21, 235, 304, 219, 1879}

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1-\cdots\right.\right.}}\cdots}{4\,3^{3/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqr
```

```
t[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(4*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^2}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{\int \frac{-a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{6a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{18a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} + \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx} - \frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{-}}\right.}{6a^{2/3}bx}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}(}{((1-\sqrt{3}}}}$$

Mathematica [C] time = 0.0571026, size = 86, normalized size = 0.15

$$\frac{x\sqrt[3]{\frac{a-bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{18a\sqrt[3]{a-bx^2}} + \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) + (x*((a - b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(18*a*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+3\,a\right)^{2}}\left(-bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{2}{3}}}{\left(bx^{2}+3\,a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-bx^{2}+a\right)^{\frac{2}{3}}}{b^{2}x^{4}+6\,abx^{2}+9\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(2/3)/(b^2*x^4 + 6*a*b*x^2 + 9*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{2}{3}}}{\left(3a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

3.114 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3} dx$

Optimal. Leaf size=818

$$\frac{(a-bx^2)^{2/3} x}{36a^2(bx^2+3a)} - \frac{x}{36a^2\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{(a-bx^2)^{2/3} x}{12a(bx^2+3a)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\, 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}{\sqrt{b}x}\right.}{72\, 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(36*a^
2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) +
 ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]
) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(
216*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]
]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*3^(1/4)*a^
(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.560126, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {412, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{(a-bx^2)^{2/3} x}{36a^2(bx^2+3a)} - \frac{x}{36a^2\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{(a-bx^2)^{2/3} x}{12a(bx^2+3a)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\, 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}{\sqrt{b}x}\right.}{72\, 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(36*a^
2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) +
 ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]
) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(
216*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]
]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*3^(1/4)*a^
(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^3}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} - \frac{\int \frac{-3a+\frac{5bx^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx}{12a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\int \frac{16a^2b+\frac{8}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{288a^3b}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{108a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{36a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\right.}{216\ 2^{2/3}a^{1}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\right.}{216\ 2^{2/3}a^{1}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)} - \frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}}{7}$$

Mathematica [C] time = 0.310927, size = 252, normalized size = 0.31

$$\frac{\frac{27x\left(-\frac{b^2x^4}{a^2}+\frac{18\left(3a+bx^2\right)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}-\frac{5bx^2}{a}+6\right)}{\left(3a+bx^2\right)^2}+\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}}{972\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] ((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2
)/(3*a)])/a^3 + (27*x*(6 - (5*b*x^2)/a - (b^2*x^4)/a^2 + (18*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x
^2)/(3*a)]))))/(3*a + b*x^2)^2)/(972*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^3}\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-b x^{2}\right)^{\frac{2}{3}}}{\left(3 a+b x^{2}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{2}{3}}}{\left(b x^{2}+3 a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

3.115 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx$

Optimal. Leaf size=849

$$\frac{(a-bx^2)^{2/3}x}{144a^3(bx^2+3a)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{(a-bx^2)^{2/3}x}{54a^2(bx^2+3a)^2} + \frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)^3} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{1296\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \cdots$$

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/3))/(54*a^
2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*a + b*x^2)) - x/(144
*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt
[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqr
t[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(
2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(
2/3)*a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]]
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(
8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.650621, antiderivative size = 849, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {412, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{(a-bx^2)^{2/3}x}{144a^3(bx^2+3a)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{(a-bx^2)^{2/3}x}{54a^2(bx^2+3a)^2} + \frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)^3} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{1296\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/3))/(54*a^
2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*a + b*x^2)) - x/(144
*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt
[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqr
t[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(
2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(
2/3)*a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]]
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(
8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
```

```
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} - \frac{\int \frac{-5a+\frac{11bx^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}\,dx}{18a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{\int \frac{64a^2b-\frac{80}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx}{864a^3b}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} - \frac{\int \frac{-368a^3b^2-48a^2b^3x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{20736a^5b^2}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{432a^3} + \frac{7\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{648a^2}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\cdots}\right.}{1296\ 2^{2/3}\sqrt{3}\cdots}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\cdots}\right.}{1296\ 2^{2/3}\sqrt{3}\cdots}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{7\,\mathrm{t}\cdots}{1296\cdots}$$

Mathematica [C] time = 0.180592, size = 265, normalized size = 0.31

$$\frac{x\left(\frac{621a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)} + \frac{9a\left(a-bx^2\right)\left(75a^2+26abx^2+3b^2x^4\right)}{\left(3a+bx^2\right)^3} + bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2}\cdots\right.\right.}{3888a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*((9*a*(a - b*x^2)*(75*a^2 + 26*a*b*x^2 + 3*b^2*x^4))/(3*a + b*x^2)^3 + b
*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(
3*a)] + (621*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3
*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*
b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/
2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])))))/(3888*a^4*(a - b*x^2)^(1/3)
)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^4}\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3\,a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - bx^2\right)^{\frac{2}{3}}}{\left(3a + bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2 + a\right)^{\frac{2}{3}}}{\left(bx^2 + 3\,a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

3.116 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx$

Optimal. Leaf size=668

$$\frac{3746304\sqrt{2}3^{3/4}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\ldots}{267995\ldots}$$

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*x^2)^(5/3)
)/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432*a*x*(a - b*x^2)^(8/
3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2)^2)/31 - (11238
912*a^5*x)/(267995*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5619456*
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) + (3746304*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.570797, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {416, 528, 388, 195, 235, 304, 219, 1879}

$$-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995}+\frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425}-\frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*x^2)^(5/3)
)/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432*a*x*(a - b*x^2)^(8/
3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2)^2)/31 - (11238
912*a^5*x)/(267995*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5619456*
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) + (3746304*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
```

```
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx = -\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2 - \frac{3\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\left(-96a^2b-48ab^2x^2\right)dx}{31b}$$

$$= -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2 + \frac{9\int \left(a-bx^2\right)^{5/3}\left(2544a\right.}{775}$$

$$= -\frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

$$= \frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425} - \frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3}{31}$$

$$= \frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995} + \frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425} - \frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(\right.$$

$$= \frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995} + \frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425} - \frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(\right.$$

$$= \frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995} + \frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425} - \frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(\right.$$

$$= \frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995} + \frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425} - \frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725} - \frac{432}{775}ax\left(\right.$$

Mathematica [C] time = 5.04916, size = 110, normalized size = 0.16

$$\frac{3\left(749658a^2b^3x^7 - 1675114a^3b^2x^5 + 6243840a^5x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right) - 5312355a^4bx^3 + 5815935a^5x + 378651ab^4\right.}{1339975\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (3*(5815935*a^5*x - 5312355*a^4*b*x^3 - 1675114*a^3*b^2*x^5 + 749658*a^2*b^
3*x^7 + 378651*a*b^4*x^9 + 43225*b^5*x^11 + 6243840*a^5*x*(1 - (b*x^2)/a)^(
1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1339975*(a - b*x^2)^(1/
3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(-b x^{2}+a\right)^{\frac{5}{3}} \left(b x^{2}+3 a\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2}+3 a\right)^{3}\left(-b x^{2}+a\right)^{\frac{5}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^{4} x^{8}+8 a b^{3} x^{6}+18 a^{2} b^{2} x^{4}-27 a^{4}\right)\left(-b x^{2}+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 8*a*b^3*x^6 + 18*a^2*b^2*x^4 - 27*a^4)*(-b*x^2 + a)^(2
/3), x)
```

Sympy [A] time = 6.24803, size = 139, normalized size = 0.21

$$27a^{\frac{14}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{18a^{\frac{8}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}-\frac{8a^{\frac{5}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}-\frac{a^{\frac{2}{3}}b^4x^9\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(14/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 18*
a**(8/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
 - 8*a**(5/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)
/a)/7 - a**(2/3)*b**4*x**9*hyper((-2/3, 9/2), (11/2,), b*x**2*exp_polar(2*I
*pi)/a)/9
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

3.117 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx$

Optimal. Leaf size=637

$$\frac{38016\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\cdots}{8645\left(\left(1\cdots\right.\right.}$$

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(5/3))/6175
 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/
25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5
7024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.483626, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {416, 388, 195, 235, 304, 219, 1879}

$$-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645}+\frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175}+\frac{38016\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(5/3))/6175
 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/
25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5
7024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = -\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{3\int \left(a-bx^2\right)^{5/3}\left(-78a^2b-34ab^2x^2\right) dx}{25b}$$

$$= -\frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) + \frac{1}{475}\left(4752a^2\right)\int \left(a-bx^2\right)^{5/3} dx$$

$$= \frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) + \frac{\left(9504a^3\right)\int}{}$$

$$= \frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645} + \frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}$$

$$= \frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645} + \frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}$$

$$= \frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645} + \frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}$$

$$= \frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645} + \frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175} - \frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}$$

Mathematica [C] time = 2.77532, size = 173, normalized size = 0.27

$$\frac{x\left(a-bx^2\right)^{2/3}\left(4b\text{Gamma}\left(-\frac{2}{3}\right)\left(3ax+bx^3\right)^2\text{HypergeometricPFQ}\left(\left\{-\frac{2}{3},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},\frac{bx^2}{a}\right)+8bx^2\text{Gamma}\left(-\frac{2}{3}\right)\left(18a\right.\right.}{105\text{Gamma}\left(-\frac{5}{3}\right)\left(1-\frac{b}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-5/3]*Hype
rgeometric2F1[-5/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[-2/3]*Hypergeometric2F1[-2/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*
x + b*x^3)^2*Gamma[-2/3]*HypergeometricPFQ[{-2/3, 3/2, 2}, {1, 9/2}, (b*x^2
)/a]))/(105*(1 - (b*x^2)/a)^(2/3)*Gamma[-5/3])
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(-bx^{2}+a\right)^{\frac{5}{3}} \left(bx^{2}+3 a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{2}+3 a\right)^{2} \left(-bx^{2}+a\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^{3} x^{6}+5 \, a b^{2} x^{4}+3 \, a^{2} b x^{2}-9 \, a^{3}\right) \left(-b x^{2}+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integrate(-(b^3*x^6 + 5*a*b^2*x^4 + 3*a^2*b*x^2 - 9*a^3)*(-b*x^2 + a)^(2/3),
 x)
```

Sympy [A] time = 4.90736, size = 131, normalized size = 0.21

$$9 a^{\frac{11}{3}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right) - a^{\frac{8}{3}} b x^{3} {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right) - a^{\frac{5}{3}} b^{2} x^{5} {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right) - \frac{a^{\frac{2}{3}} b^{3} x^{7} {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(
8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(5/
3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(2
/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

3.118 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)\,dx$

Optimal. Leaf size=608

$$\frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\cdots}{1729\left(\left(1-\cdots\right.\right.}$$

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400*Sqrt[2]*3^(3/4
)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi [A] time = 0.420145, antiderivative size = 608, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {388, 195, 235, 304, 219, 1879}

$$-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\cdots}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400*Sqrt[2]*3^(3/4
)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)dx = -\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{1}{19}(60a)\int\left(a-bx^2\right)^{5/3}dx$$

$$= \frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{1}{247}\left(600a^2\right)\int\left(a-bx^2\right)^{2/3}dx$$

$$= \frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{\left(2400a^3\right)\int\frac{1}{\sqrt[3]{a-bx^2}}dx}{1729}$$

$$= \frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}-\frac{\left(3600a^3\sqrt{-bx^2}\right)\text{Subst}}{17}$$

$$= \frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{\left(3600a^3\sqrt{-bx^2}\right)\text{Subst}}{}$$

$$= \frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}$$

Mathematica [C] time = 0.0615369, size = 68, normalized size = 0.11

$$\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(\frac{20a^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}-\left(a-bx^2\right)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (3*x*(a - b*x^2)^(2/3)*(-(a - b*x^2)^2 + (20*a^2*Hypergeometric2F1[-5/3, 1/
2, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/19
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int\left(-bx^2+a\right)^{\frac{5}{3}}\left(bx^2+3a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+3a\right)\left(-bx^2+a\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2x^4+2\,abx^2-3\,a^2\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 3.47799, size = 100, normalized size = 0.16

$$3a^{\frac{8}{3}}x\,{}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{2a^{\frac{5}{3}}bx^3\,{}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3}-\frac{a^{\frac{2}{3}}b^2x^5\,{}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3 - a**
(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

3.119 $\int \frac{\left(a-bx^2\right)^{5/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=765

$$-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}}{}$$

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqr
t[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/
3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
 + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.443618, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {416, 530, 235, 304, 219, 1879, 393}

$$\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}}{\left(\left(1-\sqrt{3}\right)\right.}}}{7bx\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqr
t[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/
3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
 + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{5/3}}{3a+bx^2}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{3\int \frac{\frac{16a^2b}{3}-\frac{32}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}dx}{7b}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{1}{7}(32a)\int \frac{1}{\sqrt[3]{a-bx^2}}dx + \left(16a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}dx$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{4\sqrt[3]{2}a^{7/6}\tanh}{3\sqrt{b}}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{4\sqrt[3]{2}a^{7/6}\tanh}{3\sqrt{b}}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}(}{}\right)}{\sqrt{3}\sqrt{b}}$$

Mathematica [C] time = 0.133257, size = 231, normalized size = 0.3

$$\frac{x\left(27\left(\frac{48a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}-a+bx^2\right)-32bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\right.}{63\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (x*(-32*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
(b*x^2)/(3*a)] + 27*(-a + b*x^2 + (48*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*
x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))))/(
63*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{b x^{2}+3 a} \left(-b x^{2}+a \right)^{\frac{5}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{5}{3}}}{b x^{2}+3 \, a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - b x^{2}\right)^{\frac{5}{3}}}{3 a + b x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2 + a\right)^{\frac{5}{3}}}{bx^2 + 3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

3.120 $\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=775

$$\frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b
]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*
a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (
11*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3
/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.436631, antiderivative size = 775, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {413, 530, 235, 304, 219, 1879, 393}

$$\frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b
]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*
a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (
11*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3
/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^2}\,dx = \frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)} + \frac{\int \frac{-2a^2b+\frac{22}{3}ab^2x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{6ab}$$

$$= \frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)} + \frac{11}{9}\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx - (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

$$= \frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right.}{3\sqrt{b}}$$

$$= \frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right.}{3\sqrt{b}}$$

$$= \frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)} - \frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{bx}}\right.}{\sqrt{3}\sqrt{b}}$$

Mathematica [C] time = 0.139529, size = 235, normalized size = 0.3

$$\frac{x\left(\frac{27\left(-\frac{9a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+2a-2bx^2\right)}{3a+bx^2}+\frac{11bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a}\right)}{81\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*((11*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
(b*x^2)/(3*a)])/a + (27*(2*a - 2*b*x^2 - (9*a^2*AppellF1[1/2, 1/3, 1, 3/2,
(b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*
x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a
)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(3*a + b*x^2
```

```
)))/(81*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^2}\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{5}{3}}}{\left(3a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3} \, dx$

Optimal. Leaf size=815

$$-\frac{\left(a-bx^2\right)^{2/3} x}{18a\left(bx^2+3a\right)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3} x}{3\left(bx^2+3a\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-$$

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))/(18*a*(3*
a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcT
an[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18
*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3
)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b
*x^2)^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) -
 (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1
 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.613432, antiderivative size = 815, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {413, 12, 471, 530, 235, 304, 219, 1879, 393}

$$-\frac{\left(a-bx^2\right)^{2/3} x}{18a\left(bx^2+3a\right)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3} x}{3\left(bx^2+3a\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))/(18*a*(3*
a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcT
an[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18
*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3
)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b
*x^2)^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) -
 (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1
 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^3}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} + \frac{\int \frac{16ab^2x^2}{3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx}{12ab}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} + \frac{1}{9}(4b)\int \frac{x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} - \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)} + \frac{\int \frac{a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{18a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} - \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)} + \frac{1}{9}\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{54a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} - \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{54\,2^{2/3}a^{5/6}\sqrt{}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} - \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{54\,2^{2/3}a^{5/6}\sqrt{}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)^2} - \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)} + \frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\sqrt{3}\right.}{18\,2}$$

Mathematica [C] time = 0.219609, size = 252, normalized size = 0.31

$$\frac{\frac{27x\left(\frac{9a(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\frac{b^2x^4}{a}+3a-4bx^2\right)}{(3a+bx^2)^2}-\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^2}}{486\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (-((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x
^2)/(3*a)])/a^2) + (27*x*(3*a - 4*b*x^2 + (b^2*x^4)/a + (9*a*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x
^2)/(3*a)]))))/(3*a + b*x^2)^2)/(486*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+3a)^3}\left(-bx^2+a\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{(bx^2+3a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - bx^2\right)^{\frac{5}{3}}}{\left(3a + bx^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2 + a\right)^{\frac{5}{3}}}{\left(bx^2 + 3\,a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=659

$$\frac{1264896\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\ldots}{8645\left(\left(1\ldots\right.\right.}$$

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^2)^(2/3)*(
3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (3*x*(
a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (3794688*a^4*x)/(8645*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (1897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3
)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (
1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.52655, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {416, 528, 388, 235, 304, 219, 1879}

$$-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}+\frac{1264896\sqrt{2}3^{3/4}a^{13/3}\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^2)^(2/3)*(
3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (3*x*(
a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (3794688*a^4*x)/(8645*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (1897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3
)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (
1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
```

```
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3 - \frac{3\int \frac{(3a+bx^2)^2(-78a^2b-50ab^2x^2)}{\sqrt[3]{a-bx^2}}\,dx}{25b}$$

$$= -\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3 + \frac{9\int \frac{(3a+bx^2)(1632a^3b^2+1344a^2b^3x^2)}{\sqrt[3]{a-bx^2}}\,dx}{475b^2}$$

$$= -\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175} - \frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

$$= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225} - \frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175} - \frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 -$$

$$= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225} - \frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175} - \frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 -$$

$$= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225} - \frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175} - \frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 -$$

$$= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225} - \frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175} - \frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 -$$

Mathematica [C] time = 5.05644, size = 98, normalized size = 0.15

$$\frac{3x\left(184044a^2b^2x^4+2108160a^4\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+727830a^3bx^2-941085a^4+27482ab^3x^6+1729b^4x^8\right)}{43225\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-941085*a^4 + 727830*a^3*b*x^2 + 184044*a^2*b^2*x^4 + 27482*a*b^3*x^6
 + 1729*b^4*x^8 + 2108160*a^4*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3,
1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(b x^{2}+3 a\right)^{4} \frac{1}{\sqrt[3]{-b x^{2}+a}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+3 a\right)^{4}}{\left(-b x^{2}+a\right)^{\frac{1}{3}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b^{4} x^{8}+12\, a b^{3} x^{6}+54\, a^{2} b^{2} x^{4}+108\, a^{3} b x^{2}+81\, a^{4}\right)\left(-b x^{2}+a\right)^{\frac{2}{3}}}{b x^{2}-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4
)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A] time = 4.45997, size = 165, normalized size = 0.25

$$81a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+36a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{54a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{12a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}+\frac{b^4x\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)
```

```
[Out] 81*a**(11/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 36*a
**(8/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 54*a
**(5/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 +
 12*a**(2/3)*b**3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a
)/7 + b**4*x**9*hyper((1/3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)/(9*a
**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^4}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

3.123 $\int \frac{\left(3a+bx^2\right)^3}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=628

$$\frac{71712\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{2}{1729\left(\left(1-\right.\right.}$$

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (215136*a^3*x)/(1
729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (107568*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.439609, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {416, 528, 388, 235, 304, 219, 1879}

$$-\frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{71712\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (215136*a^3*x)/(1
729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (107568*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{3\int \frac{(3a+bx^2)(-60a^2b-36ab^2x^2)}{\sqrt[3]{a-bx^2}}\,dx}{19b}$$

$$= -\frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{9\int \frac{888a^3b^2+584a^2b^3x^2}{\sqrt[3]{a-bx^2}}\,dx}{247b^2}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{(71712a}{}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{(107568}{}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{(107568}{}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{}{1729\left(\left(1\right.\right.}$$

Mathematica [C] time = 5.04694, size = 88, normalized size = 0.14

$$\frac{3\left(23904a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+7041a^2bx^3-8343a^3x+1211ab^2x^5+91b^3x^7\right)}{1729\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*(-8343*a^3*x + 7041*a^2*b*x^3 + 1211*a*b^2*x^5 + 91*b^3*x^7 + 23904*a^3*
x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1729
*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^3 \frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^3}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^3x^6+9\,ab^2x^4+27\,a^2bx^2+27\,a^3\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b*x^2 - a), x)
```

Sympy [A] time = 3.45422, size = 129, normalized size = 0.21

$$27a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(8/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**
(5/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**(
2/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 + b*
*3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^3}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

3.124 $\int \frac{\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=597

$$\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{324}{91\left(\left(1-\sqrt{3}\right)\right.}$$

```
[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
- (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1620*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.380998, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {416, 388, 235, 304, 219, 1879}

$$-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
- (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1620*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3\int \frac{-42a^2b-22ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{13b}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{1}{91}\left(1080a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{\left(1620a^2\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx\ldots}\right.}{91bx}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{\left(1620a^2\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a\ldots}\right.}{91bx}$$

$$= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1620\sqrt[4]{3}\sqrt{2+\ldots}}{\ldots}$$

Mathematica [C] time = 4.43264, size = 158, normalized size = 0.26

$$\frac{x\sqrt[3]{1-\frac{bx^2}{a}}\left(4b\left(3ax+bx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{4}{3},\frac{3}{2},2\right\},\left\{1,\frac{9}{2}\right\},\frac{bx^2}{a}\right)+8bx^2\left(18a^2+9abx^2+b^2x^4\right)\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{9}{2};\frac{b}{}\right.\right.}{315a\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/3)*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Hypergeometr
ic2F1[1/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^2*x^4)*Hy
pergeometric2F1[4/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x + b*x^3)^2*Hypergeom
etricPFQ[{4/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]))/(315*a*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2x^4 + 6\,abx^2 + 9\,a^2\right)\left(-bx^2 + a\right)^{\frac{2}{3}}}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A] time = 2.54655, size = 94, normalized size = 0.16

$$9a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + 2a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
```

```
[Out] 9*a**(5/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**(
2/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + b**2*x*
*5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^2}{\left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=568

$$\frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{\ldots}}}{\ldots}$$

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.304862, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {388, 235, 304, 219, 1879}

$$\frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(
```

```
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
```

```
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{1}{7}(24a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{\left(36a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} + \frac{\left(36a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx} - \frac{\left(36\sqrt{2\left(2+\sqrt{3}\right)}a^{4/3}\sqrt{-bx^2}\right.}{\ldots}$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-b\ldots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\ldots\right.}}}{7bx\sqrt{-\frac{\sqrt[3]{a}\ldots}{\left(\left(1-\ldots\right.\right.}}}$$

Mathematica [C] time = 0.0243245, size = 62, normalized size = 0.11

$$\frac{3x\left(8a\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{7\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-a + b*x^2 + 8*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (bx^2+3a)\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 3\,a}{\left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(bx^2 + 3\,a\right)\left(-bx^2 + a\right)^{\frac{2}{3}}}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A] time = 1.56947, size = 60, normalized size = 0.11

$$3a^{\frac{2}{3}}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{bx^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
```

```
[Out] 3*a**(2/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + b*x**3
*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 3\,a}{\left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.030783, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```


```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```


Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-b\ldots}\right.}\right.}{2\,2^{2/3}a^{5/6}\sqrt{b}}$$

Mathematica [C] time = 0.0293798, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*
a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+3\,a}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=787

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}-\frac{}{24a^2\left(\left(1-\right.\right.}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/
3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(
a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcT
anh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x
)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt
[b]) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4
)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.438898, antiderivative size = 787, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {414, 530, 235, 304, 219, 1879, 393}

$$\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{\left(\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/
3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(
a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcT
anh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x
)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt
[b]) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4
)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} - \frac{\int \frac{-7ab-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{24a^2b}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{72a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{4a}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}} + \cdots$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}} + \cdots$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} - \frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{\cdots}\right.}{\cdots}\right)}{8\,2^{2/3}\sqrt{3}a\cdots}$$

<u>Mathematica</u> [C] time = 0.153498, size = 234, normalized size = 0.3

$$\frac{x\left(\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27\left(\frac{a-bx^2}{a^2}+\frac{63F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}\right)}{3a+bx^2}\right)}{648\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*
x^2)/(3*a)])/a^3 + (27*((a - b*x^2)/a^2 + (63*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^
2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(3*a + b*x^2))
```

```
)/(648*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^2}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^2\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}\,dx$

Optimal. Leaf size=818

$$\frac{5\left(a-bx^2\right)^{2/3}x}{288a^3\left(bx^2+3a\right)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3}x}{48a^2\left(bx^2+3a\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{\sqrt{b}x}\right.}{144\,2^{2/3}\sqrt{3}a^{17/6}\cdots}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)^(2/3))/(2
88*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a
^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2
)^(1/3)))/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh
[(Sqrt[b]*x)/Sqrt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]
*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(17/6)
*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(19
2*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.543291, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{5\left(a-bx^2\right)^{2/3}x}{288a^3\left(bx^2+3a\right)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3}x}{48a^2\left(bx^2+3a\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{\sqrt{b}x}\right.}{144\,2^{2/3}\sqrt{3}a^{17/6}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)^(2/3))/(2
88*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a
^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2
)^(1/3)))/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh
[(Sqrt[b]*x)/Sqrt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]
*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(17/6)
*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(19
2*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} - \frac{\int \frac{-15ab+\frac{5b^2x^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx}{48a^2b}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x\left(a-bx^2\right)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{\int \frac{100a^2b^2+\frac{20}{3}ab^3x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{1152a^4b^2}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x\left(a-bx^2\right)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{864a^3} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{72a^2}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x\left(a-bx^2\right)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx}\right.}{\sqrt{bx}}\right.}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x\left(a-bx^2\right)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx}\right.}{\sqrt{bx}}\right.}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x\left(a-bx^2\right)^{2/3}}{288a^3\left(3a+bx^2\right)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{5\tan^{-1}\left(\right.}{144\ 2^{2/3}\sqrt{3}a}$$

Mathematica [C] time = 0.161985, size = 255, normalized size = 0.31

$$\frac{x\left(\frac{6075a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)} + 5bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right) + \frac{27a(}{}\right.}{7776a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*((27*a*(a - b*x^2)*(21*a + 5*b*x^2))/(3*a + b*x^2)^2 + 5*b*x^2*(1 - (b*x
^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + (6075*
a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(
9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-Appel
lF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/
2, (b*x^2)/a, -(b*x^2)/(3*a)])))))/(7776*a^4*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^3}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^3\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

3.129 $\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=623

$$-\frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{20\ldots}{91\left(\left(1-\sqrt{3}\ldots\right.\right.}$$

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2*x)/(91*((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (669
6*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.434653, antiderivative size = 623, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {413, 528, 388, 235, 304, 219, 1879}

$$\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\ldots\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2*x)/(91*((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (669
6*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{4/3}}\,dx = \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{\left(3a+bx^2\right)\left(6a^2b+18ab^2x^2\right)}{\sqrt[3]{a-bx^2}}\,dx}{2ab}$$

$$= \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} + \frac{9\int \frac{-132a^3b^2-188a^2b^3x^2}{\sqrt[3]{a-bx^2}}\,dx}{26ab^2}$$

$$= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} - \frac{1}{91}\left(6696a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} + \frac{\left(10044a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\right.}{91bx}$$

$$= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} - \frac{\left(10044a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\right.}{91}$$

$$= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}} + \frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right.}$$

Mathematica [C] time = 5.04996, size = 76, normalized size = 0.12

$$-\frac{3x\left(2232a^2\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-3051a^2+132abx^2+7b^2x^4\right)}{91\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-3051*a^2 + 132*a*b*x^2 + 7*b^2*x^4 + 2232*a^2*(1 - (b*x^2)/a)^(1/3)
*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^3}{\left(-bx^2 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3x^6 + 9\,ab^2x^4 + 27\,a^2bx^2 + 27\,a^3\right)\left(-bx^2 + a\right)^{\frac{2}{3}}}{b^2x^4 - 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a + bx^2\right)^3}{\left(a - bx^2\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^3}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=592

$$-\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{162\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

```
[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3) + (324*a
*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
- b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.367632, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {413, 388, 235, 304, 219, 1879}

$$-\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3) + (324*a
*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
- b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{4/3}}\,dx = \frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{6a^2b+10ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{2ab}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}} - \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}} + \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}} - \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx} + \frac{\left(162\sqrt{2}\right.}{}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}} + \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-}\right.}{}$$

Mathematica [C] time = 5.04354, size = 62, normalized size = 0.1

$$-\frac{3x\left(36a\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-57a+bx^2\right)}{7\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-57*a + b*x^2 + 36*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/
2, 3/2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{4}+6\,abx^{2}+9\,a^{2}\right)\left(-bx^{2}+a\right)^{\frac{2}{3}}}{b^{2}x^{4}-2\,abx^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*
x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3a+bx^{2}\right)^{2}}{\left(a-bx^{2}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{2}+3\,a\right)^{2}}{\left(-bx^{2}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

3.131 $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=561

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\cdots}{\cdots}$$

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
 + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.304341, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {385, 235, 304, 219, 1879}

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
 + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
```

```
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx = \frac{6x}{\sqrt[3]{a-bx^2}} - 3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} - \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx} + \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{-bx^2}\right)\text{Subst}\,(\ldots}{bx}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2\ldots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\ldots}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-b\ldots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\ldots}\right)\ldots}}}$$

Mathematica [C] time = 0.0168319, size = 53, normalized size = 0.09

$$\frac{6x-3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (6*x - 3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
])/(a - b*x^2)^(1/3)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(b x^{2}+3 a\right) \left(-b x^{2}+a\right)^{-\frac{4}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b x^{2} + 3 \, a}{{\left(-b x^{2} + a\right)}^{\frac{4}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + 3 \, a\right)} {\left(-b x^{2} + a\right)}^{\frac{2}{3}}}{b^{2} x^{4} - 2 \, a b x^{2} + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [A] time = 3.80125, size = 60, normalized size = 0.11

$$\frac{3x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[3]{a}}+\frac{bx^3{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)
```

```
[Out] 3*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/3) + b*x**3
*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

3.132 $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=776

$$-\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{}{8a^2\left(\left(1\right.\right.}$$

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]
*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[
b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)
*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3
)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.443219, antiderivative size = 776, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {414, 530, 235, 304, 219, 1879, 393}

$$\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{3^{3/4}\left(\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]
*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[
b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)
*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3
)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx = \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3\int \frac{-\frac{ab}{3}-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{8a^2b}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{8a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{4a}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}} + \text{—}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}} + \text{—}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\ldots\right.}{\sqrt{b}\ldots}\right.}{8\,2^{2/3}\sqrt{3}a^{11\ldots}}$$

Mathematica [C] time = 0.129158, size = 226, normalized size = 0.29

$$\frac{x\left(27\left(\frac{1}{a^2}-\frac{3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)-\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}\right)}{72\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(
b*x^2)/(3*a)])/a^3) + 27*(a^(-2) - (3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)
/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))))/(72*(
a - b*x^2)^(1/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{b x^{2}+3 a}\left(-b x^{2}+a\right)^{-\frac{4}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+3 a\right)\left(-b x^{2}+a\right)^{\frac{4}{3}}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a-b x^{2}\right)^{\frac{4}{3}}\left(3 a+b x^{2}\right)}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3a\right)\left(-bx^2 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

3.133 $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=807

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{bx}}\right.}{16\ 2^{2/3}\sqrt{3}a^{17/6}}$$

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)) +
 x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(8*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rubi [A] time = 0.533981, antiderivative size = 807, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{bx}}\right.}{16\ 2^{2/3}\sqrt{3}a^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)) +
 x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(8*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^2}\,dx = \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{\int \frac{-7ab+\frac{5b^2x^2}{3}}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx}{24a^2b}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{\int \frac{-\frac{8}{3}a^2b^2+\frac{16}{9}ab^3x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{64a^4b^2}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{36a^3} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{8a^2}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{bx}}\right.}{16\ 2^{2/3}\sqrt{3}a^{17/6}}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{bx}}\right.}{16\ 2^{2/3}\sqrt{3}a^{17/6}}$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} + \frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan}{16\ 2^{2/}}$$

Mathematica [C] time = 0.172227, size = 236, normalized size = 0.29

$$\frac{x\left(\frac{27a\left(\frac{9a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+7a+2bx^2\right)}{3a+bx^2}-2bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}{648a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-2*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(
b*x^2)/(3*a)] + (27*a*(7*a + 2*b*x^2 + (9*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^
2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(3*a + b*x^2))
)/(648*a^4*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

3.134 $\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx$

Optimal. Leaf size=849

$$-\frac{19\left(a-bx^2\right)^{2/3}x}{1152a^4\left(bx^2+3a\right)}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)^2}+\ldots$$

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a - b*x^2)^
(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a^4*(3*a + b*x^2)) +
(19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(
Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*
ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(864*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3)
+ 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b]) + (19*Sqrt[2
 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sq
rt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.653338, antiderivative size = 849, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 530, 235, 304, 219, 1879, 393}

$$-\frac{19\left(a-bx^2\right)^{2/3}x}{1152a^4\left(bx^2+3a\right)}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a - b*x^2)^
(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a^4*(3*a + b*x^2)) +
(19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(
Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*
ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(864*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3)
+ 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b]) + (19*Sqrt[2
 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sq
rt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
```

```
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx = \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} - \frac{\int \frac{-15ab+\frac{11b^2x^2}{3}}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^2}\,dx}{48a^2b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{\int \frac{-6a^2b^2-\frac{170}{9}ab^3x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}\,dx}{128a^4b^2}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{19x\left(a-bx^2\right)^{2/3}}{1152a^4\left(3a+bx^2\right)} + \frac{\int \frac{\frac{296a^3\cdots}{3}}{\sqrt[3]{a-\cdots}}}{3\cdots}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{19x\left(a-bx^2\right)^{2/3}}{1152a^4\left(3a+bx^2\right)} - \frac{19\int \cdots}{345\cdots}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{19x\left(a-bx^2\right)^{2/3}}{1152a^4\left(3a+bx^2\right)} + \frac{7\tan\cdots}{288\ 2\cdots}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{19x\left(a-bx^2\right)^{2/3}}{1152a^4\left(3a+bx^2\right)} + \frac{7\tan\cdots}{288\ 2\cdots}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)} - \frac{19x\left(a-bx^2\right)^{2/3}}{1152a^4\left(3a+bx^2\right)} + \frac{\cdots}{1152a^{\cdots}}$$

Mathematica [C] time = 0.230172, size = 256, normalized size = 0.3

$$\frac{x\left(\frac{27a\left(\frac{333a^2(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+273a^2+140abx^2+19b^2x^4\right)}{(3a+bx^2)^2}-19bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\right.\right.}{31104a^5\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(-19*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
(b*x^2)/(3*a)] + (27*a*(273*a^2 + 140*a*b*x^2 + 19*b^2*x^4 + (333*a^2*(3*a
+ b*x^2)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -(b*x^2)/(3*a)]))))/(3*a + b*x^2)^2))/(31104*a^5*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^3}\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)^3\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3\,a\right)^3 \left(-bx^2 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=653

$$\frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36}{91\left(\left(1-\sqrt{3}\right.\right.}$$

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/1
3 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2)^3)/(2*
(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (18468*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12312*Sqrt[2]*3^(3/4)*a^(7/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.512337, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 526, 528, 388, 235, 304, 219, 1879}

$$-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/1
3 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2)^3)/(2*
(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (18468*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12312*Sqrt[2]*3^(3/4)*a^(7/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
```

```
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{(3a+bx^2)^2(-12a^2b+20ab^2x^2)}{(a-bx^2)^{4/3}}\,dx}{8ab}$$

$$= -\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{9\int \frac{(3a+bx^2)(-48a^3b^2-48a^2b^3x^2)}{\sqrt[3]{a-bx^2}}\,dx}{16a^2b^2}$$

$$= -\frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{27\int \frac{768a^4b^3+640a^3b^4x^2}{\sqrt[3]{a-bx^2}}\,dx}{208a^2b^3}$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{1}{91}(1231$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{(18468a}{}$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{(18468a}{}$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{}{91((1-}$$

Mathematica [C] time = 5.07125, size = 96, normalized size = 0.15

$$-\frac{3\left(-4104a^2x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-4743a^2bx^3+1647a^3x+177ab^2x^5+7b^3x^7\right)}{91(a-bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3*(1647*a^3*x - 4743*a^2*b*x^3 + 177*a*b^2*x^5 + 7*b^3*x^7 - 4104*a^2*x*(
a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
```

```
]))/(91*(a - b*x^2)^(4/3))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^4 \left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3a\right)^4}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^4x^8+12\,ab^3x^6+54\,a^2b^2x^4+108\,a^3bx^2+81\,a^4\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4
)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a+bx^2\right)^4}{\left(a-bx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**4/(a - b*x**2)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^4}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=596

$$\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{}}}{}$$

[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(4/3))
 - (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (162*3^(1/4)*
Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])

Rubi [A] time = 0.426372, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {413, 21, 388, 235, 304, 219, 1879}

$$\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}}{}$$

Antiderivative was successfully verified.

[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]

```
[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(4/3))
- (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (162*3^(1/4)*
Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(3a+bx^2\right)^3}{\left(a-bx^2\right)^{7/3}}\,dx = \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} - \frac{3\int \frac{\left(3a+bx^2\right)\left(-12a^2b+12ab^2x^2\right)}{\left(a-bx^2\right)^{4/3}}\,dx}{8ab}$$

$$= \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} + \frac{9}{2}\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} + \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} - \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} + \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx} - \frac{(162}{}$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}} - \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 5.06674, size = 83, normalized size = 0.14

$$\frac{81a^2x + 108ax\left(a-bx^2\right)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right) + 90abx^3 - 3b^2x^5}{7\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (81*a^2*x + 90*a*b*x^3 - 3*b^2*x^5 + 108*a*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1
/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(7*(a - b*x^2)^(4/3))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^3 \left(-bx^2+a\right)^{-\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3a\right)^3}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^3x^6+9\,ab^2x^4+27\,a^2bx^2+27\,a^3\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=44

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

```
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

Rubi [A] time = 0.0172769, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {413, 383}

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{-12a^2b+4ab^2x^2}{(a-bx^2)^{4/3}}\,dx}{8ab}$$
$$= \frac{9x}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}$$

Mathematica [A] time = 5.02633, size = 24, normalized size = 0.55

$$\frac{9ax-3bx^3}{(a-bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

Maple [A] time = 0.004, size = 24, normalized size = 0.6

$$3\,\frac{x(-bx^2+3\,a)}{(-bx^2+a)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x)
```

```
[Out] 3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

Maxima [A] time = 1.14021, size = 45, normalized size = 1.02

$$\frac{3(bx^3-3\,ax)}{(bx^2-a)(-bx^2+a)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

Fricas [A] time = 1.77402, size = 90, normalized size = 2.05

$$-\frac{3\left(bx^3-3\,ax\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^2x^4-2\,abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="giac")

[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

3.138 $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=590

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),4\sqrt{3}-7\right)}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*x)/(4*a*((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.366449, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {385, 199, 235, 304, 219, 1879}

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*x)/(4*a*((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
```

```
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

<u>Rule 385</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

<u>Rule 199</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

<u>Rule 235</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

<u>Rule 304</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

<u>Rule 219</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{3a+bx^2}{\left(a-bx^2\right)^{7/3}}\,dx = \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{3}{2}\int \frac{1}{\left(a-bx^2\right)^{4/3}}\,dx$$

$$= \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}-\frac{3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{4a}$$

$$= \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{8abx}$$

$$= \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}-\frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{8abx}+\frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\right.}{}$$

$$= \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\cdots}}{8a\cdots}$$

Mathematica [C] time = 0.0380983, size = 74, normalized size = 0.13

$$\frac{-3x\left(a-bx^2\right)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+15ax-9bx^3}{4a\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (15*a*x - 9*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1
[1/3, 1/2, 3/2, (b*x^2)/a])/(4*a*(a - b*x^2)^(4/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (bx^2+3\,a)\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b
*x^2 - a^3), x)
```

Sympy [A] time = 11.2917, size = 60, normalized size = 0.1

$$\frac{3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{4}{3}}}+\frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

```
[Out] 3*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(4/3) + b*x**3
*hyper((3/2, 7/3), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

3.139 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$

Optimal. Leaf size=796

$$\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/3)) + (21*
x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7
*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(32*Sqrt[2]*a^(8/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.536772, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/3)) + (21*
x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7
*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(32*Sqrt[2]*a^(8/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}dx = \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{3\int \frac{\frac{23ab}{3}+\frac{5b^2x^2}{3}}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}dx}{32a^2b}$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{9\int \frac{-\frac{68}{9}a^2b^2-\frac{28}{9}ab^3x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}dx}{256a^4b^2}$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} - \frac{7\int \frac{1}{\sqrt[3]{a-bx^2}}dx}{64a^3} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}dx}{16a^2}$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} -$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} -$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{}}$$

Mathematica [C] time = 0.205683, size = 248, normalized size = 0.31

$$\frac{x\left(27a\left(\frac{9a-7bx^2}{a-bx^2}-\frac{51a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}\right)-7bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a}\right.}{576a^4\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-7*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(
b*x^2)/(3*a)] + 27*a*((9*a - 7*b*x^2)/(a - b*x^2) - (51*a^2*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x
^2)/(3*a)]))))))/(576*a^4*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+3a}\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3a\right)\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a-bx^2)^{\frac{7}{3}}\,(3a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

3.140 $\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx$

Optimal. Leaf size=827

$$\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(bx^2+3a\right)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{5x}{384a^3\left(a-bx^2\right)^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{128\;2^{2/3}a^{23/\cdots}}$$

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(1/3)) + x/
(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*a^4*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x
)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[
b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcT
anh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128*2^(2
/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(512*3^(3/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqrt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)])
```

Rubi [A] time = 0.63431, antiderivative size = 827, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 530, 235, 304, 219, 1879, 393}

$$\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(bx^2+3a\right)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{5x}{384a^3\left(a-bx^2\right)^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{128\;2^{2/3}a^{23/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(1/3)) + x/
(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*a^4*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x
)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[
b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcT
anh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128*2^(2
/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(512*3^(3/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqrt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
```

```
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)^2}\,dx = \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{\int \frac{-7ab+\frac{11b^2x^2}{3}}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx}{24a^2b}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{\int \frac{-\frac{194}{3}a^2b^2-\frac{50}{9}ab^3x^2}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx}{256a^4b^2}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{3\int \frac{\frac{344a^3b^3}{9}+\frac{632}{27}a^2b^4}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}}{2048a^6b^3}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{79\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{2304a^4} +$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} + \frac{79}{768a^4\left(\left(1-\sqrt{3}\right)\right.}$$

Mathematica [C] time = 0.241185, size = 259, normalized size = 0.31

$$\frac{x\left(\frac{27a\left(\frac{299a^2-148abx^2-79b^2x^4}{a-bx^2}-\frac{387a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}\right)}{3a+bx^2}-79bx^2\sqrt[3]{1-\frac{bx^2}{a}}F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx}{3a}\right.\right.}{20736a^5\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-79*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
(b*x^2)/(3*a)] + (27*a*((299*a^2 - 148*a*b*x^2 - 79*b^2*x^4)/(a - b*x^2) -
(387*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -(b*x^2)/(3*a)]))))/(3*a + b*x^2)))/(20736*a^5*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

3.141 $\int \frac{1}{\left(-3a-bx^2\right)\sqrt[3]{-a+bx^2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a
]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)^(1/
3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
 + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - Ar
cTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1
/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0727463, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {393}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a
]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)^(1/
3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
 + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - Ar
cTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1
/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
```

```
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(-3a-bx^2\right)\sqrt[3]{-a+bx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{bx}}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\ }{\sqrt{a}}\right)}{2\,2^{2/}}$$

Mathematica [C] time = 0.124108, size = 163, normalized size = 0.65

$$-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{bx^2-a}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] (-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((-a + b*x^2
)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(
3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + A
ppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-3\,a}\frac{1}{\sqrt[3]{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2+3\,a\right)\left(bx^2-a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

```
[Out] -Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2+3\,a\right)\left(bx^2-a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[b]*
x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt
[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/(S
qrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.0287481, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```


```
[Out] -ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[b]*
x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt
[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/(S
qrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```


Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3a-bx^2\right)\sqrt[3]{a+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

Mathematica [C] time = 0.151177, size = 166, normalized size = 0.82

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{\left(3a-bx^2\right)\sqrt[3]{a+bx^2}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/((3*a - b*x
^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)
/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)]
- AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)])))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2+3\,a}\frac{1}{\sqrt[3]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(bx^2-3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-3a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

```
[Out] -Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(bx^2-3\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d]) +
(Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*d*x^
2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)]/(2*
2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x^2
)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi [A] time = 0.0424418, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {392}

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```


```
[Out] -ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d]) +
(Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*d*x^
2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)]/(2*
2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x^2
)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```


Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+\right.}{\sqrt{d}x}\right.}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Mathematica [C] time = 0.14757, size = 153, normalized size = 0.75

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{\left(c-dx^2\right)\sqrt[3]{c+3dx^2}\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c - d*x^2)*(c
 + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c]
+ 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x^2)/c, (d*x^2)/c] - AppellF1[3
/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x^2)/c])))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-dx^2+c}\frac{1}{\sqrt[3]{3\,dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

```
[Out] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,dx^2+c\right)^{\frac{1}{3}}\left(dx^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-c\sqrt[3]{c+3dx^2}+dx^2\sqrt[3]{c+3dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
```

```
[Out] -Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,dx^2+c\right)^{\frac{1}{3}}\left(dx^2-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.0289102, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right.}\right.}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Mathematica [C] time = 0.0296787, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*
a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+3\,a}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*(
c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqr
t[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqr
t[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c
 - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi [A] time = 0.0287804, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*(
c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqr
t[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqr
t[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c
 - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
```

```
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c-3d}\right.}\right.}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Mathematica [C] time = 0.15142, size = 156, normalized size = 0.76

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\left(c+dx^2\right)\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c - 3*d*x^2
)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/
c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3*d*x^2)/c, -((d*x^2)/c)] + App
ellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c, -((d*x^2)/c)])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\frac{1}{\sqrt[3]{-3\,dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

```
[Out] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2 + c\right)\left(-3\,dx^2 + c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c - 3dx^2}\left(c + dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

```
[Out] Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2 + c\right)\left(-3\,dx^2 + c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.020868, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```


```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```


Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.0397392, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(
-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0.001, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 10.8668, size = 5544, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
```

```
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi [A] time = 0.0139981, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```


```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```


Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.0350786, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")

[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [B] time = 8.78884, size = 4852, normalized size = 44.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")

[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
```

```
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=96

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3
))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) + (
1 + x)^(2/3)])/(2*2^(2/3))
```

Rubi [A] time = 0.0183386, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1008}

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3
))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) + (
1 + x)^(2/3)])/(2*2^(2/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}} - \frac{\log\left(3+x^2\right)}{2\ 2^{2/3}} + \frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.144073, size = 143, normalized size = 1.49

$$-\frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(x^2*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/6 - (27*x*AppellF1[1/2, 1/3, 1,
3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2
, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[
3/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{3-x}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 54.8404, size = 775, normalized size = 8.07

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}\left(x^4+3x^3+3x^2+9x\right)\left(-x^2+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6-18x^5-117x^4-36x^3+207x^2+54x-\right.\right.}{6\left(x^6+54x^5+171x^4+108x^3-81x^2-162x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 3*x^3 +
3*x^2 + 9*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^6 - 18*x^5 - 117*x^4 - 36*x^3 +
207*x^2 + 54*x - 27) + 12*(x^5 + 19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x^2
+ 1)^(1/3))/(x^6 + 54*x^5 + 171*x^4 + 108*x^3 - 81*x^2 - 162*x - 27)) - 1/2
4*4^(2/3)*log((6*4^(2/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^4 + 18*x
^3 + 24*x^2 - 18*x - 9) - 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))/(x^4
+ 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 3) + 6*4^(1/3)*(-x^2 + 1)^
(1/3)*(x + 1) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx-\int-\frac{3}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x) - Integral(-
3/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=95

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1 +
 x)^(1/3)])/(2*2^(2/3))
```

Rubi [A] time = 0.0171041, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1008}

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1 +
 x)^(1/3)])/(2*2^(2/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+x}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.0978211, size = 143, normalized size = 1.51

$$\frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x^2*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/6 - (27*x*AppellF1[1/2, 1/3, 1, 3
/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2,
 x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3
/2, 4/3, 1, 5/2, x^2, -x^2/3])))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{3+x}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 55.3616, size = 911, normalized size = 9.59

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\left(x^4-3\,x^3+3\,x^2-9\,x\right)\left(-x^2+1\right)^{\frac{2}{3}}+12\,(-1)^{\frac{1}{3}}\left(x^5-19\,x^4+42\,x^3-6\,x^2-\right.\right.}{6\left(x^6-54\,x^5+171\,x^4-108\,x^3-8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1)
^(2/3)*(x^4 - 3*x^3 + 3*x^2 - 9*x)*(-x^2 + 1)^(2/3) + 12*(-1)^(1/3)*(x^5 -
19*x^4 + 42*x^3 - 6*x^2 - 27*x + 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 + 18*x^
5 - 117*x^4 + 36*x^3 + 207*x^2 - 54*x - 27))/(x^6 - 54*x^5 + 171*x^4 - 108*
x^3 - 81*x^2 + 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*(-1)^
(1/3)*(x^2 - 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 - 18*x^3 + 24*
x^2 + 18*x - 9) - 6*(x^3 - 7*x^2 + 3*x + 3)*(-x^2 + 1)^(1/3))/(x^4 + 6*x^2
+ 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3)
*(x - 1) + 4^(2/3)*(-1)^(1/3)*(x^2 + 3) - 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+3}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan[
(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) - (S
qrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi [A] time = 0.0270012, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {394}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```



```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan[
(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) - (S
qrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*Sqrt[3]*a^(5/6)*d)
```


Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{bx}}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{bx}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{bx}}\right)}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] time = 0.161904, size = 169, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\left(9a+bx^2\right)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)])/(d*(a +
 b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
-(b*x^2)/(9*a)] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -(b*x^2
)/(9*a)] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(9*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2+a}}\left(9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sq
rt[b]*x)])/(12*a^(5/6)*d)
```

Rubi [A] time = 0.0260154, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {395}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```


```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sq
rt[b]*x)])/(12*a^(5/6)*d)
```


Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

Mathematica [C] time = 0.170368, size = 167, normalized size = 1.09

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\left(9a-bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(a - b*
x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2
)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-bx^2+a}}\left(-9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6)*S
qrt[b]*x)])/(12*a^(5/6)*d)
```

Rubi [A] time = 0.0282874, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {395}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6)*S
qrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

Mathematica [C] time = 0.122239, size = 168, normalized size = 1.11

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{bx^2-a}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(9*a -
b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^
2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2-a}}\left(-9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)/
d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2-a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt[b]*ArcTan
[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) +
(Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi [A] time = 0.0288537, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {394}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt[b]*ArcTan
[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) +
(Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqrt[b]*
x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{bx}}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{bx}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)}{\sqrt{bx}}\right)}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] time = 0.158877, size = 172, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\left(9a+bx^2\right)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)])/(d*(-a
- b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
 -(b*x^2)/(9*a)] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -(b*x^
2)/(9*a)] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(9*a)])))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-bx^2-a}}\left(9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2-a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2-a\right)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan[
(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) - (S
qrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0338085, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {394}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```


```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan[
(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) - (S
qrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*2^(5/6)*Sqrt[3]*d)
```


Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Mathematica [C] time = 0.145696, size = 148, normalized size = 0.98

$$-\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{bx^2+2}\left(bx^2+18\right)\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2)/2, -(b*x^2)/18])/(d*(2 + b*x^2
)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2)/2, -(b*x^2)/1
8] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2)/2, -(b*x^2)/18] + 3*AppellF
1[3/2, 4/3, 1, 5/2, -(b*x^2)/2, -(b*x^2)/18])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2+2}}\left(18\frac{d}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
```

```
[Out] int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*
2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6)*S
qrt[b]*x)])/(12*2^(5/6)*d)
```

Rubi [A] time = 0.0239209, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {395}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```


```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*
2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6)*S
qrt[b]*x)])/(12*2^(5/6)*d)
```


Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}$$

Mathematica [C] time = 0.162218, size = 148, normalized size = 1.01

$$\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(bx^2-18\right)\sqrt[3]{bx^2-2}\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)+27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-18 + b*x^2)
*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18] +
 b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2, (b*x^2)/18] + 3*AppellF1[3/2,
 4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2-2}}\left(-18\,\frac{d}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
```

```
[Out] int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-2\right)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{1}{bx^2 \sqrt[3]{bx^2-2} - 18 \sqrt[3]{bx^2-2}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-2\right)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}(6d+dx^2)}\,dx$

Optimal. Leaf size=123

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1/
3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^(1
/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi [A] time = 0.0204762, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1/
3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^(1
/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+3x^2}\left(6d+dx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

Mathematica [C] time = 0.112846, size = 136, normalized size = 1.11

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\left(x^2+6\right)\sqrt[3]{3x^2+2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6])/(d*(6 + x^2)*(2 + 3*x
^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6] + x^2*(AppellF
1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (-3*
x^2)/2, -x^2/6])))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+6d}\frac{1}{\sqrt[3]{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x)
```

```
[Out] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+6d\right)\left(3x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}\left(-6d+dx^2\right)}\,dx$

Optimal. Leaf size=123

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] -ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) - ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3))^2/
(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0167803, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {395}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] -ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) - ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3))^2/
(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2-3x^2}\left(-6d+dx^2\right)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Mathematica [C] time = 0.119382, size = 136, normalized size = 1.11

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\left(x^2-6\right)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)^(1/3)*(-6
 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/2
, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
x^2/6])))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2-6d}\frac{1}{\sqrt[3]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2-6d\right)\left(-3x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{2-3x^2}-6\sqrt[3]{2-3x^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] Integral(1/(x**2*(2 - 3*x**2)**(1/3) - 6*(2 - 3*x**2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2-6\,d\right)\left(-3\,x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}\left(-6d+dx^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))^2
/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0170262, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {395}

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))^2
/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\left(-6d+dx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Mathematica [C] time = 0.101261, size = 136, normalized size = 1.14

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\left(x^2-6\right)\sqrt[3]{3x^2-2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*(-2 + 3*x^
2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/
2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
 x^2/6])))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2-6\,d}\frac{1}{\sqrt[3]{3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x)
```

```
[Out] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2-6\,d\right)\left(3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2-2}-6\sqrt[3]{3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}\left(6d+dx^2\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] -ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2)^(
1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)*(2^
(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi [A] time = 0.0171224, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {394}

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] -ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2)^(
1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)*(2^
(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\left(6d+dx^2\right)}\,dx = -\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

Mathematica [C] time = 0.106694, size = 136, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-3x^2-2}\left(x^2+6\right)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6])/(d*(-2 - 3*x^2)^(1/3)
*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6] + x^2*(Appell
F1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (-3
*x^2)/2, -x^2/6])))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+6\,d}\frac{1}{\sqrt[3]{-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x)
```

```
[Out] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+6\,d\right)\left(-3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

3.160 $\int \frac{1}{\sqrt[3]{1+x^2}(9+x^2)}\,dx$

Optimal. Leaf size=70

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rubi [A] time = 0.0088051, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {394}

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+x^2}\left(9+x^2\right)}\,dx = \frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right) + \frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.0934875, size = 124, normalized size = 1.77

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{x^2+1}\left(x^2+9\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -x^2/9])/((1 + x^2)^(1/3)*(9 + x^2)
*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -x^2/9] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, -x^2, -x^2/9] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -x^2/9])))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+9}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

```
[Out] int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+9\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 14.6608, size = 4397, normalized size = 62.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 + 32*sqrt(3)*x^3
+ 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*
x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3) - 243)
/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 89
1*x^2 + 18*(3*x^4 - 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sq
rt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 +
 81*x) + 81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/36*
arctan((384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5 - 1259712*x^3 + 36
*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 + sqrt(3)*(x^10 + 549*x^8 -
8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x^2 + 1)^(2/3) + sq
rt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 6816879*x^4 + 6022998*x
^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 3773304*x^6 - 6908733*x
^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*x^2 + 12*sqrt(3)*(
11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3) + 6*sqrt(3)*(43*x^
11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 + 177147*x) + 6*(453
*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^2 + sqrt(3)*(x^11
+ 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147*x) + 531441)*(x^2
 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 - 32*sqrt
(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sqrt(3)*(x^5 + 10*x^3 + 9*x)
+ 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3
) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 6423*x^9 + 225018*x^7
 - 1106622*x^5 - 1541835*x^3 + 3*sqrt(3)*(37*x^10 - 675*x^8 + 34722*x^6 - 9
7686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3) - 8503056*x)/(x^1
2 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^4 - 9565938*x^2 +
4782969)) + 1/36*arctan(-(384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5
- 1259712*x^3 + 36*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 - sqrt(3)*
(x^10 + 549*x^8 - 8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x
^2 + 1)^(2/3) - sqrt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 68168
79*x^4 + 6022998*x^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 37733
04*x^6 - 6908733*x^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*
```

```
x^2 - 12*sqrt(3)*(11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3)
- 6*sqrt(3)*(43*x^11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 +
177147*x) + 6*(453*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^
2 - sqrt(3)*(x^11 + 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147
*x) + 531441)*(x^2 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 1
8*(3*x^4 + 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^
5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) +
81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 642
3*x^9 + 225018*x^7 - 1106622*x^5 - 1541835*x^3 - 3*sqrt(3)*(37*x^10 - 675*x
^8 + 34722*x^6 - 97686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3)
 - 8503056*x)/(x^12 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^
4 - 9565938*x^2 + 4782969)) - 1/36*arctan(6*(11*x^5 + 30*x^3 + 6*(23*x^3 +
27*x)*(x^2 + 1)^(2/3) + (x^5 - 240*x^3 - 81*x)*(x^2 + 1)^(1/3) - 81*x)/(x^6
 - 1971*x^4 - 1701*x^2 - 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

```
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}(9+bx^2)}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*Sq
rt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rubi [A] time = 0.0163418, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```


```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*Sq
rt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(4*Sqrt[3]*Sqrt[b])
```


Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+bx^2}\left(9+bx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

Mathematica [C] time = 0.108447, size = 137, normalized size = 1.32

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{bx^2+1}\left(bx^2+9\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -(b*x^2)/9])/((1 + b*x^2)^(1/3)
*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -(b*x^2)/9] + 2*b*x^
2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -(b*x^2)/9] + 3*AppellF1[3/2, 4/3,
1, 5/2, -(b*x^2), -(b*x^2)/9])))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+9}\frac{1}{\sqrt[3]{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

```
[Out] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+9\right)\left(bx^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2+1}\left(bx^2+9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
```

```
[Out] Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+9\right)\left(bx^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

3.162 $\int \frac{1}{\sqrt[3]{1-x^2}(9-x^2)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 - A
rcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rubi [A] time = 0.0112752, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {395}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 - A
rcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(9-x^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}} + \frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right) - \frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

Mathematica [C] time = 0.0451872, size = 125, normalized size = 1.69

$$\frac{\sqrt[3]{\frac{x-1}{x-3}}\sqrt[3]{\frac{x+1}{x-3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{x-3},-\frac{2}{x-3}\right) - \sqrt[3]{\frac{x-1}{x+3}}\sqrt[3]{\frac{x+1}{x+3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2}{x+3},\frac{4}{x+3}\right)}{4\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] (((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 + x)
)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x), 4/(3 + x)])/(4*(1 - x^2)^(1
/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+9}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(-x^2+9),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(-x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(x^2-9\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] time = 6.55866, size = 749, normalized size = 10.12

$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{36\sqrt{3}\left(x^4-32x^3-42x^2+9\right)\left(-x^2+1\right)^{\frac{2}{3}}+12\sqrt{3}\left(x^5+27x^4-210x^3-54x^2+81x+27\right)\left(-x^2+1\right)^{\frac{1}{3}}}{3\left(x^6+108x^5-1647x^4-1080x^3+891x^2+972\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*arctan(1/3*(36*sqrt(3)*(x^4 - 32*x^3 - 42*x^2 + 9)*(-x^2 + 1)
^(2/3) + 12*sqrt(3)*(x^5 + 27*x^4 - 210*x^3 - 54*x^2 + 81*x + 27)*(-x^2 + 1
)^(1/3) + sqrt(3)*(x^6 - 108*x^5 - 567*x^4 + 1080*x^3 + 459*x^2 - 972*x - 4
05))/(x^6 + 108*x^5 - 1647*x^4 - 1080*x^3 + 891*x^2 + 972*x + 243)) - 1/72*
log((x^3 + 33*x^2 + 18*(-x^2 + 1)^(2/3)*(x + 1) - 6*(x^2 + 6*x - 3)*(-x^2 +
 1)^(1/3) - 9*x - 9)/(x^3 + 9*x^2 + 27*x + 27)) + 1/36*log(-(x^3 - 33*x^2 +
 18*(-x^2 + 1)^(2/3)*(x - 1) + 6*(x^2 - 6*x - 3)*(-x^2 + 1)^(1/3) - 9*x + 9
)/(x^3 + 9*x^2 + 27*x + 27))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x^2\sqrt[3]{1-x^2}-9\sqrt[3]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] -Integral(1/(x**2*(1 - x**2)**(1/3) - 9*(1 - x**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(x^2-9\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

3.163 $\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}+\frac{x\sqrt{c^2x^2-1}}{2d\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] (x*Sqrt[-1 + c^2*x^2])/(2*d*(d - c^2*d*x^2)^(3/2)) + (Sqrt[-1 + c^2*x^2]*Ar
cTanh[c*x])/(2*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.0192063, antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\sqrt{c^2x^2-1}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[-1 + c^2*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
 + c^2*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{\sqrt{-1+c^2x^2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+c^2x^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{d-c^2dx^2}}\\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+c^2x^2}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0255409, size = 57, normalized size = 0.72

$$\frac{\left(c^2x^2-1\right)\tanh^{-1}(cx)-cx}{2cd^2\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d -
 c^2*d*x^2])
```

Maple [A] time = 0.02, size = 94, normalized size = 1.2

$$\frac{\ln(cx-1)\,x^2c^2-\ln(cx+1)\,x^2c^2+2\,cx-\ln(cx-1)+\ln(cx+1)}{4\,d^3c\,(cx-1)\,(cx+1)}\sqrt{-\left(c^2x^2-1\right)d}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*x^2*c^2-ln(c*x+1)*x
^2*c^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^3/c/(c*x-1)/(c*x+1)
```

Maxima [A] time = 0.99042, size = 95, normalized size = 1.2

$$-\frac{x}{2\left(c^2\sqrt{-d}d^2x^2-\sqrt{-d}d^2\right)}-\frac{\sqrt{-d}\log\left(cx+1\right)}{4\,cd^3}+\frac{\sqrt{-d}\log\left(cx-1\right)}{4\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(c^2*sqrt(-d)*d^2*x^2 - sqrt(-d)*d^2) - 1/4*sqrt(-d)*log(c*x + 1)/(c
*d^3) + 1/4*sqrt(-d)*log(c*x - 1)/(c*d^3)
```

Fricas [A] time = 2.30977, size = 662, normalized size = 8.38

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\left(c^5d^3x^4-2\,c^3d^3x^2+cd^3\right)},\ \frac{2\sqrt{-c^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3), 1/4*(2*sqrt(-c^2
*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arcta
n(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c
^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\left(cx-1\right)\left(cx+1\right)}}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(sqrt((c*x - 1)*(c*x + 1))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2-1}}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 - 1)/(-c^2*d*x^2 + d)^(5/2), x)
```

3.164 $\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{dx\sqrt{c^2x^2-1}}{2\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2c\sqrt{d-c^2dx^2}}$$

```
[Out] (d*x*Sqrt[-1 + c^2*x^2])/(2*(d - c^2*d*x^2)^(3/2)) + (Sqrt[-1 + c^2*x^2]*Ar
cTanh[c*x])/(2*c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.0176594, antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(c^2x^2-1\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\tanh^{-1}(cx)}{2cd^2\left(c^2x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (x*(d - c^2*d*x^2)^(3/2))/(2*d^2*(1 - c^2*x^2)*(-1 + c^2*x^2)^(3/2)) + ((d
- c^2*d*x^2)^(3/2)*ArcTanh[c*x])/(2*c*d^2*(-1 + c^2*x^2)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(d-c^2dx^2\right)^{3/2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\left(-1+c^2x^2\right)^{3/2}}\\ &= \frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(-1+c^2x^2\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\left(-1+c^2x^2\right)^{3/2}}\\ &= \frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(-1+c^2x^2\right)^{3/2}}+\frac{\left(d-c^2dx^2\right)^{3/2}\tanh^{-1}(cx)}{2cd^2\left(-1+c^2x^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0190215, size = 54, normalized size = 0.73

$$\frac{\left(c^2x^2-1\right)\tanh^{-1}(cx)-cx}{2c\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2
*d*x^2])
```

Maple [A] time = 0.016, size = 94, normalized size = 1.3

$$\frac{\ln(cx-1)\,x^2c^2-\ln(cx+1)\,x^2c^2+2\,cx-\ln(cx-1)+\ln(cx+1)}{4\,cd\,(cx-1)\,(cx+1)}\sqrt{-\left(c^2x^2-1\right)d}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*x^2*c^2-ln(c*x+1)*x
^2*c^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d/c/(c*x-1)/(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2dx^2+d}\left(c^2x^2-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

Fricas [A] time = 2.32747, size = 645, normalized size = 8.72

$$\left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\left(c^5dx^4-2\,c^3dx^2+cd\right)},\ \frac{2\sqrt{-c^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d*x^4 - 2*c^3*d*x^2 + c*d), 1/4*(2*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arctan(2*sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c^5*d*x
^4 - 2*c^3*d*x^2 + c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx-1)\,(cx+1)\right)^{\frac{3}{2}}\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(3/2)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(((c*x - 1)*(c*x + 1))**(3/2)*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c^2dx^2+d}\left(c^2x^2-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

3.165 $\int \frac{1}{\sqrt{-1+c^2x^2}\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{x\sqrt{c^2x^2-1}}{2\left(d-c^2dx^2\right)^{3/2}}-\frac{\sqrt{c^2x^2-1}\tanh^{-1}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] -(x*Sqrt[-1 + c^2*x^2])/(2*(d - c^2*d*x^2)^(3/2)) - (Sqrt[-1 + c^2*x^2]*Arc
Tanh[c*x])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.0168311, antiderivative size = 91, normalized size of antiderivative = 1.2, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 199, 208}

$$\frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{c^2x^2-1}}+\frac{\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{2cd^2\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[d - c^2*d*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[-1 + c^2*x^2]) + (Sqrt[d
- c^2*d*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+c^2x^2}\left(d-c^2dx^2\right)^{3/2}}\,dx &= \frac{\sqrt{d-c^2dx^2}\int \frac{1}{\left(d-c^2dx^2\right)^2}\,dx}{\sqrt{-1+c^2x^2}}\\ &= \frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{-1+c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{-1+c^2x^2}}\\ &= \frac{x\sqrt{d-c^2dx^2}}{2d^2\left(1-c^2x^2\right)\sqrt{-1+c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{2cd^2\sqrt{-1+c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0287236, size = 57, normalized size = 0.75

$$\frac{\left(1-c^2x^2\right)\tanh^{-1}(cx)+cx}{2cd\sqrt{c^2x^2-1}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (c*x + (1 - c^2*x^2)*ArcTanh[c*x])/(2*c*d*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d
*x^2])
```

Maple [A] time = 0.014, size = 94, normalized size = 1.2

$$-\frac{\ln(cx-1)x^2c^2-\ln(cx+1)x^2c^2+2cx-\ln(cx-1)+\ln(cx+1)}{4d^2c(cx-1)(cx+1)}\sqrt{-\left(c^2x^2-1\right)d}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*x^2*c^2-ln(c*x+1)*
x^2*c^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^2/c/(c*x-1)/(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

Fricas [B] time = 2.03795, size = 664, normalized size = 8.74

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx+\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\left(c^5d^2x^4-2\,c^3d^2x^2+cd^2\right)},-\frac{2\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x + (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^
4 + 3*c^2*x^2 - 1)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2), -1/4*(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arc
tan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/
(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx-1)(cx+1)}\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((c*x - 1)*(c*x + 1))*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

3.166 $\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=328

$$-\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\cdots\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots$$

```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[c + d*x^2]
) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d) + (b*x*Sqrt[
a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c]*(2*b^2*c^2 - 7*a*b*c*d - 3*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
- (c^(3/2)*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rubi [A] time = 0.315506, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {416, 528, 531, 418, 492, 411}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{b}+7ac-\frac{2bc^2}{d}\right)}{15\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}(bc-9a\cdots}{15d^{3/2}\sqrt{c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```


```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[c + d*x^2]
) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d) + (b*x*Sqrt[
a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c]*(2*b^2*c^2 - 7*a*b*c*d - 3*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
- (c^(3/2)*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx=\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}+\frac{\int\frac{\sqrt{c+dx^2}\left(-a(bc-5ad)-2b(bc-3ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{5d}$$

$$=-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}+\frac{\int\frac{-abc(bc-9ad)-b\left(2b^2c^2-7abcd-\right.}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15bd}$$

$$=-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{(ac(bc-9ad))\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+}}}{15d}$$

$$=\frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}}-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)}{5d}$$

$$=\frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}}-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)}{5d}$$

Mathematica [C] time = 0.447919, size = 243, normalized size = 0.74

$$\frac{-2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2+7abcd-\right.}{15d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d + b*(c + 3*d*x^2)) - I*c*(-2*
b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(b^2*c^2 - 4*a*b*c*d
 + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A] time = 0.033, size = 543, normalized size = 1.7

$$\frac{1}{\left(15\,bd x^4+15\,ad x^2+15\,bc x^2+15\,ac\right)d^2}\sqrt{b x^2+a}\sqrt{d x^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+9\sqrt{-\frac{b}{a}}x^5abd^3+4\sqrt{-\frac{b}{a}}x^5b^2cd^2+6\sqrt{-\frac{b}{a}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+9*(-b/a)^(
1/2)*x^5*a*b*d^3+4*(-b/a)^(1/2)*x^5*b^2*c*d^2+6*(-b/a)^(1/2)*x^3*a^2*d^3+10
*(-b/a)^(1/2)*x^3*a*b*c*d^2+(-b/a)^(1/2)*x^3*b^2*c^2*d+6*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-8
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a*b*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-2*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*b^2*c^3+6*(-b/a)^(1/2)*x*a^2*c*d^2+(-b/a)^(1/2)*x*a*b*c^2*d)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^2+a\right)^{\frac{3}{2}}\sqrt{d x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^2+a\right)^{\frac{3}{2}}\sqrt{d x^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2\right)^{\frac{3}{2}} \sqrt{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

3.167 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=249

$$\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{x\sqrt{a+bx^2}(ad+bc)}{3b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\Big(}{3b\sqrt{d}\sqrt{c+dx^2}}$$

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.178589, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {417, 531, 418, 492, 411}

$$\frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{x\sqrt{a+bx^2}(ad+bc)}{3b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\Big(\tan^{-1}\Big(}{3b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
```

```
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2}{3}\int \frac{ac+\frac{1}{2}(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$= \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{1}{3}(2ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + \frac{1}{3}(bc+ad)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,d$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(c(bc\ -}{}$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} - \frac{\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} +$$

Mathematica [C] time = 0.237942, size = 198, normalized size = 0.8

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+bc}{3d\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*
(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.012, size = 328, normalized size = 1.3

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^(1/2)*x^
3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2+(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^2+a}\sqrt{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

3.168 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/2}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0925536, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx &= c\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}\\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0496046, size = 86, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.013, size = 101, normalized size = 0.5

$$\frac{c}{bdx^4 + adx^2 + bcx^2 + ac}\sqrt{bx^2 + a}\sqrt{dx^2 + c}\sqrt{\frac{bx^2 + a}{a}}\sqrt{\frac{dx^2 + c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.169 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(
Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Rubi [A] time = 0.0168172, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {411}

$$\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(
Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Mathematica [C] time = 0.279228, size = 133, normalized size = 1.58

$$\frac{x\left(c+dx^2\right)+\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}}}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(c + d*x^2) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/Sqrt[b/a])/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.031, size = 181, normalized size = 2.2

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)a}\left(x^3d\sqrt{-\frac{b}{a}}-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}+\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] (x^3*d*(-b/a)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)+EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*((
d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)+x*c*(-b/a)^(1/2))*(d*x^2+c)^(1/2)*(b*
x^2+a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

3.170 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*Sqrt[
b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (c^
(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^2*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rubi [A] time = 0.120085, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {412, 525, 418, 411}

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*Sqrt[
b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (c^
(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^2*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
```

```
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2c-dx^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{3a}$$

$$= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)} + \frac{(2bc-ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx}{3a(bc-ad)}$$

$$= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} + \frac{(2bc-ad)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3a^2(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.443624, size = 243, normalized size = 1.03

$$\frac{-2ic\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(2a^2d+ab\left(dx^2-3c\right)-2b^2c\right.}{3a^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(2*a^2*d - 2*b^2*c*x^2 + a*b*(-3*c + d*x^2)) + I*c
*(-2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*(a + b*x^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(-(b*c) + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d
*x^2])
```

Maple [B] time = 0.036, size = 617, normalized size = 2.6

$$\frac{1}{(3\,ad-3\,bc)\,a^2}\left(x^5abd^2\sqrt{-\frac{b}{a}}-2\,x^5b^2cd\sqrt{-\frac{b}{a}}+2\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^2abcd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-2\,\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(x^5*a*b*d^2*(-b/a)^(1/2)-2*x^5*b^2*c*d*(-b/a)^(1/2)+2*EllipticF(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)-2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^2*c^2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c
*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*x^2*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*x^3*a^2*d
^2*(-b/a)^(1/2)-2*x^3*a*b*c*d*(-b/a)^(1/2)-2*x^3*b^2*c^2*(-b/a)^(1/2)+2*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)-2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*
c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*x*a^2*c*d*(-
b/a)^(1/2)-3*x*a*b*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)
/a^2/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
^2 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

3.171 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

Optimal. Leaf size=309

$$-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\right.}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*Sqrt[c + d
*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 13*a*b*c*d +
3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]) - (2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.227773, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {412, 527, 525, 418, 411}

$$\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```


```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*Sqrt[c + d
*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 13*a*b*c*d +
3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]) - (2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{7/2}}\,dx = \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-4c-3dx^2}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx}{5a}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{c(8bc-9ad)+d(4bc-3ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{(2cd(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)^2} + \frac{\left(8b^2c^2-13abc\right.}{15}$$

$$= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{\left(8b^2c^2-13abcd+3a^2d^2\right)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{15a^{5/2}\sqrt{b}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Mathematica [C] time = 0.531225, size = 285, normalized size = 0.92

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(3a^2d^2-13abcd+8b^2c^2\right)+3a^2(bc-ad)^2+a\left(a+bx^2\right)(ad-bc)(3ad-4bc)\right)+ic\sqrt{\frac{bx^2}{a}+1}\left(a\right.}{15a^3\sqrt{\frac{b}{a}}\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-4*b*c +
3*a*d)*(a + b*x^2) + (8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^2) +
I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((8*b^2*c^2 - 13*
a*b*c*d + 3*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b
^2*c^2 + 17*a*b*c*d - 9*a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.042, size = 1411, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x)
```

```
[Out] 1/15*(3*x^7*a^2*b^2*d^3*(-b/a)^(1/2)-13*x^7*a*b^3*c*d^2*(-b/a)^(1/2)-18*x^3
*a^2*b^2*c^2*d*(-b/a)^(1/2)-26*x*a^3*b*c^2*d*(-b/a)^(1/2)+8*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)+9*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*
d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a^2*b^2*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-3*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*x^5*b^4*c^3*(-b/a)^(1/2)+9*x^3*a^4*d^3*(-b/
a)^(1/2)+9*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-30*x^5*a^2*b^2*c*d^2*(-b/a)^(1/2)+7*x^5
*a*b^3*c^2*d*(-b/a)^(1/2)-17*x^3*a^3*b*c*d^2*(-b/a)^(1/2)+16*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)-16*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-17*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a^3*b*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+13*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-
3*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)-17*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4
*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*x^7*b^4*c^2*d*(-b/a)
^(1/2)+20*x^3*a*b^3*c^3*(-b/a)^(1/2)+9*x*a^4*c*d^2*(-b/a)^(1/2)+15*x*a^2*b^
2*c^3*(-b/a)^(1/2)+9*x^5*a^3*b*d^3*(-b/a)^(1/2)+13*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*x^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+1
8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)-34*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a
^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-6*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)+26*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2))/(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)^2/a^
3/(b*x^2+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2
*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

3.172 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=410

$$-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}+$$

```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(35*b^2*
d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/
(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c +
a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
5*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.438785, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {416, 528, 531, 418, 492, 411}

$$-\frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{2\sqrt{c}\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```


```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(35*b^2*
d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/
(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c +
a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
```

```
5*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}dx = \frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{\int \frac{\left(a+bx^2\right)^{3/2}\left(c(7bc-ad)+2d(4bc-ad)x^2\right)}{\sqrt{c+dx^2}}dx}{7b}$$

$$= \frac{2(4bc-ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{\int \frac{\sqrt{a+bx^2}\left(3acd(9bc-ad)+\right.}{\sqrt{c}}}{3}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2(4bc-ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx}{}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2(4bc-ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx}{}$$

$$= -\frac{2(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c}$$

$$= -\frac{2(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c}$$

Mathematica [C] time = 0.583425, size = 302, normalized size = 0.74

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2bcd^2+a^3d^3-11ab^2c^2d+2b^3c^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)}{35bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8*d*x^2) +
b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 - 5*a*b^2*c^2*d - 5*a
```

```
^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8*a^2
*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A] time = 0.018, size = 780, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*x^9*b^3*d^4+13*(-b/a)^
(1/2)*x^7*a*b^2*d^4+13*(-b/a)^(1/2)*x^7*b^3*c*d^3+9*(-b/a)^(1/2)*x^5*a^2*b*
d^4+38*(-b/a)^(1/2)*x^5*a*b^2*c*d^3+9*(-b/a)^(1/2)*x^5*b^3*c^2*d^2+(-b/a)^(
1/2)*x^3*a^3*d^4+26*(-b/a)^(1/2)*x^3*a^2*b*c*d^3+26*(-b/a)^(1/2)*x^3*a*b^2*
c^2*d^2+(-b/a)^(1/2)*x^3*b^3*c^3*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+8*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2-
11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a*b^2*c^3*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+10*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c
^2*d^2+10*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b^2*c^3*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1/2)*x*a^3*c*d^3+17*(
-b/a)^(1/2)*x*a^2*b*c^2*d^2+(-b/a)^(1/2)*x*a*b^2*c^3*d)/b/d^2/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^4 + (bc + ad)x^2 + ac\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2\right)^{\frac{3}{2}} \left(c + dx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{3}{2}}\left(dx^2 + c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

3.173 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=336

$$\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{15b^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2+\cdots\right.}{15b^2\cdots}$$

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c + d*
x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b) + (d*x*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d -
 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.280561, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {416, 528, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{15b^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(9\cdots}{15b\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```


```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c + d*
x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b) + (d*x*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d -
 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx = \frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{\int \frac{\sqrt{a+bx^2}\left(c(5bc-ad)+2d(3bc-ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5b}$$

$$=\frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{\int \frac{acd(9bc-ad)+d\left(3b^2c^2+7abcd-2a^2\right.}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15bd}$$

$$=\frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{(ac(9bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx}}}{15b}$$

$$=\frac{\left(3b^2c^2+7abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}}+\frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{}}{5b}$$

$$=\frac{\left(3b^2c^2+7abcd-2a^2d^2\right)x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}}+\frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{}}{5b}$$

Mathematica [C] time = 0.410342, size = 246, normalized size = 0.73

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+2abcd-3b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-7abcd-3b^2\right.}{15bd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*b*c + a*d + 3*b*d*x^2) + I*c*(-3*
b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-3*b^2*c^2 + 2*a*b*c*d
+ a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.014, size = 545, normalized size = 1.6

$$\frac{1}{15\,d\left(bd x^4+ad x^2+bc x^2+ac\right)b}\sqrt{b x^2+a}\sqrt{d x^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+4\sqrt{-\frac{b}{a}}x^5abd^3+9\sqrt{-\frac{b}{a}}x^5b^2cd^2+\sqrt{-\frac{b}{a}}x^3a^2d^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+4*(-b/a)^(
1/2)*x^5*a*b*d^3+9*(-b/a)^(1/2)*x^5*b^2*c*d^2+(-b/a)^(1/2)*x^3*a^2*d^3+10*(
-b/a)^(1/2)*x^3*a*b*c*d^2+6*(-b/a)^(1/2)*x^3*b^2*c^2*d+((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+2*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a*b*c^2*d-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+3*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b^2*c^3+(-b/a)^(1/2)*x*a^2*c*d^2+6*(-b/a)^(1/2)*x*a*b*c^2*d)/d/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b x^2+a}\left(d x^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b x^2+a}\left(d x^2+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

3.174 $\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3b^2\sqrt{c+dx^2}}-\frac{2\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-}\right.}{3b^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (d*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*
d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(3*a*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.164806, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {416, 531, 418, 492, 411}

$$\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3b^2\sqrt{c+dx^2}}-\frac{2\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{3ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```


```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (d*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*
d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(3*a*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} + \frac{\int \frac{c(3bc-ad)+2d(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b}$$

$$= \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} + \frac{(2d(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b} + \frac{(c(3bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} + \frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} - \frac{2\sqrt{c}\sqrt{d}(2bc-ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3b^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.225573, size = 199, normalized size = 0.73

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-}{3b\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(-2*b*c + a*d)*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Maple [A] time = 0.014, size = 330, normalized size = 1.2

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^(1/2)*x^
3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+4*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*b*c^2+(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/b/(-b/a)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

3.175 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=267

$$\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}(bc-2ad)}{ab^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{ab^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) + ((b*c - a*
d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sq
rt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(a*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.158601, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {413, 531, 418, 492, 411}

$$-\frac{dx\sqrt{a+bx^2}(bc-2ad)}{ab^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```



```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) + ((b*c - a*
d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sq
rt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(a*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{b} - \frac{(d(bc-2ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ab}$$

$$= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(cd(bc-$$

$$= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.270166, size = 191, normalized size = 0.72

$$\frac{(bc-ad)\left(x\sqrt{\frac{b}{a}}(c+dx^2) - ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right) - ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad-bc)E\left(i\sinh\right.}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2)
 - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.021, size = 332, normalized size = 1.2

$$\frac{1}{b\left(bdx^4+adx^2+bcx^2+ac\right)a}\left(-\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd-ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+\sqrt{\frac{bx^2}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] (-(-b/a)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d-a*c*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+2
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b*c^2-(-b/a)^(1/2)*x*a*c*d+x*b*c^2*(-b/a)^(1/2))*(d*x^
2+c)^(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,(dx^2+c)^{\frac{3}{2}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.176 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c + a*d)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3
*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (
c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a^2*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])
```

Rubi [A] time = 0.135841, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {413, 525, 418, 411}

$$\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c + a*d)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3
*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (
c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a^2*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{c(2bc+ad)+d(bc+2ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\left(a+bx^2\right)^{3/2}} - \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ab} + \frac{(2(bc+ad))\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\left(a+bx^2\right)^{3/2}} + \frac{2(bc+ad)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{3a^2b\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.462401, size = 232, normalized size = 1.01

$$\frac{-ic\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(a^2d+ab\left(3c+2dx^2\right)+2b^2\right.}{3a^3\left(\frac{b}{a}\right)^{3/2}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(a^2*d + 2*b^2*c*x^2 + a*b*(3*c + 2*d*x^2)) + (2*I
)*c*(b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b*c + a*d)*(a + b*x^2)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)])/(3*a^3*(b/a)^(3/2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.021, size = 607, normalized size = 2.7

$$\frac{1}{3a^2b}\left(2x^5abd^2\sqrt{-\frac{b}{a}}+2x^5b^2cd\sqrt{-\frac{b}{a}}+\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^2abcd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}+2\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*x^5*a*b*d^2*(-b/a)^(1/2)+2*x^5*b^2*c*d*(-b/a)^(1/2)+EllipticF(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^2*c^2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b
*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*x^2*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x^3*a^2*d
^2*(-b/a)^(1/2)+5*x^3*a*b*c*d*(-b/a)^(1/2)+2*x^3*b^2*c^2*(-b/a)^(1/2)+Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*
c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a^2*c*d*(-b/
a)^(1/2)+3*x*a*b*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/a^2/(-b/a)^(1/2)/(b*x^2+
a)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.177 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$

Optimal. Leaf size=315

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right.}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2*b*c + a*d
)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 3*a*b*c*d
 - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d
)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.282366, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {413, 527, 525, 418, 411}

$$\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2x\sqrt{c}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2*b*c + a*d
)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 3*a*b*c*d
 - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d
)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{7/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{c(4bc+ad)+d(3bc+2ad)x^2}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx}{5ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\left(a+bx^2\right)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-c(bc-ad)(8bc+ad)-2d(bc-ad)(2bc+ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2b(bc-ad)}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\left(a+bx^2\right)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\left(a+bx^2\right)^{3/2}} - \frac{(cd(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2b(bc-ad)} + \frac{\left(8b^2c^2-3abcd-\right.}{15a^2}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\left(a+bx^2\right)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\left(a+bx^2\right)^{3/2}} + \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Mathematica [C] time = 0.580328, size = 285, normalized size = 0.9

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(-2a^2d^2-3abcd+8b^2c^2\right)+3a^2(bc-ad)^2+2a\left(a+bx^2\right)(ad+2bc)(bc-ad)\right)-ic\left(a+bx^2\right)^2}{15a^4\left(\frac{b}{a}\right)^{3/2}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 2*a*(b*c - a*d)*(2*b*c + a*
d)*(a + b*x^2) + (8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*(a + b*x^2)^2) - I*c*(
a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((-8*b^2*c^2 + 3*a*b*c
*d + 2*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*b^2*c^2
 - 7*a*b*c*d - a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(1
5*a^4*(b/a)^(3/2)*(b*c - a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.027, size = 1414, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x)
```

```
[Out] -1/15*(-2*x^7*a^2*b^2*d^3*(-b/a)^(1/2)-3*x^7*a*b^3*c*d^2*(-b/a)^(1/2)+7*x^3
*a^2*b^2*c^2*d*(-b/a)^(1/2)-11*x*a^3*b*c^2*d*(-b/a)^(1/2)+8*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a^2*b^2*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)+8*x^5*b^4*c^3*(-b/a)^(1/2)-x^3*a^4*d^3*(-b/a)^(
1/2)-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)-10*x^5*a^2*b^2*c*d^2*(-b/a)^(1/2)+17*x^5*a*b^
3*c^2*d*(-b/a)^(1/2)-17*x^3*a^3*b*c*d^2*(-b/a)^(1/2)+16*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)-16*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)-7*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b
*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+3*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^3*b*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)-7*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a*b^3*c
^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*x^7*b^4*c^2*d*(-b/a)^(1/2)+2
0*x^3*a*b^3*c^3*(-b/a)^(1/2)-x*a^4*c*d^2*(-b/a)^(1/2)+15*x*a^2*b^2*c^3*(-b/
a)^(1/2)-6*x^5*a^3*b*d^3*(-b/a)^(1/2)+3*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*x^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)-14*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+4*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+6*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2))/(d*x^2+c)^(1/2)/a^3/(a*d-b*c)/(-b/a)^(1/2)/(b*x^2+a)^(5/
2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b
^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

3.178 $\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$

Optimal. Leaf size=235

$$\frac{2\sqrt{2}\sqrt{bx^2+2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}+\frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}-\frac{\sqrt{2}(3b+2d)\sqrt{bx^2+2}E\left(\cdots\right.}{3b\sqrt{d}\sqrt{dx^2+\cdots}}$$

```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^2]*
Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*
Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.134702, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {417, 531, 418, 492, 411}

$$\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}+\frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}+\frac{2\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}(3b+2d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\cdots\right.\right.}{3b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```


```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^2]*
Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*
Sqrt[3 + d*x^2])
```


Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
```

```
, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} + \frac{2}{3}\int \frac{6+\frac{1}{2}(3b+2d)x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$$
$$= \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} + 4\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx + \frac{1}{3}(3b+2d)\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$$
$$= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}} + \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} + \frac{2\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{(-3b}{}$$
$$= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}} + \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} - \frac{\sqrt{2}(3b+2d)\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

Mathematica [C] time = 0.107454, size = 127, normalized size = 0.54

$$\frac{i\sqrt{3}(3b-2d)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)+\sqrt{b}dx\sqrt{bx^2+2}\sqrt{dx^2+3}-i\sqrt{3}(3b+2d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] - I*Sqrt[3]*(3*b + 2*d)*Ellipt
icE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] + I*Sqrt[3]*(3*b - 2*d)*El
lipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d)
```

Maple [A] time = 0.028, size = 303, normalized size = 1.3

$$\frac{1}{\left(3\,bdx^4+9\,bx^2+6\,dx^2+18\right)b}\sqrt{bx^2+2}\sqrt{dx^2+3}\left(x^5b^2d\sqrt{-d}+3\,x^3b^2\sqrt{-d}+2\,x^3bd\sqrt{-d}+3\,\sqrt{2}\text{EllipticF}\left(1/3\,x\sqrt{3}\sqrt{-d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(x^5*b^2*d*(-d)^(1/2)+3*x^3*b^2*(-d)^(1
/2)+2*x^3*b*d*(-d)^(1/2)+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2
```

```
^(1/2)*3^(1/2)*(1/d*b)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)-2*2^(1/2)*E
llipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*d*(b*x
^2+2)^(1/2)*(d*x^2+3)^(1/2)+3*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/
2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+2*2^(1/2
)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*d*(
b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+6*x*b*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*d*x^2+6
)/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2+2}\sqrt{dx^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + 2}\sqrt{dx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

3.179 $\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx$

Optimal. Leaf size=38

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-1\right)}{\sqrt{3}}+\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x$$

```
[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]
```

Rubi [A] time = 0.009185, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {248, 195, 221}

$$\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x+\frac{2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx &= \int \sqrt{6-24x^4}\,dx\\ &= \sqrt{\frac{2}{3}}x\sqrt{1-4x^4}+4\int \frac{1}{\sqrt{6-24x^4}}\,dx\\ &= \sqrt{\frac{2}{3}}x\sqrt{1-4x^4}+\frac{2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0116507, size = 22, normalized size = 0.58

$$\sqrt{6}x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] Sqrt[6]*x*Hypergeometric2F1[-1/2, 1/4, 5/4, 4*x^4]
```

Maple [B] time = 0.026, size = 75, normalized size = 2.

$$-\frac{\sqrt{2}}{36\,x^4-9}\sqrt{-6\,x^2+3}\sqrt{2\,x^2+1}\left(\sqrt{2}\sqrt{3}\sqrt{-6\,x^2+3}\sqrt{2\,x^2+1}\mathrm{EllipticF}\left(x\sqrt{2},i\right)-12\,x^5+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x)
```

```
[Out] -1/9*(-6*x^2+3)^(1/2)*2^(1/2)*(2*x^2+1)^(1/2)*(2^(1/2)*3^(1/2)*(-6*x^2+3)^(
1/2)*(2*x^2+1)^(1/2)*EllipticF(x*2^(1/2),I)-12*x^5+3*x)/(4*x^4-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{6}\int\sqrt{1-2x^2}\sqrt{2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(6)*Integral(sqrt(1 - 2*x**2)*sqrt(2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

3.180 $\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx$

Optimal. Leaf size=20

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rubi [A] time = 0.0038428, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {22}

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx &= \sqrt{\frac{2}{3}}\int \left(3+6x^2\right)\,dx\\ &= \sqrt{6}x+2\sqrt{\frac{2}{3}}x^3\end{aligned}$$

Mathematica [A] time = 0.0019961, size = 15, normalized size = 0.75

$$\sqrt{6}\left(\frac{2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*(x + (2*x^3)/3)
```

Maple [C] time = 0.001, size = 38, normalized size = 1.9

$$\frac{x\left(2\,x^{2}+3\right)}{6\,x^{2}+3}\sqrt{4\,x^{2}+2}\sqrt{6\,x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{6\,x^{2}+3}\sqrt{4\,x^{2}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2), x)
```

Fricas [B] time = 1.76583, size = 85, normalized size = 4.25

$$\frac{\left(2\,x^{3}+3\,x\right)\sqrt{6\,x^{2}+3}\sqrt{4\,x^{2}+2}}{3\left(2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

Sympy [A] time = 6.09365, size = 17, normalized size = 0.85

$$\frac{2\sqrt{6}x^3}{3} + \sqrt{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

Giac [A] time = 1.08836, size = 23, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\sqrt{2}\left(2x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

3.181 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rubi [A] time = 0.0810888, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx &= 2\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx\\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{\left(3+dx^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0083341, size = 37, normalized size = 0.2

$$\frac{\sqrt{2}E\left(\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

Maple [A] time = 0.016, size = 37, normalized size = 0.2

$$\sqrt{2}\text{EllipticE}\left(\frac{x\sqrt{3}}{3}\sqrt{-d}, \frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*2^(1/
2)/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c]
)) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sq
rt[c + d*x^2])
```

Rubi [A] time = 0.0577295, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {423, 426, 424, 421, 419}

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c]
)) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sq
rt[c + d*x^2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
```

```
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx &= -\frac{\int \frac{\sqrt{c+dx^2}}{\sqrt{4-x^2}}\,dx}{d}-\frac{(-c-4d)\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx}{d}\\
&= -\frac{\sqrt{c+dx^2}\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{4-x^2}}\,dx}{d\sqrt{1+\frac{dx^2}{c}}}-\frac{\left((-c-4d)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}\\
&= -\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}+\frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0380096, size = 60, normalized size = 0.66

$$\frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -c/(4*d)])/(Sqrt[-
(d/c)]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 78, normalized size = 0.9

$$\frac{1}{d}\left(c\text{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+4\,\text{EllipticF}\left(x/2,2\sqrt{-\frac{d}{c}}\right)d-c\text{EllipticE}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{dx^2+c}{c}}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (c*EllipticF(1/2*x,2*(-d/c)^(1/2))+4*EllipticF(1/2*x,2*(-d/c)^(1/2))*d-c*El
lipticE(1/2*x,2*(-d/c)^(1/2)))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x-2\right)\left(x+2\right)}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.183 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=150

$$\frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c\left(x^2+4\right)}}}+\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c\left(x^2+4\right)}}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))]) + (4*S
qrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(
c + d*x^2)/(c*(4 + x^2))])
```

Rubi [A] time = 0.0591805, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}+\frac{4\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c\left(x^2+4\right)}}}-\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c\left(x^2+4\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```



```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))]) + (4*S
qrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(
c + d*x^2)/(c*(4 + x^2))])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx &= 4\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx+\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}-\frac{4\int \frac{\sqrt{c+dx^2}}{(4+x^2)^{3/2}}\,dx}{d}\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}
\end{aligned}$$

Mathematica [A] time = 0.0311412, size = 60, normalized size = 0.4

$$\frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], c/(4*d)])/(Sqrt[-(
d/c)]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 53, normalized size = 0.4

$$2\frac{1}{\sqrt{dx^2+c}}\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)\sqrt{\frac{dx^2+c}{c}}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 2*EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(
1/2)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.184 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}} \, dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rubi [A] time = 0.0068373, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}} \, dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.006326, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Maple [A] time = 0.015, size = 23, normalized size = 1.2

$$\frac{\sqrt{2}}{6}\left(\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)+2\,\mathrm{EllipticE}\left(x,1/2\,\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(EllipticF(x,1/2*6^(1/2))+2*EllipticE(x,1/2*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.21761, size = 34, normalized size = 1.7

$$\begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.185 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rubi [A] time = 0.0076009, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {424}

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0041986, size = 21, normalized size = 1.

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] time = 0.027, size = 18, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.43051, size = 36, normalized size = 1.71

$$\begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{1}{6}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 1/6)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + 4}}{\sqrt{-3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.186 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Rubi [A] time = 0.0070226, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0045136, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Maple [A] time = 0.02, size = 29, normalized size = 1.5

$$-\frac{\sqrt{2}}{12}\left(5\,\mathrm{EllipticF}\left(2\,x,1/4\,\sqrt{6}\right)-8\,\mathrm{EllipticE}\left(2\,x,1/4\,\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/12*2^(1/2)*(5*EllipticF(2*x,1/4*6^(1/2))-8*EllipticE(2*x,1/4*6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.34122, size = 34, normalized size = 1.7

$$\begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{8}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 8/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.187 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] EllipticE[ArcSin[x], -1]
```

Rubi [A] time = 0.0052178, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {424}

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

```
[Out] EllipticE[ArcSin[x], -1]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E\left(\sin^{-1}(x)\middle|-1\right)$$

Mathematica [A] time = 0.003282, size = 4, normalized size = 1.

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

```
[Out] EllipticE[ArcSin[x], -1]
```

Maple [A] time = 0.012, size = 5, normalized size = 1.3

$$\mathrm{EllipticE}\left(x,i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] EllipticE(x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-x^2+1}}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-x^2 + 1)/(x^2 - 1), x)
```

Sympy [B] time = 2.13838, size = 10, normalized size = 2.5

$$\begin{cases} E(\operatorname{asin}(x)|-1) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((elliptic_e(asin(x), -1), (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

3.188 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0064151, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0036869, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] time = 0.016, size = 19, normalized size = 1.

$$\frac{\sqrt{3}}{3}\text{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.35063, size = 36, normalized size = 1.8

$$\begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.189 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rubi [A] time = 0.0064591, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {424}

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]

[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx=\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0041913, size = 21, normalized size = 1.

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] time = 0.018, size = 19, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i}{6}\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.51391, size = 37, normalized size = 1.76

$$\begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle| -\frac{1}{6}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.190 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}} \, dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rubi [A] time = 0.0072809, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}} \, dx = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0046723, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Maple [A] time = 0.023, size = 19, normalized size = 1.

$$\frac{\sqrt{3}}{3}\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{2i}{3}\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{4x^2+1}\sqrt{-3x^2+2}}{3x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [A] time = 4.46413, size = 36, normalized size = 1.8

$$\begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.191 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=13

$$2\text{EllipticF}\left(\sin^{-1}(x),-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rubi [A] time = 0.0137832, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {423, 424, 248, 221}

$$2F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]

[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}}\,dx - \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx\\ &= -E\left(\sin^{-1}(x)\middle|-1\right) + 2\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -E\left(\sin^{-1}(x)\middle|-1\right) + 2F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.0031746, size = 12, normalized size = 0.92

$$-iE\left(i\sinh^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (-I)*EllipticE[I*ArcSinh[x], -1]
```

Maple [A] time = 0.008, size = 14, normalized size = 1.1

$$-\mathrm{EllipticE}\left(x,i\right) + 2\,\mathrm{EllipticF}\left(x,i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

3.192 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{5\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 + (5*EllipticF[ArcSin[x], -3/2])/(3
*Sqrt[2])
```

Rubi [A] time = 0.0202918, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {423, 424, 419}

$$\frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 + (5*EllipticF[ArcSin[x], -3/2])/(3
*Sqrt[2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-x^2}}\,dx\right)+\frac{5}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)+\frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}$$

Mathematica [C] time = 0.0050336, size = 27, normalized size = 0.87

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3])/Sqrt[3]
```

Maple [A] time = 0.014, size = 27, normalized size = 0.9

$$\frac{\left(5\,\mathrm{EllipticF}\left(x,i/2\sqrt{6}\right)-2\,\mathrm{EllipticE}\left(x,i/2\sqrt{6}\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*(5*EllipticF(x,1/2*I*6^(1/2))-2*EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.193 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{7}{3}\sqrt{2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 + (7*Sqrt[2]*EllipticF[ArcSin[x/2],
 -6])/3
```

Rubi [A] time = 0.0205367, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {423, 424, 419}

$$\frac{7}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 + (7*Sqrt[2]*EllipticF[ArcSin[x/2],
 -6])/3
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{4-x^2}}\,dx\right) + \frac{14}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right) + \frac{7}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Mathematica [C] time = 0.0044379, size = 27, normalized size = 0.77

$$-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] time = 0.023, size = 31, normalized size = 0.9

$$\frac{\sqrt{2}}{3}\left(7\,\mathrm{EllipticF}\left(x/2, i\sqrt{6}\right) - \mathrm{EllipticE}\left(\frac{x}{2}, i\sqrt{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*(7*EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.194 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{11\text{EllipticF}\left(\sin^{-1}(2x),-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)|-\frac{3}{8}\right)$$

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSin[2*x], -3
/8])/(6*Sqrt[2])
```

Rubi [A] time = 0.0199491, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {423, 424, 419}

$$\frac{11F\left(\sin^{-1}(2x)|-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)|-\frac{3}{8}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSin[2*x], -3
/8])/(6*Sqrt[2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = -\left(\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-4x^2}}\,dx\right) + \frac{11}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx$$
$$= -\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right) + \frac{11F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{6\sqrt{2}}$$

Mathematica [C] time = 0.0044684, size = 27, normalized size = 0.77

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -8/3])/Sqrt[3]
```

Maple [A] time = 0.018, size = 31, normalized size = 0.9

$$-\frac{\left(-11\,\mathrm{EllipticF}\left(2\,x, i/4\sqrt{6}\right) + 8\,\mathrm{EllipticE}\left(2\,x, i/4\sqrt{6}\right)\right)\sqrt{2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/12*(-11*EllipticF(2*x,1/4*I*6^(1/2))+8*EllipticE(2*x,1/4*I*6^(1/2)))*2^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(2x-1)(2x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.195 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}} \, dx$

Optimal. Leaf size=131

$$\frac{\sqrt{3x^2+2}\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) + (Sqrt[2 +
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)
/(1 + x^2)])
```

Rubi [A] time = 0.0408642, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {422, 418, 492, 411}

$$\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+1}}+\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```


```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) + (Sqrt[2 +
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)
/(1 + x^2)])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx &= \int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx + \int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx \\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} - \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\left(1+x^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0038823, size = 27, normalized size = 0.21

$$
-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[3]
```

Maple [A] time = 0.015, size = 30, normalized size = 0.2

$$-\frac{i}{6}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)+2\,\mathrm{EllipticE}\left(ix,1/2\,\sqrt{6}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/6*I*(EllipticF(I*x,1/2*6^(1/2))+2*EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.196 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=136

$$\frac{2\sqrt{2}\sqrt{3x^2+2}\text{EllipticF}\left(\tan^{-1}\left(\frac{x}{2}\right),-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}+\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)]) + (2*Sqrt[2
]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^
2)/(4 + x^2)])
```

Rubi [A] time = 0.0442971, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {422, 418, 492, 411}

$$\frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+4}}+\frac{2\sqrt{2}\sqrt{3x^2+2}F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)]) + (2*Sqrt[2
]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^
2)/(4 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx &= 4\int \frac{1}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx + \int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx \\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} + \frac{2\sqrt{2}\sqrt{2+3x^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}} - \frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\left(4+x^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}} + \frac{2\sqrt{2}\sqrt{2+3x^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0039403, size = 27, normalized size = 0.2

$$
-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] time = 0.013, size = 26, normalized size = 0.2

$$-\frac{i}{3}\left(5\,\mathrm{EllipticF}\left(i/2x,\sqrt{6}\right)+\mathrm{EllipticE}\left(\frac{i}{2}x,\sqrt{6}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*(5*EllipticF(1/2*I*x,6^(1/2))+EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 4}}{\sqrt{3x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + 4}}{\sqrt{3\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.197 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\tan^{-1}(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}+\frac{4\sqrt{3x^2+2}\,x}{3\sqrt{4x^2+1}}-\frac{2\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3*x^2]*Elli
pticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
+ (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)
/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rubi [A] time = 0.0497495, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{4\sqrt{3x^2+2}\,x}{3\sqrt{4x^2+1}}+\frac{\sqrt{3x^2+2}\,F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}-\frac{2\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3*x^2]*Elli
pticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
+ (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)
/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx &= 4\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} - \frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\left(1+4x^2\right)^{3/2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{2\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}
\end{aligned}
$$

Mathematica [C] time = 0.0045624, size = 27, normalized size = 0.18

$$
-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3])/Sqrt[3]
```

Maple [C] time = 0.023, size = 20, normalized size = 0.1

$$-\frac{i}{3}\mathrm{EllipticE}\left(\frac{i}{2}x\sqrt{6}, \frac{2\sqrt{6}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*EllipticE(1/2*I*x*6^(1/2),2/3*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.198 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[-1 + 2*x^
2])
```

Rubi [A] time = 0.014525, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {427, 424}

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[-1 + 2*x^
2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\int \frac{\sqrt{1-x^2}}{\sqrt{1-2x^2}}\,dx}{\sqrt{-1+2x^2}}$$
$$= \frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+2x^2}}$$

Mathematica [A] time = 0.0240141, size = 35, normalized size = 0.88

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/Sqrt[-2 + 4*x^2]
```

Maple [A] time = 0.01, size = 32, normalized size = 0.8

$$\frac{\mathrm{EllipticF}\left(x,\sqrt{2}\right)+\mathrm{EllipticE}\left(x,\sqrt{2}\right)}{2}\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(x,2^(1/2))+EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

3.199 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(3bc-7ad)\left(15a^2d^2-11abcd+8b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-\right.}{105d^3}$$

```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a + b*x^2])/
(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d + 71*a^2*d^2)*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*b*(b*c - 2*a*d)*x*(a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(
7*d) + (8*Sqrt[c]*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^
(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(3*
b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.431295, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {416, 528, 531, 418, 492, 411}

$$\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}-\frac{8x\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^3\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}(3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```



```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a + b*x^2])/
(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d + 71*a^2*d^2)*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*b*(b*c - 2*a*d)*x*(a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(
7*d) + (8*Sqrt[c]*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^
(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(3*
```

```
b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{\left(a+bx^2\right)^{3/2}\left(-a(bc-7ad)-6b(bc-2ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{7d}$$

$$= -\frac{6b(bc-2ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{\sqrt{a+bx^2}\left(a\left(6b^2c^2-17abcd+35a^2d^2\right)+\cdots\right.}{\sqrt{c+dx^2}}}{35d^2}$$

$$= \frac{b\left(24b^2c^2-71abcd+71a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{6b(bc-2ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx\left(a\cdots\right.}{\cdots}$$

$$= \frac{b\left(24b^2c^2-71abcd+71a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{6b(bc-2ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx\left(a\cdots\right.}{\cdots}$$

$$= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} + \frac{b\left(24b^2c^2-71abcd+71a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c\cdots}}{105d^3}$$

$$= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} + \frac{b\left(24b^2c^2-71abcd+71a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c\cdots}}{105d^3}$$

Mathematica [C] time = 1.48904, size = 321, normalized size = 0.76

$$\frac{-i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(353a^2b^2c^2d^2-298a^3bcd^3+105a^4d^4-208ab^3c^3d+48b^4c^4\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+bd\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(122*a^2*d^2 + a*b*d*(-89*c + 66*d
*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - (8*I)*b*c*(-6*b^3*c^3 + 23
*a*b^2*c^2*d - 33*a^2*b*c*d^2 + 22*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(48*b^4*c^4 - 2
08*a*b^3*c^3*d + 353*a^2*b^2*c^2*d^2 - 298*a^3*b*c*d^3 + 105*a^4*d^4)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 852, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*(-b/a)^(1/2)*x^9*b^4*d^4+81*(-b/a
)^(1/2)*x^7*a*b^3*d^4-3*(-b/a)^(1/2)*x^7*b^4*c*d^3+188*(-b/a)^(1/2)*x^5*a^2
*b^2*d^4-26*(-b/a)^(1/2)*x^5*a*b^3*c*d^3+6*(-b/a)^(1/2)*x^5*b^4*c^2*d^2+122
*(-b/a)^(1/2)*x^3*a^3*b*d^4+99*(-b/a)^(1/2)*x^3*a^2*b^2*c*d^3-83*(-b/a)^(1/
2)*x^3*a*b^3*c^2*d^2+24*(-b/a)^(1/2)*x^3*b^4*c^3*d+105*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*d^4-298*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^3*b*c*d^3+353*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2-208*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*b^4*c^4+176*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3-264*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2+184*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b
^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*b^4*c^4+122*(-b/a)^(1/2)*x*a^3*b*c*d^3-89*(-b/a)^(1/2)*x
*a^2*b^2*c^2*d^2+24*(-b/a)^(1/2)*x*a*b^3*c^3*d)/d^4/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{7}{2}}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{bx^{2}+a}}{\sqrt{dx^{2}+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/sqrt(d
*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^{2}\right)^{\frac{7}{2}}}{\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(7/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{\sqrt{dx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

3.200 $\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=344

$$\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15d^2\sqrt{c+dx^2}}-\frac{\sqrt{c}}{}$$

```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2*Sqrt[c +
d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) +
(b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 23*a*
b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (Sqrt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.282602, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {416, 528, 531, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15d^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```


```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2*Sqrt[c +
d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) +
(b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 23*a*
b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (Sqrt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{\sqrt{a+bx^2}\left(-a(bc-5ad)-4b(bc-2ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5d}$$

$$= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{a\left(4b^2c^2-11abcd+15a^2d^2\right)+b\left(8b^2c^2-23ab\right.}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15d^2}$$

$$= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\left(a\left(4b^2c^2-11abcd+15a^2d^2\right)\right)\int}{15d^2}$$

$$= \frac{\left(8b^2c^2-23abcd+23a^2d^2\right)x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+}}{5d}$$

$$= \frac{\left(8b^2c^2-23abcd+23a^2d^2\right)x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+}}{5d}$$

Mathematica [C] time = 0.478191, size = 260, normalized size = 0.76

$$\frac{-i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(-34a^2bcd^2+15a^3d^3+27ab^2c^2d-8b^3c^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}}{15d^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 11*a*d + 3*b*d*x^2) - I*
b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-8*b^3*c^3 + 27*
a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.017, size = 615, normalized size = 1.8

$$\frac{1}{15\,d^{3}\left(bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac\right)}\sqrt{b\,x^{2}+a}\sqrt{d\,x^{2}+c}\left(3\sqrt{-\frac{b}{a}}x^{7}b^{3}d^{3}+14\sqrt{-\frac{b}{a}}x^{5}ab^{2}d^{3}-\sqrt{-\frac{b}{a}}x^{5}b^{3}cd^{2}+11\sqrt{-\frac{b}{a}}x^{3}a^{2}bd^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^3*d^3+14*(-b/a)^
(1/2)*x^5*a*b^2*d^3-(-b/a)^(1/2)*x^5*b^3*c*d^2+11*(-b/a)^(1/2)*x^3*a^2*b*d^
3+10*(-b/a)^(1/2)*x^3*a*b^2*c*d^2-4*(-b/a)^(1/2)*x^3*b^3*c^2*d+15*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
3*d^3-34*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a^2*b*c*d^2+27*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+23*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b*c*d^2-23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+11*(-b/a)^(1/2)*x*a^2*
b*c*d^2-4*(-b/a)^(1/2)*x*a*b^2*c^2*d)/d^3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{\sqrt{d\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

3.201 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}}{3d}$$

```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.160054, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {416, 531, 418, 492, 411}

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```


```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{\int \frac{-a(bc-3ad)-2b(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}$$

$$= \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{(a(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} - \frac{(2b(bc-2ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d}$$

$$= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{\sqrt{c}(bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} +$$

$$= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{2\sqrt{c}(bc-2ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} -$$

Mathematica [C] time = 0.336415, size = 216, normalized size = 0.83

$$\frac{-i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-5abcd+2b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+bdx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+}}{3d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - (2*I)*b*c*(-(b*c) + 2*a*d)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^
2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.015, size = 399, normalized size = 1.5

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,bcx^2+3\,ac\right)d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5b^2d^2+\sqrt{-\frac{b}{a}}x^3abd^2+\sqrt{-\frac{b}{a}}x^3b^2cd+3\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b^2*d^2+(-b/a)^(1/2)*
x^3*a*b*d^2+(-b/a)^(1/2)*x^3*b^2*c*d+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^2-5*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b^2*c^2+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*x*a*b*c*d)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

3.202 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.090015, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```


```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0498443, size = 86, normalized size = 0.44

$$
\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.012, size = 158, normalized size = 0.8

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)d}\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(a\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\mathrm{EllipticF}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.203 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0186419, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0545042, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(x\sqrt{-\frac{b}{a}}\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.015, size = 100, normalized size = 1.2

$$\frac{1}{bdx^4+adx^2+bcx^2+ac}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.204 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}(bc-ad)}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan\right.}{a\sqrt{c+dx^2}(bc-a}$$

```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*Sqrt[c + d*
x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c
- a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.143031, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {414, 21, 422, 418, 492, 411}

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx\right.}{a\left(c+dx\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*Sqrt[c + d*
x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c
- a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} +$$

$$= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} -$$

Mathematica [A] time = 0.235924, size = 112, normalized size = 0.41

$$\frac{\frac{ad\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}} - bx\left(c+dx^2\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*x*(c + d*x^2)) + (a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/Sqrt[-(d/c)])/(a*(-(b*c) + a*d)*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.024, size = 248, normalized size = 0.9

$$\frac{1}{a\left(ad-bc\right)\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-x^3bd\sqrt{-\frac{b}{a}} + \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ad\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}} - \mathrm{EllipticF}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-x^3*b*d*(-b/a)^(1/2)+EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-x*b*c*(-b/a)^(1/2))
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/a/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^2dx^6+\left(b^2c+2\,abd\right)x^4+a^2c+\left(2\,abc+a^2d\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4
 + a^2*c + (2*a*b*c + a^2*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

3.205 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx}{3a\left(a+b\right.}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sqrt[b]*(b*c
 - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/
(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.146316, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {414, 525, 418, 411}

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sqrt[b]*(b*c
 - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/
(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2bc+3ad-bdx^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{(d(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)^2} + \frac{(2b(bc-2ad))\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,d}{3a(bc-ad)^2}$$

$$= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2\sqrt{b}(bc-2ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}} - \frac{\sqrt{c}\sqrt{d}(bc-}{3a^2}$$

Mathematica [C] time = 0.592447, size = 261, normalized size = 1.02

$$\frac{-i\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-5abcd+2b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+bx\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(-5a^2d+ab\left(3\right.\right.}{3a^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(-5*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 4*d*x^2)) -
 (2*I)*b*c*(-(b*c) + 2*a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*b^2*c^2 - 5*a*b*
c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*(a
 + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.028, size = 752, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(-4*x^5*a*b^2*d^2*(-b/a)^(1/2)+2*x^5*b^3*c*d*(-b/a)^(1/2)+3*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)-5*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^2*c*d*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^2*b^3*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+4*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*x^2*a*b^2*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^3*c^2*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)-5*x^3*a^2*b*d^2*(-b/a)^(1/2)-x^3*a*b^2*c*d*(-b/a)^(1
/2)+2*x^3*b^3*c^2*(-b/a)^(1/2)+3*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
a^3*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-5*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a^2*b*c*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)+4*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a
*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-5*x*a^2*b*c*d*(-b/a)^(1/2)
+3*x*a*b^2*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/(a*d-b*c)^2/(-b/a)^(1/2)/a^2/(
```

```
b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^3dx^8+\left(b^3c+3\,ab^2d\right)x^6+3\left(ab^2c+a^2bd\right)x^4+a^3c+\left(3\,a^2bc+a^3d\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x
^6 + 3*(a*b^2*c + a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

3.206 $\int \frac{1}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=334

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15a^{5/2}\sqrt{a+bx^2}(bc-ad)^3}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*(b*c - 2*a
*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (Sqrt[b]*
(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 1
1*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.277528, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {414, 527, 525, 418, 411}

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\tan^{-1}\right.}{15a^{5/2}\sqrt{a+bx^2}(bc-ad)^3\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*(b*c - 2*a
*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (Sqrt[b]*
(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 1
1*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-4bc+5ad-3bdx^2}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx}{5a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{8b^2c^2-19abcd+15a^2d^2+4bd(bc-2ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}}{15a^2(bc-ad)^2}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{\left(d\left(4b^2c^2-11abcd+15a^2d^2\right)\right)\int}{15a^2(bc-ad)^3}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\sqrt{b}\left(8b^2c^2-23abcd+23a^2d^2\right)}{15a^{5/2}(bc-ad)^3}$$

Mathematica [C] time = 0.620821, size = 301, normalized size = 0.9

$$\frac{bx\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(23a^2d^2-23abcd+8b^2c^2\right)+3a^2(bc-ad)^2+4a\left(a+bx^2\right)(bc-2ad)(bc-ad)\right)+i\sqrt{\frac{bx^2}{a}+1}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 4*a*(b*c - 2*a*d)*(b*c -
a*d)*(a + b*x^2) + (8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*(a + b*x^2)^2) + I
*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*(8*b^2*c^2 - 23
*a*b*c*d + 23*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8
*b^3*c^3 + 27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^3*(a + b*x^2)^
(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.031, size = 1607, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(-15*x*a^2*b^3*c^3*(-b/a)^(1/2)+15*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a^5*d^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-23*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x^4*a*b^4*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)-34*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^3*c*d^2*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-46*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*x^2*a^2*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-68*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)+27*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a*b^4*c^2*d
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+46*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*x^2*a^3*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+30*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^4*b*d^3*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)-16*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^4*c^
3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+23*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^4*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-23*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)-34*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*b*c*d^2*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+27*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*a^3*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-23*x^7*a^2*b^3*d
^3*(-b/a)^(1/2)-34*x*a^4*b*c*d^2*(-b/a)^(1/2)-13*x^3*a^3*b^2*c*d^2*(-b/a)^(
1/2)+43*x^3*a^2*b^3*c^2*d*(-b/a)^(1/2)+35*x^5*a^2*b^3*c*d^2*(-b/a)^(1/2)+23
*x^7*a*b^4*c*d^2*(-b/a)^(1/2)-8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x
^4*b^5*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+8*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a^2*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-8*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^3*c^3*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)+15*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^3*b^2*
d^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+16*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*x^2*a*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+54*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^3*c^2*d*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)-8*x^7*b^5*c^2*d*(-b/a)^(1/2)-54*x^5*a^3*b^2*d^3*(-b/a)
^(1/2)-34*x^3*a^4*b*d^3*(-b/a)^(1/2)-20*x^3*a*b^4*c^3*(-b/a)^(1/2)-8*x^5*b^
5*c^3*(-b/a)^(1/2)+41*x*a^3*b^2*c^2*d*(-b/a)^(1/2)+8*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*x^4*b^5*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+3*x
^5*a*b^4*c^2*d*(-b/a)^(1/2)+23*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^
4*a^2*b^3*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2))/(d*x^2+c)^(1/2)/(a
*d-b*c)^3/(-b/a)^(1/2)/a^3/(b*x^2+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^4dx^{10}+\left(b^4c+4\,ab^3d\right)x^8+2\left(2\,ab^3c+3\,a^2b^2d\right)x^6+a^4c+2\left(3\,a^2b^2c+2\,a^3bd\right)x^4+\left(4\,a^3bc+a^4d\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^4*d*x^10 + (b^4*c + 4*a*b^3*d)*
x^8 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^6 + a^4*c + 2*(3*a^2*b^2*c + 2*a^3*b*d)
*x^4 + (4*a^3*b*c + a^4*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

3.207 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=445

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+\right.}{15cd^3}$$

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a + b
*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(5/2))/(c*d*
Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sq
rt[c + d*x^2])/(5*c*d^2) - ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(7/2)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.405257, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {413, 528, 531, 418, 492, 411}

$$-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{15cd^3}+\frac{x\sqrt{a+bx^2}\left(103a^2bcd^2-15a^3d^3-128ab^2c^2d+48b^3c^3\right)}{15cd^3\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a + b
*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(5/2))/(c*d*
Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sq
rt[c + d*x^2])/(5*c*d^2) - ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
```

```
b*c)/(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(7/2)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\left(c+dx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{\left(a+bx^2\right)^{3/2}\left(abc+b(6bc-5ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} + \frac{b(6bc-5ad)x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5cd^2} + \frac{\int \frac{\sqrt{a+bx^2}\left(-2abc(3bc-5ad)-b\left(24b^2c^2-43\right.\right.}{\sqrt{c+dx^2}}}{5cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b\left(24b^2c^2-43abcd+15a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} + \frac{b(6bc-5ad)x\left(a+\right.}{5c}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b\left(24b^2c^2-43abcd+15a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} + \frac{b(6bc-5ad)x\left(a+\right.}{5c}$$

$$= \frac{\left(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3\right)x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b\left(24b^2c^2-43\right.}{}$$

$$= \frac{\left(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3\right)x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b\left(24b^2c^2-43\right.}{}$$

Mathematica [C] time = 1.1849, size = 318, normalized size = 0.71

$$\frac{4ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(41a^2bcd^2-15a^3d^3-38ab^2c^2d+12b^3c^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-45*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*c*d*(61*c
+ 16*d*x^2) - 3*b^3*c*(8*c^2 + 2*c*d*x^2 - d^2*x^4)) + I*b*c*(-48*b^3*c^3 +
 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*b*c*(1
2*b^3*c^3 - 38*a*b^2*c^2*d + 41*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*
Sqrt[b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.043, size = 755, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^4*c*d^3+19*(-b/a
)^(1/2)*x^5*a*b^3*c*d^3-6*(-b/a)^(1/2)*x^5*b^4*c^2*d^2+15*(-b/a)^(1/2)*x^3*
a^3*b*d^4-29*(-b/a)^(1/2)*x^3*a^2*b^2*c*d^3+55*(-b/a)^(1/2)*x^3*a*b^3*c^2*d
^2-24*(-b/a)^(1/2)*x^3*b^4*c^3*d+60*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3-164*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c
^2*d^2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a*b^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4-15*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^3+103*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a^2*b^2*c^2*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4+15*x*a^4*d^4
*(-b/a)^(1/2)-45*(-b/a)^(1/2)*x*a^3*b*c*d^3+61*(-b/a)^(1/2)*x*a^2*b^2*c^2*d
^2-24*(-b/a)^(1/2)*x*a*b^3*c^3*d)/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^
(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{7}{2}}}{\left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(7/2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{7}{2}}}{\left(dx^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.208 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{2b\sqrt{c}\sqrt{a+bx^2}(2bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{3cd^2\sqrt{c+dx^2}}+\frac{\sqrt{a+bx^2}\left(3a^2d^2\right.}{3}$$

```
[Out] -((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c*d^2*Sqrt[c +
 d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) + (b*(4*
b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^2) + ((8*b^2*c^2 - 1
3*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.266402, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {413, 528, 531, 418, 492, 411}

$$-\frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{3cd^2\sqrt{c+dx^2}}+\frac{\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c*d^2*Sqrt[c +
 d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) + (b*(4*
b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^2) + ((8*b^2*c^2 - 1
3*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{\sqrt{a+bx^2}\left(abc+b(4bc-3ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} + \frac{\int \frac{-2abc(2bc-3ad)-b\left(8b^2c^2-13abcd+3a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{3cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} - \frac{(2ab(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d^2}$$

$$= -\frac{\left(8b^2c^2-13abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx}}{3cd^2}$$

$$= -\frac{\left(8b^2c^2-13abcd+3a^2d^2\right)x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx}}{3cd^2}$$

Mathematica [C] time = 0.454262, size = 256, normalized size = 0.74

$$\frac{-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(9a^2d^2-17abcd+8b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-13ab\right.}{3cd^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(4*c + d*x^2)) +
 I*b*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(8*b^2*c^2
- 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

Maple [A] time = 0.023, size = 539, normalized size = 1.6

$$\frac{1}{\left(3\,bd{x}^{4}+3\,ad{x}^{2}+3\,bc{x}^{2}+3\,ac\right){d}^{3}c}\sqrt{b{x}^{2}+a}\sqrt{d{x}^{2}+c}\left(\sqrt{-{\frac{b}{a}}}{x}^{5}{b}^{3}c{d}^{2}+3\,\sqrt{-{\frac{b}{a}}}{x}^{3}{a}^{2}b{d}^{3}-5\,\sqrt{-{\frac{b}{a}}}{x}^{3}a{b}^{2}c{d}^{2}+4\,\sqrt{-{\frac{b}{a}}}{x}^{3}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b^3*c*d^2+3*(-b/a)^(1
/2)*x^3*a^2*b*d^3-5*(-b/a)^(1/2)*x^3*a*b^2*c*d^2+4*(-b/a)^(1/2)*x^3*b^3*c^2
*d+9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b*c*d^2-17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3-3*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2
*b*c*d^2+13*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+3*x*a^3*d^3*(-b/a)^(1/2)-6*
(-b/a)^(1/2)*x*a^2*b*c*d^2+4*(-b/a)^(1/2)*x*a*b^2*c^2*d)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)/d^3/c/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{d^2x^4+2\,cdx^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d^2*x
^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.209 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a}\cdots}{\cdots}$$

```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a*d)*x
*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.15181, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {413, 531, 418, 492, 411}

$$\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}(2\cdots}{cd\sqrt{c+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a*d)*x
*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(ab)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{d} + \frac{(b(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(2bc-ad)\int}{}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{}}{}$$

Mathematica [C] time = 0.279338, size = 196, normalized size = 0.76

$$\frac{(ad-bc)\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-2bc)E\left(\right.}{cd^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^
2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Maple [A] time = 0.02, size = 345, normalized size = 1.3

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)d^2c}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^3abd^2-\sqrt{-\frac{b}{a}}x^3b^2cd+2\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^3*a*b*d^2-(-b/a)^(1/2)*x^3*
b^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+x
*a^2*d^2*(-b/a)^(1/2)-(-b/a)^(1/2)*x*a*b*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
/d^2/c/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{d^2x^4+2\,cdx^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0165276, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {411}

$$\frac{\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```


```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.307038, size = 136, normalized size = 1.62

$$\frac{\frac{x(a+bx^2)}{c}+\frac{ia\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)\right)}{d}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2))/c + (I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/d)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.015, size = 188, normalized size = 2.2

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)dc}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(x^3bd\sqrt{-\frac{b}{a}}+\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-\text{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(x^3*b*d*(-b/a)^(1/2)+EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
+x*a*d*(-b/a)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d/c/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\left(dx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\left(dx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

3.211 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.151967, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {414, 21, 422, 418, 492, 411}

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```


```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} + \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= \frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} - \frac{d\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.230638, size = 112, normalized size = 0.58

$$\frac{\frac{bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}} - dx\left(a+bx^2\right)}{c\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(d*x*(a + b*x^2)) + (b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*(b*c - a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 144, normalized size = 0.7

$$\frac{1}{c\left(ad-bc\right)\left(bdx^4+adx^2+bcx^2+ac\right)}\left(x^3bd\sqrt{-\frac{b}{a}} - \mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}} + xad\sqrt{-\frac{b}{a}}\right)\sqrt{dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (x^3*b*d*(-b/a)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a*d*(-b/a)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2
+a)^(1/2)/c/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4
 + a*c^2 + (b*c^2 + 2*a*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\left(dx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.212 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=242

$$-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a}{a(c}}}$$

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.11983, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {414, 525, 418, 411}

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{\int \frac{ad-bdx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx}{a(bc-ad)}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{(2bd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} + \frac{(d(bc+ad))\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{a(bc-ad)^2}$$

$$= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(bc+ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} - \frac{2b\sqrt{c}\cdots}{a\cdots}$$

Mathematica [C] time = 0.673031, size = 224, normalized size = 0.93

$$\frac{\sqrt{\frac{b}{a}}\left(ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+x\sqrt{\frac{b}{a}}\left(a^2d^2+abd^2x^2+b^2c\left(c+dx^2\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\right.}{bc\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) + I*b*c
*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c - a
*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.024, size = 354, normalized size = 1.5

$$\frac{1}{ac\left(ad-bc\right)^2\left(bdx^4+adx^2+bcx^2+ac\right)}\left(\sqrt{-\frac{b}{a}}x^3abd^2+\sqrt{-\frac{b}{a}}x^3b^2cd-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] ((-b/a)^(1/2)*x^3*a*b*d^2+(-b/a)^(1/2)*x^3*b^2*c*d-((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^
2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+x*a^2*d^2*(-b/a)^(1/2)+x*b^2*c^2*(-b/a)
^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/a/(-b/a)^(1/2)/(a*d-b*c)^2/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^2d^2x^8+2\left(b^2cd+abd^2\right)x^6+\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(abc^2+a^2cd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^
2)*x^6 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c
*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

3.213 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=323

$$-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*(b*c - 3*a
*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(2*
b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
*a^2*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.264476, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {414, 527, 525, 418, 411}

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2bx(bc-3ad)}{3a^2\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(b}{3a^2\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```



```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*(b*c - 3*a
*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(2*
b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
*a^2*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} - \frac{\int \frac{-2bc+3ad-3bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\int \frac{ad(bc+3ad)+2b\cdots}{\sqrt{a+bx^2}\left(c+\cdots\right.}}{3a^2(bc-\cdots}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{(bd(bc-9ad))\cdots}{3a(b\cdots}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}\left(2b^2c^2-7\cdots\right.}{3\cdots}$$

<u>**Mathematica**</u> **[C]** time = 1.03374, size = 337, normalized size = 1.04

$$\frac{2ibc\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+x\sqrt{\frac{b}{a}}\left(a^2b^2d\left(8c^2+8cdx^2+3\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 2*b^4*c^2*x^2*(c + d*x^2) + a^2
*b^2*d*(8*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + a*b^3*c*(-3*c^2 + 4*c*d*x^2 + 7*d^
2*x^4)) + I*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + (2*I)*b*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3
*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

<u>**Maple**</u> **[B]** time = 0.031, size = 964, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-3*x^5*a^2*b^2*d^3*(-b/a)^(1/2)-7*x^5*a*b^3*c*d^2*(-b/a)^(1/2)+2*x^5*
b^4*c^2*d*(-b/a)^(1/2)+6*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*
b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-8*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*x^2*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+
2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^4*c^3*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)+3*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^
2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+7*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*x^2*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^4*c^3*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)-6*x^3*a^3*b*d^3*(-b/a)^(1/2)-8*x^3*a^2*b^2*c*d^2*(-b/a)^
(1/2)-4*x^3*a*b^3*c^2*d*(-b/a)^(1/2)+2*x^3*b^4*c^3*(-b/a)^(1/2)+6*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)-8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+3*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+7*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-3*x*a^4*d^3*(-b/a)^(1/2)-8*x*a^2*b^
2*c^2*d*(-b/a)^(1/2)+3*x*a*b^3*c^3*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/(a*d-b*c)^
3/(-b/a)^(1/2)/a^2/c/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{b^3d^2x^{10}+\left(2\,b^3cd+3\,ab^2d^2\right)x^8+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^6+a^3c^2+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^4+\left(3\,a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b^3*d^2*x^10 + (2*b^3*c*d + 3*a*b
^2*d^2)*x^8 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^6 + a^3*c^2 + (3*a*b^
2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^4 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

3.214 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0190631, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.054663, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(x\sqrt{-\frac{b}{a}}\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0., size = 100, normalized size = 1.2

$$\frac{1}{bdx^4+adx^2+bcx^2+ac}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{bdx^4+(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.215 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*
x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0537777, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {421, 419}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*
x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{c+dx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0605808, size = 87, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(x\sqrt{\frac{b}{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], -((
a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 106, normalized size = 1.2

$$-\frac{1}{bdx^4-adx^2+bcx^2-ac}\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/
a)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/(b*d*x^4-a*d*x^2+b*c*
x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2 + a}\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a}\sqrt{dx^2 + c}}{bdx^4 + (bc - ad)x^2 - ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*x^4 + (b*c - a*d)*x^2 - a*c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - bx^2}\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2 + a}\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.216 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0524197, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {421, 419}

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}$$
$$= \frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] time = 0.0649762, size = 89, normalized size = 1.02

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(x\sqrt{-\frac{b}{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
-((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.028, size = 106, normalized size = 1.2

$$-\frac{1}{bdx^4+adx^2-bcx^2-ac}\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\sqrt{bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/
c)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(b*d*x^4+a*d*x^2-b*c*
x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{bx^2 + a}\sqrt{-dx^2 + c}}{bdx^4 - (bc - ad)x^2 - ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)/(b*d*x^4 - (b*c - a*d)*x^2 - a*c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^2}\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.217 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0570123, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {421, 419}

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}$$
$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}$$
$$= \frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] time = 0.0614795, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], (a*
d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.026, size = 108, normalized size = 1.2

$$\frac{1}{bdx^4-adx^2-bcx^2+ac}\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)/c
)^(1/2)*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(b*d*x^4-a*d*x^2-b*c*
x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}{bdx^4-(bc+ad)x^2+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)/(b*d*x^4 - (b*c + a*d)*x^2 + a*c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.218 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rubi [A] time = 0.0070182, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx=\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0050385, size = 12, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Maple [A] time = 0.026, size = 14, normalized size = 1.2

$$\frac{\mathrm{EllipticF}\left(x, \frac{i}{2}\sqrt{10}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}{5\,x^4-3\,x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)/(5*x^4 - 3*x^2 - 2), x)
```

Sympy [A] time = 4.44315, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(x\right)\middle| -\frac{5}{2}\right)}{2} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -5/2)/2, (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5 \, x^{2} + 2} \sqrt{-x^{2} + 1}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-2\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rubi [A] time = 0.007433, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)|-2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{F\left(\sin^{-1}(x)|-2\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.030298, size = 58, normalized size = 5.8

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-2x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I/2)*Sqrt[1 - x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/2]
)/Sqrt[1 + x^2 - 2*x^4]
```

Maple [A] time = 0.023, size = 14, normalized size = 1.4

$$\frac{\text{EllipticF}\left(x, i\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}{2\left(2\,x^4-x^2-1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)/(2*x^4 - x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{1-x^2}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.0068817, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx=\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0047674, size = 12, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Maple [A] time = 0.015, size = 14, normalized size = 1.2

$$\frac{\operatorname{EllipticF}\left(x, \frac{i}{2}\sqrt{6}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}{3\,x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)/(3*x^4 - x^2 - 2), x)
```

Sympy [A] time = 4.46093, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle| -\frac{3}{2}\right)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -3/2)/2, (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.221 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-1\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rubi [A] time = 0.0052985, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {248, 221}

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2-2x^4}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0191893, size = 10, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] time = 0.018, size = 10, normalized size = 1.

$$\frac{\text{EllipticF}(x,i)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}{2\,\left(x^4-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)/(x^4 - 1), x)
```

Sympy [B] time = 8.39547, size = 73, normalized size = 7.3

$$-\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}+\frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
(0,)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3/2)) + sqrt(2)*meijerg(((-1/4, 0,
 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi**(
3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.222 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rubi [A] time = 0.0065566, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.0194594, size = 18, normalized size = 1.5

$$-i\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[2]], -2]
```

Maple [A] time = 0.022, size = 14, normalized size = 1.2

$$\frac{\text{EllipticF}\left(x, \frac{i}{2}\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+2}\sqrt{-x^2+1}}{x^4+x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 2)*sqrt(-x^2 + 1)/(x^4 + x^2 - 2), x)
```

Sympy [A] time = 2.17779, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{1}{2}\right)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -1/2)/2, (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.223 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rubi [A] time = 0.0074622, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.005636, size = 12, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Maple [A] time = 0.022, size = 13, normalized size = 1.1

$$\frac{\sqrt{2}}{2}\mathrm{EllipticF}\left(x,\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2+2}\sqrt{-x^2+1}}{x^4-3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)/(x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.224 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rubi [A] time = 0.0022928, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {22, 206}

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{\int \frac{1}{1-x^2}\,dx}{\sqrt{2}}$$
$$= \frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Mathematica [B] time = 0.0048565, size = 26, normalized size = 3.25

$$-\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

Maple [A] time = 0.04, size = 8, normalized size = 1.

$$\frac{\text{Artanh}\,(x)\,\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [B] time = 1.98489, size = 169, normalized size = 21.12

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{x^6+5x^4-2\sqrt{2}\left(x^3+x\right)\sqrt{-x^2+1}\sqrt{-2x^2+2}-5x^2-1}{x^6-3x^4+3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(x^6 + 5*x^4 - 2*sqrt(2)*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2*
x^2 + 2) - 5*x^2 - 1)/(x^6 - 3*x^4 + 3*x^2 - 1))
```

Sympy [A] time = 2.66071, size = 22, normalized size = 2.75

$$-\sqrt{2}\left(\begin{cases}-\frac{\operatorname{acoth}(x)}{2} & \text{for}\ x^2>1\\ -\frac{\operatorname{atanh}(x)}{2} & \text{for}\ x^2<1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(2)*Piecewise((-acoth(x)/2, x**2 > 1), (-atanh(x)/2, x**2 < 1))
```

Giac [B] time = 1.0861, size = 26, normalized size = 3.25

$$\frac{1}{4}\sqrt{2}\log(x+1)-\frac{1}{4}\sqrt{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x + 1) - 1/4*sqrt(2)*log(x - 1)
```

3.225 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Rubi [A] time = 0.0068764, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)|\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx=\frac{F\left(\sin^{-1}(x)|\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0050175, size = 12, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Maple [A] time = 0.017, size = 13, normalized size = 1.1

$$\frac{\sqrt{2}}{2}\mathrm{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}{3\,x^4-5\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)/(3*x^4 - 5*x^2 + 2), x)
```

Sympy [A] time = 4.37675, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{3}{2}\right)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), 3/2)/2, (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.226 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),2\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rubi [A] time = 0.0073752, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)|2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{F\left(\sin^{-1}(x)|2\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0089306, size = 10, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Maple [A] time = 0.019, size = 11, normalized size = 1.1

$$\frac{\text{EllipticF}\left(x, \sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}{2\left(2\,x^4-3\,x^2+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)/(2*x^4 - 3*x^2 + 1), x)
```

Sympy [A] time = 8.18606, size = 39, normalized size = 3.9

$$\frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for } x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqrt(2
)/2) & (x < sqrt(2)/2)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.227 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rubi [A] time = 0.006778, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {419}

$$\frac{F\left(\sin^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]

[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx=\frac{F\left(\sin^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0049479, size = 12, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Maple [A] time = 0.025, size = 13, normalized size = 1.1

$$\frac{\sqrt{2}}{2}\mathrm{EllipticF}\left(x, \frac{\sqrt{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}{5\,x^4-7\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)/(5*x^4 - 7*x^2 + 2), x)
```

Sympy [A] time = 4.36036, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{5}{2}\right)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), 5/2)/2, (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.228 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 5*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0094425, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {418}

$$\frac{\sqrt{5x^2+2}\,F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 5*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0203425, size = 19, normalized size = 0.37

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 5/2])/Sqrt[2]
```

Maple [A] time = 0.032, size = 17, normalized size = 0.3

$$-\frac{i}{2}\text{EllipticF}\left(ix,\frac{\sqrt{10}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*10^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x^2+2}\sqrt{x^2+1}}{5\,x^4+7\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)/(5*x^4 + 7*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(5*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.229 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{2x^2+1}\,\text{EllipticF}\left(\tan^{-1}(x),-1\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(1 +
 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0092925, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {418}

$$\frac{\sqrt{2x^2+1}F\left(\tan^{-1}(x)|-1\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(1 +
 2*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}F\left(\tan^{-1}(x)|-1\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0277553, size = 17, normalized size = 0.35

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 2])/Sqrt[2]
```

Maple [A] time = 0.02, size = 15, normalized size = 0.3

$$-\frac{i}{2}\text{EllipticF}\left(ix,\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x^2+2}\sqrt{x^2+1}}{2\left(2\,x^4+3\,x^2+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(4*x^2 + 2)*sqrt(x^2 + 1)/(2*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{x^2+1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.230 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0087415, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {418}

$$\frac{\sqrt{3x^2+2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 3*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{2+3x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0195597, size = 19, normalized size = 0.37

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 3/2])/Sqrt[2]
```

Maple [A] time = 0.013, size = 17, normalized size = 0.3

$$-\frac{i}{2}\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*6^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,x^2+2}\sqrt{x^2+1}}{3\,x^4+5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.231 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

```
[Out] ArcTan[x]/Sqrt[2]
```

Rubi [A] time = 0.0023531, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {22, 203}

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]

[Out] ArcTan[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \sqrt{2}\int \frac{1}{2+2x^2}\,dx$$
$$= \frac{\tan^{-1}(x)}{\sqrt{2}}$$

Mathematica [A] time = 0.0029739, size = 8, normalized size = 1.

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Maple [A] time = 0.027, size = 8, normalized size = 1.

$$\frac{\arctan(x)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] 1/2*arctan(x)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [B] time = 1.90796, size = 97, normalized size = 12.12

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2x^2+2}\sqrt{x^2+1}x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

Sympy [A] time = 2.82725, size = 8, normalized size = 1.

$$\frac{\sqrt{2}\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*atan(x)/2
```

Giac [C] time = 1.19743, size = 31, normalized size = 3.88

$$\frac{1}{4}i\sqrt{2}\log(ix-1)-\frac{1}{4}i\sqrt{2}\log(-ix-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*log(I*x - 1) - 1/4*I*sqrt(2)*log(-I*x - 1)
```

3.232 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.00898, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {418}

$$\frac{\sqrt{x^2+2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0182237, size = 19, normalized size = 0.4

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 1/2])/Sqrt[2]
```

Maple [C] time = 0.022, size = 15, normalized size = 0.3

$$-i\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] -I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{x^4+3x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.233 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0061151, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.0183213, size = 19, normalized size = 1.9

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [A] time = 0.022, size = 14, normalized size = 1.4

$$\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-x^2+2}}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-x^2 + 2)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

3.234 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-1\right)}{\sqrt{2}}$$

[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]

Rubi [A] time = 0.0047369, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {248, 221}

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{2-2x^4}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0118819, size = 10, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] time = 0.006, size = 10, normalized size = 1.

$$\frac{\text{EllipticF}\,(x,i)\,\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}{2\left(x^4-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)/(x^4 - 1), x)
```

Sympy [B] time = 7.75348, size = 76, normalized size = 7.6

$$\frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4, 0, 1/4, 1/2, 3/
4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi
**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.235 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0073399, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx=\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0060708, size = 20, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] time = 0.016, size = 19, normalized size = 1.

$$\frac{\sqrt{3}}{3}\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}{3\,x^4+x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)/(3*x^4 + x^2 - 2), x)
```

Sympy [A] time = 4.81374, size = 36, normalized size = 1.8

$$\begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.236 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rubi [A] time = 0.0066735, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {419}

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

Mathematica [A] time = 0.0062921, size = 16, normalized size = 1.

$$\frac{1}{2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Maple [A] time = 0.019, size = 15, normalized size = 0.9

$$\frac{\text{EllipticF}\left(x\sqrt{2},\frac{i}{2}\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}{2\left(2\,x^4+x^2-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)/(2*x^4 + x^2 - 1), x)
```

Sympy [A] time = 8.5156, size = 41, normalized size = 2.56

$$\frac{\sqrt{2}\left(\begin{cases}\frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for } x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sqrt(
2)/2) & (x < sqrt(2)/2)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.237 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rubi [A] time = 0.0091487, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}}$$

Mathematica [A] time = 0.0065552, size = 20, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Maple [A] time = 0.035, size = 19, normalized size = 1.

$$\frac{\sqrt{5}}{5}\text{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{i}{5}\sqrt{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}{5\,x^4+3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)/(5*x^4 + 3*x^2 - 2), x)
```

Sympy [A] time = 4.56159, size = 36, normalized size = 1.8

$$\begin{cases} \frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle| -\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/5, (x > -sqrt(10)/5
) & (x < sqrt(10)/5)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.238 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0144636, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx}{\sqrt{-1+x^2}} = \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0220841, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] time = 0.029, size = 37, normalized size = 1.2

$$-\frac{i}{5}\text{EllipticF}\left(\frac{i}{2}x\sqrt{10},\frac{i}{5}\sqrt{10}\right)\sqrt{5}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] -1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*I*10^(1/2))*(-x^2+1)^(1/2)*5^(1/2)/(x
^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x^2+2}\sqrt{x^2-1}}{5\,x^4-3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)/(5*x^4 - 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0146774, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx}{\sqrt{-1+x^2}} = \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0268143, size = 30, normalized size = 1.

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] time = 0.029, size = 34, normalized size = 1.1

$$-\frac{i}{2}\text{EllipticF}\left(ix\sqrt{2},\frac{i}{2}\sqrt{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x*2^(1/2),1/2*I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x^2+2}\sqrt{x^2-1}}{2\left(2\,x^4-x^2-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(4*x^2 + 2)*sqrt(x^2 - 1)/(2*x^4 - x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.240 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0132576, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx}{\sqrt{-1+x^2}} = \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0208477, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] time = 0.026, size = 37, normalized size = 1.2

$$-\frac{i}{3}\text{EllipticF}\left(\frac{i}{2}x\sqrt{6},\frac{i}{3}\sqrt{6}\right)\sqrt{3}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))*(-x^2+1)^(1/2)*3^(1/2)/(x^2
-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,x^2+2}\sqrt{x^2-1}}{3\,x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)/(3*x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.241 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)$$

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rubi [A] time = 0.0109653, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {253, 222}

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{-1+x^2}\sqrt{2+2x^2}}$$
$$= \frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)$$

Mathematica [C] time = 0.0095928, size = 46, normalized size = 1.84

$$\frac{x\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)}{\sqrt{x^2-1}\sqrt{2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[-1 + x^2]*Sqr
t[2 + 2*x^2])
```

Maple [C] time = 0.019, size = 30, normalized size = 1.2

$$-\frac{i}{2}\mathrm{EllipticF}\left(ix,i\right)\sqrt{2}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,I)*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^2+2}\sqrt{x^2-1}}{2\left(x^4-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2*x^2 + 2)*sqrt(x^2 - 1)/(x^4 - 1), x)
```

Sympy [C] time = 8.34769, size = 75, normalized size = 3.

$$\frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), exp_polar(2*I*pi)/x**4)/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4,
0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi*
*(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.242 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0131919, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0202462, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] time = 0.021, size = 34, normalized size = 1.1

$$-i\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},i\sqrt{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] -I*EllipticF(1/2*I*x*2^(1/2),I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2-1}}{x^4+x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 - 1)/(x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.243 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=12

$$-\text{EllipticF}\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)$$

```
[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]
```

Rubi [A] time = 0.0065863, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {420}

$$-F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = -F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Mathematica [B] time = 0.0221592, size = 47, normalized size = 3.92

$$\frac{\sqrt{1-x^2}\sqrt{1-\frac{x^2}{2}}\,\text{EllipticF}\left(\sin^{-1}(x),\frac{1}{2}\right)}{\sqrt{-x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[x], 1/2])/Sqrt[-2 + 3*x^2 -
 x^4]
```

Maple [A] time = 0.02, size = 28, normalized size = 2.3

$$\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},\sqrt{2}\right)\sqrt{-x^2+1}\,\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] EllipticF(1/2*x*2^(1/2),2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{x^2-1}\sqrt{-x^2+2}}{x^4-3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 - 1)*sqrt(-x^2 + 2)/(x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

3.244 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

```
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rubi [A] time = 0.0033172, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {23, 207}

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{-1+x^2}\int \frac{1}{-1+x^2}\,dx}{\sqrt{2-2x^2}}$$
$$= -\frac{\sqrt{-1+x^2}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Mathematica [A] time = 0.0105619, size = 40, normalized size = 1.38

$$\frac{\left(x^2-1\right)\left(\log(1-x)-\log(x+1)\right)}{2\sqrt{2}\sqrt{-\left(x^2-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-1 + x^2)*(Log[1 - x] - Log[1 + x]))/(2*Sqrt[2]*Sqrt[-(-1 + x^2)^2])
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$\frac{\sqrt{2}\mathrm{Artanh}\left(x\right)}{2}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)*arctanh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [A] time = 1.83012, size = 97, normalized size = 3.34

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{-2\,x^2+2}x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{1-x^2}\sqrt{x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(x**2 - 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.245 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0147927, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]

[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0232915, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{3}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/(Sqrt[3]*Sqrt[-1 + x^2]
)
```

Maple [A] time = 0.022, size = 29, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}{3\,x^4-5\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)/(3*x^4 - 5*x^2 + 2), x)
```

Sympy [C] time = 4.80676, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{\sqrt{3}iF\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(3)*I*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.246 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0150831, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}} = \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.029037, size = 36, normalized size = 1.2

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),\frac{1}{2}\right)}{2\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2]*x], 1/2])/(2*Sqrt[-1 + x^2])
```

Maple [A] time = 0.023, size = 27, normalized size = 0.9

$$\frac{\text{EllipticF}\left(x,\sqrt{2}\right)\sqrt{2}}{2}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}{2\left(2\,x^4-3\,x^2+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)/(2*x^4 - 3*x^2 + 1), x)
```

Sympy [A] time = 8.98279, size = 42, normalized size = 1.4

$$\frac{\sqrt{2}\left(\begin{cases}-\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\ x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqr
t(2)/2) & (x < sqrt(2)/2)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.247 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0141692, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}\,F\left(\sin^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}} = \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0241583, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], 2/5])/(Sqrt[5]*Sqrt[-1 + x^2]
)
```

Maple [A] time = 0.023, size = 29, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\text{EllipticF}\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}{5\,x^4-7\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)/(5*x^4 - 7*x^2 + 2), x)
```

Sympy [C] time = 4.58751, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{\sqrt{5}iF\left(\operatorname{asin}{\left(\frac{\sqrt{10}x}{2}\right)}\middle|\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(5)*I*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)
/5) & (x < sqrt(10)/5)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.248 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 5*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0104685, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{5x^2+2}\,F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```



```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 5*x^2)/(1 + x^2)])
```


Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{2+5x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0257513, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{x^2+1}\text{EllipticF}\left(i\sinh^{-1}(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 5/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] time = 0.026, size = 36, normalized size = 0.7

$$\frac{i}{5}\sqrt{5}\text{EllipticF}\left(\frac{i}{2}x\sqrt{10},\frac{\sqrt{10}}{5}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] 1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*10^(1/2))/(x^2+1)^(1/2)*5^(1/2)*(-x^2-
1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{5x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{5x^2+2}\sqrt{-x^2-1}}{5x^4+7x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)/(5*x^4 + 7*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(5*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2x^2+1}\,\text{EllipticF}\left(\tan^{-1}(x),-1\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(1
+ 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0101914, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{2x^2+1}\,F\left(\tan^{-1}(x)|-1\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(1
+ 2*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx=\frac{\sqrt{1+2x^2}\,F\left(\tan^{-1}(x)|-1\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0279036, size = 37, normalized size = 0.73

$$-\frac{i\sqrt{x^2+1}\text{EllipticF}\left(i\sinh^{-1}(x),2\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] time = 0.021, size = 33, normalized size = 0.7

$$\frac{i}{2}\text{EllipticF}\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x*2^(1/2),1/2*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4x^2+2}\sqrt{-x^2-1}}{2\left(2x^4+3x^2+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)/(2*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{3x^2+2}\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0094579, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 3*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0228308, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{x^2+1}\text{EllipticF}\left(i\sinh^{-1}(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 3/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] time = 0.024, size = 36, normalized size = 0.7

$$\frac{i}{3}\sqrt{3}\text{EllipticF}\left(\frac{i}{2}x\sqrt{6},\frac{\sqrt{6}}{3}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))/(x^2+1)^(1/2)*3^(1/2)*(-x^2-1)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}{3\,x^4+5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(3*x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.251 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}} dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rubi [A] time = 0.0035139, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {23, 203}

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2+2x^2}\int \frac{1}{2+2x^2}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}\tan^{-1}(x)}{\sqrt{2}\sqrt{-1-x^2}}$$

Mathematica [A] time = 0.0096165, size = 26, normalized size = 0.93

$$\frac{(x^2+1)\tan^{-1}(x)}{\sqrt{2}\sqrt{-(x^2+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

Maple [A] time = 0.007, size = 24, normalized size = 0.9

$$-\frac{\arctan(x)\sqrt{2}}{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] -1/2*(-x^2-1)^(1/2)*2^(1/2)/(x^2+1)^(1/2)*arctan(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [B] time = 1.84894, size = 258, normalized size = 9.21

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}x+\sqrt{2}\left(x^4-1\right)\right)}{x^4+2x^2+1}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}x-\sqrt{2}\left(x^4-1\right)\right)}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x + sqrt(2)*(x^4 - 1))/
(x^4 + 2*x^2 + 1)) - 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x
- sqrt(2)*(x^4 - 1))/(x^4 + 2*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2-1}\sqrt{x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.252 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(2 +
 x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0087737, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {418}

$$\frac{\sqrt{x^2+2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(2 +
 x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0351139, size = 53, normalized size = 1.08

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-\left(x^2+1\right)\left(x^2+2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x], 1/2])/(Sqrt[2]*Sq
rt[-((1 + x^2)*(2 + x^2))])
```

Maple [C] time = 0.019, size = 33, normalized size = 0.7

$$\frac{i}{2}\sqrt{2}\text{EllipticF}\left(ix,\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+2}\sqrt{-x^2-1}}{x^4+3x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 2)*sqrt(-x^2 - 1)/(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.253 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rubi [A] time = 0.0156464, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {421, 419}

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-1-x^2}}$$

Mathematica [C] time = 0.0248291, size = 39, normalized size = 1.26

$$-\frac{i\sqrt{x^2+1}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] time = 0.02, size = 34, normalized size = 1.1

$$\frac{i}{2}\text{EllipticF}\left(ix,\frac{i}{2}\sqrt{2}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*2^(1/2))/(x^2+1)^(1/2)*2^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+2}\sqrt{-x^2-1}}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(-x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.254 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-\frac{1}{x^4}}\,x^2\text{EllipticF}\left(\csc^{-1}(x),-1\right)}{\sqrt{2-2x^2}\sqrt{-x^2-1}}$$

```
[Out] -((Sqrt[1 - x^(-4)]*x^2*EllipticF[ArcCsc[x], -1])/(Sqrt[2 - 2*x^2]*Sqrt[-1
- x^2]))
```

Rubi [A] time = 0.0122466, antiderivative size = 65, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {253, 222}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{2\sqrt{-x^2-1}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
 1/2])/(2*Sqrt[-1 - x^2]*Sqrt[1 - x^2])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{2\sqrt{-1-x^2}\sqrt{1-x^2}}$$

Mathematica [C] time = 0.0132972, size = 48, normalized size = 1.14

$$\frac{x\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)}{\sqrt{2-2x^2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[2 - 2*x^2]*Sq
rt[-1 - x^2])
```

Maple [A] time = 0.009, size = 30, normalized size = 0.7

$$\frac{i}{2}\mathrm{EllipticF}\left(ix,i\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,I)*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}{2\left(x^4-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)/(x^4 - 1), x)
```

Sympy [A] time = 7.92956, size = 73, normalized size = 1.74

$$\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle| {\frac{1}{x^4}} \right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle| {\frac{e^{-2i\pi}}{x^4}} \right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4), (
0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*meijerg(((-1/4, 0, 1/4, 1/2, 3/4),
(1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3
/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.255 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Rubi [A] time = 0.0156102, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {421, 419}

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

Mathematica [A] time = 0.0238062, size = 40, normalized size = 1.

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Maple [A] time = 0.019, size = 34, normalized size = 0.9

$$\frac{i}{2}\text{EllipticF}\left(ix,\frac{i}{2}\sqrt{6}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*6^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}{3\,x^4+x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)/(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x**2)*sqrt(-x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.256 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}} dx$

Optimal. Leaf size=36

$$\frac{\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Rubi [A] time = 0.0152354, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {421, 419}

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}} = \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

Mathematica [A] time = 0.0298108, size = 36, normalized size = 1.

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Maple [A] time = 0.019, size = 34, normalized size = 0.9

$$\frac{i}{2}\text{EllipticF}\left(ix,i\sqrt{2}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}{2\left(2\,x^4+x^2-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)/(2*x^4 + x^2 - 1), x)
```

Sympy [A] time = 8.71455, size = 44, normalized size = 1.22

$$\frac{\sqrt{2}\left(\begin{cases}-\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\ x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sq
rt(2)/2) & (x < sqrt(2)/2)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.257 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Rubi [A] time = 0.0150192, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {421, 419}

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)\middle|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

Mathematica [A] time = 0.0265133, size = 40, normalized size = 1.

$$\frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Maple [A] time = 0.023, size = 34, normalized size = 0.9

$$\frac{i}{2}\text{EllipticF}\left(ix,\frac{i}{2}\sqrt{10}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*10^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}{5\,x^4+3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)/(5*x^4 + 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 5*x**2)*sqrt(-x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.258 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}} dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0534156, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\ &= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\\ &= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0563871, size = 87, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.01, size = 106, normalized size = 1.2

$$\frac{a}{-bdx^4-adx^2+bcx^2+ac}\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}\text{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(
d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{bx^2+a}\sqrt{-dx^2+c}}{dx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)/(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.259 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}} dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0524295, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\
&= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\\
&= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0483519, size = 90, normalized size = 1.

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.015, size = 171, normalized size = 1.9

$$\frac{1}{\left(bdx^4+adx^2-bcx^2-ac\right)d}\left(-a\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bc\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bc\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/
2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))*(-b*x^
2-a)^(1/2)*(-d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)/(b*d*x
^4+a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 - a}\sqrt{-dx^2 + c}}{dx^2 - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(-d*x^2 + c)/(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a - bx^2}}{\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.260 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}} \, dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.0493846, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\ &= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\ &= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0504629, size = 88, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] time = 0.012, size = 168, normalized size = 1.9

$$\frac{1}{\left(bdx^4+adx^2-bcx^2-ac\right)d}\sqrt{bx^2+a}\sqrt{dx^2-c}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\left(a\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bc\mathrm{EllipticF}\left(x\sqrt{\cdot\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*(a
*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2),
(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))/(b*d*x^4+
a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 - c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 - c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.261 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.0538916, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}$$

$$= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Mathematica [A] time = 0.0429131, size = 91, normalized size = 1.

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A] time = 0.01, size = 110, normalized size = 1.2

$$\frac{a}{bdx^4+adx^2-bcx^2-ac}\sqrt{-bx^2-a}\sqrt{dx^2-c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (-b*x^2-a)^(1/2)*(d*x^2-c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d
/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-bx^2 - a}}{\sqrt{dx^2 - c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a - bx^2}}{\sqrt{-c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{dx^2 - c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.262 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0521836, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}} \\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} \\
&= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0557897, size = 88, normalized size = 1.

$$
\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.012, size = 109, normalized size = 1.2

$$
\frac{a}{bdx^4-adx^2-bcx^2+ac}\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/
2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(
d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}{dx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)/(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.263 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.0528436, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\ &= \frac{\left(\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}\\ &= \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0513023, size = 89, normalized size = 1.

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.018, size = 165, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2-bcx^2+ac\right)d}\sqrt{bx^2-a}\sqrt{-dx^2+c}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\left(a\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*
(a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2),(
a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4-a*d*
x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 - a}}{\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{bx^2 - a}\sqrt{-dx^2 + c}}{dx^2 - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*x^2 - a)*sqrt(-d*x^2 + c)/(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a + bx^2}}{\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 - a}}{\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.264 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.054621, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\ &= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0474373, size = 89, normalized size = 1.

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] time = 0.012, size = 166, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2-bcx^2+ac\right)d}\left(-a\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bc\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\surd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),
(a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2+a)^(
1/2)*(d*x^2-c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)/(b*d*x^4-a*d
*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 - c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{-c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 - c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.265 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}} \, dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.0519834, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {427, 426, 424}

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\ &= \frac{\left(\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\ &= \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.0463408, size = 90, normalized size = 1.

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A] time = 0.009, size = 111, normalized size = 1.2

$$\frac{a}{-bdx^4+adx^2+bcx^2-ac}\sqrt{bx^2-a}\sqrt{dx^2-c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x^2-c)^(1/2
)*a*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/
a/d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.266 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0844847, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```



```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0485831, size = 86, normalized size = 0.44

$$
\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] time = 0., size = 158, normalized size = 0.8

$$\frac{1}{\left(bdx^4 + adx^2 + bcx^2 + ac\right)d}\sqrt{bx^2 + a}\sqrt{dx^2 + c}\sqrt{\frac{bx^2 + a}{a}}\sqrt{\frac{dx^2 + c}{c}}\left(a\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)d - bc\text{EllipticF}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.267 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} dx$

Optimal. Leaf size=203

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0886766, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```


```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx &= -\left(a\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx\right)-b\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}-c\int \frac{\sqrt{-a-bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.044747, size = 89, normalized size = 0.44

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.014, size = 104, normalized size = 0.5

$$\frac{a}{bdx^4+adx^2+bcx^2+ac}\sqrt{-bx^2-a}\sqrt{dx^2+c}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d
/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a - bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.268 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{\sqrt{c}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.0918711, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```


```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + c\int \frac{\sqrt{a+bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

Mathematica [A] time = 0.0506476, size = 89, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] time = 0.013, size = 108, normalized size = 0.5

$$\frac{a}{-bdx^4 - adx^2 - bcx^2 - ac}\sqrt{bx^2 + a}\sqrt{-dx^2 - c}\sqrt{\frac{dx^2 + c}{c}}\sqrt{\frac{bx^2 + a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-
d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-dx^2 - c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{bx^2 + a}\sqrt{-dx^2 - c}}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 - c)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-c - dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.269 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=212

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.103342, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx &= -\left(a\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\right)-b\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\\ &= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+c\int \frac{\sqrt{-a-bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx\\ &= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

Mathematica [A] time = 0.0437262, size = 92, normalized size = 0.43

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] time = 0.013, size = 165, normalized size = 0.8

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)d}\left(-a\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bc\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2-a
)^(1/2)*(-d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^2-a}\sqrt{-dx^2-c}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a - bx^2}}{\sqrt{-c - dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 - a}}{\sqrt{-dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.270 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2])
```

Rubi [A] time = 0.124948, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```


```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2])
```


Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx}{d} + \frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{d}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0498888, size = 89, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.012, size = 164, normalized size = 0.9

$$\frac{1}{\left(bdx^4-adx^2+bcx^2-ac\right)d}\left(-a\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bc\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bc\text{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2)
,(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))*(-b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(b*d*x^4-a
*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.271 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
- (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqr
t[c + d*x^2])
```

Rubi [A] time = 0.12297, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
- (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqr
t[c + d*x^2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx}{d} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{c+dx^2}}\,dx}{d}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

$$= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0462591, size = 90, normalized size = 0.47

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.014, size = 109, normalized size = 0.6

$$\frac{a}{-bdx^4+adx^2-bcx^2+ac}\sqrt{bx^2-a}\sqrt{dx^2+c}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x^2+c)^(1/
2)*a*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-b*
c/a/d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 - a}}{\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.272 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
+ (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt
[-c - d*x^2])
```

Rubi [A] time = 0.128482, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
+ (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt
[-c - d*x^2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx}{d} + \frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{-c-dx^2}}\,dx}{d}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-c-dx^2}}$$

$$= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

$$= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

Mathematica [A] time = 0.0539043, size = 92, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] time = 0.009, size = 111, normalized size = 0.6

$$\frac{a}{bdx^4-adx^2+bcx^2-ac}\sqrt{-bx^2+a}\sqrt{-dx^2-c}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2
)*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/
(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{-dx^2-c}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(-d*x^2 - c)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-dx^2 - c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.273 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c
])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]
*Sqrt[-c - d*x^2])
```

Rubi [A] time = 0.130931, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {423, 427, 426, 424, 421, 419}

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```


```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c
])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]
*Sqrt[-c - d*x^2])
```


Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule steps

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx}{d} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{-c-dx^2}}\,dx}{d}$$
$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-c-dx^2}}$$
$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$
$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\right.}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

Mathematica [A] time = 0.0447508, size = 93, normalized size = 0.47

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] time = 0.012, size = 167, normalized size = 0.8

$$\frac{1}{\left(bdx^4-adx^2+bcx^2-ac\right)d}\sqrt{bx^2-a}\sqrt{-dx^2-c}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(a\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bc\text{EllipticF}\left(x\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2-a)^(1/2)*(-d*x^2-c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(
a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),(
-a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))/(b*d*x^4-a*
d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{bx^2-a}\sqrt{-dx^2-c}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*x^2 - a)*sqrt(-d*x^2 - c)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c - d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 - a}}{\sqrt{-dx^2 - c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.274 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] time = 0.0478082, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [A] time = 0.0538489, size = 87, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.013, size = 106, normalized size = 1.2

$$\frac{c}{-bdx^4+adx^2-bcx^2+ac}\sqrt{dx^2+c}\sqrt{-bx^2+a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(
b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{dx^2+c}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)/(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.275 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] time = 0.0517018, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```


```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```


Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

Mathematica [A] time = 0.0479414, size = 90, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [B] time = 0.013, size = 171, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2+bcx^2-ac\right)b}\left(-ad\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-c\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+ad\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2)
,(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))*(-d*x^
2-c)^(1/2)*(-b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)/(b*d*x
^4-a*d*x^2+b*c*x^2-a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a}\sqrt{-dx^2 - c}}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(-d*x^2 - c)/(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c - dx^2}}{\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.276 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] time = 0.0467954, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\\ &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\end{aligned}$$

Mathematica [A] time = 0.0456159, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [B] time = 0.011, size = 168, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2+bcx^2-ac\right)b}\sqrt{dx^2+c}\sqrt{bx^2-a}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\left(ad\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+c\text{EllipticF}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(a
*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),(-
b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))/(b*d*x^4-
a*d*x^2+b*c*x^2-a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.277 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] time = 0.0500475, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}
\end{aligned}
$$

Mathematica [A] time = 0.0418449, size = 91, normalized size = 1.

$$
\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.01, size = 110, normalized size = 1.2

$$
\frac{c}{bdx^4-adx^2+bcx^2-ac}\sqrt{-dx^2-c}\sqrt{bx^2-a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.278 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rubi [A] time = 0.0517505, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\\ &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\end{aligned}$$

Mathematica [A] time = 0.0555552, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] time = 0.012, size = 164, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2-bcx^2+ac\right)b}\left(-ad\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+c\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+ad\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(b
*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2+c)^(
1/2)*(-b*x^2+a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)/(b*d*x^4-a*
d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)/(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}}{\sqrt{-bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.279 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rubi [A] time = 0.0506423, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\\ &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\end{aligned}$$

Mathematica [A] time = 0.0470068, size = 89, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.017, size = 110, normalized size = 1.2

$$\frac{c}{bdx^4-adx^2-bcx^2+ac}\sqrt{dx^2-c}\sqrt{-bx^2+a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a}\sqrt{dx^2 - c}}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 - c)/(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c + dx^2}}{\sqrt{a - bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.280 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rubi [A] time = 0.0507299, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}$$

$$= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

$$= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Mathematica [A] time = 0.0480825, size = 89, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.012, size = 110, normalized size = 1.2

$$\frac{c}{-bdx^4+adx^2+bcx^2-ac}\sqrt{-dx^2+c}\sqrt{bx^2-a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(
b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}}{\sqrt{bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.281 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rubi [A] time = 0.0519902, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {427, 426, 424}

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}
\end{aligned}$$

Mathematica [A] time = 0.0409493, size = 90, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] time = 0.012, size = 167, normalized size = 1.9

$$\frac{1}{\left(bdx^4-adx^2-bcx^2+ac\right)b}\sqrt{dx^2-c}\sqrt{bx^2-a}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}\left(ad\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-c\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(b*c
/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4-a*d*x
^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c + dx^2}}{\sqrt{-a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.282 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0881219, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```


```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx &= c\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}\\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0427399, size = 86, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.001, size = 101, normalized size = 0.5

$$\frac{c}{bdx^4 + adx^2 + bcx^2 + ac}\sqrt{bx^2 + a}\sqrt{dx^2 + c}\sqrt{\frac{bx^2 + a}{a}}\sqrt{\frac{dx^2 + c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.283 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
 d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.0955676, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {422, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```



```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
 d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx &= -\left(c\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx\right)-d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx\\
&= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(cd)\int \frac{\sqrt{a+bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx}{b}\\
&= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\end{aligned}$$

Mathematica [A] time = 0.0446347, size = 89, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.013, size = 161, normalized size = 0.8

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)b}\left(-ad\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+c\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+ad\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),
(b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2-c
)^(1/2)*(b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c - dx^2}}{\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.284 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}} \, dx$

Optimal. Leaf size=214

$$-\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[-a -
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[-a - b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0968212, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {422, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[-a -
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[-a - b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx &= c\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(cd)\int \frac{\sqrt{-a-bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{b} \\
&= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0503971, size = 89, normalized size = 0.42

$$
\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.013, size = 162, normalized size = 0.8

$$\frac{1}{\left(bdx^4+adx^2+bcx^2+ac\right)b}\sqrt{dx^2+c}\sqrt{-bx^2-a}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\left(ad\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-c\text{EllipticF}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(a
*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2),(b*
c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2-a}\sqrt{dx^2+c}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(d*x^2 + c)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a - bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{-bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.285 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=222

$$\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*Sqrt[-a - b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.10265, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```



```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*Sqrt[-a - b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx &= -\left(c\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\right)-d\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\\
&= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}+\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{(cd)\int \frac{\sqrt{-a-bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx}{b}\\
&= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}
\end{aligned}
$$

Mathematica [A] time = 0.0478383, size = 92, normalized size = 0.41

$$
\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.012, size = 111, normalized size = 0.5

$$\frac{c}{-bdx^4 - adx^2 - bcx^2 - ac}\sqrt{-dx^2 - c}\sqrt{-bx^2 - a}\sqrt{\frac{bx^2 + a}{a}}\sqrt{\frac{dx^2 + c}{c}}\text{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)*(-d*x^2-c)^(1/2)*(-b*x^2-a)^(
1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{-bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2 - a}\sqrt{-dx^2 - c}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c - dx^2}}{\sqrt{-a - bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{-bx^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.286 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sq
rt[c - d*x^2])
```

Rubi [A] time = 0.120646, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sq
rt[c - d*x^2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}}$$

$$= -\frac{\left(d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$= -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{b}{a}\right.}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] time = 0.0497271, size = 89, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.011, size = 164, normalized size = 0.9

$$\frac{1}{\left(bdx^4+adx^2-bcx^2-ac\right)b}\left(-ad\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-c\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+ad\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(
-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))*(-d*x^2+c
)^(1/2)*(b*x^2+a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(b*d*x^4+a
*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.287 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid -\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
- (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[-c + d*x^2])
```

Rubi [A] time = 0.121549, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid -\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid -\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
- (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[-c + d*x^2])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c+dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}}$$

$$= \frac{\left(d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

Mathematica [A] time = 0.0464153, size = 90, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.014, size = 109, normalized size = 0.6

$$\frac{c}{-bdx^4-adx^2+bcx^2+ac}\sqrt{dx^2-c}\sqrt{bx^2+a}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)*(d*x^2-c)^(1/2)*(b*x^2+a)^(1/
2)*c*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-a*
d/b/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.288 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
+ (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqr
t[c - d*x^2])
```

Rubi [A] time = 0.125359, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```


```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
+ (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqr
t[c - d*x^2])
```


Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}}$$

$$= \frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\right.}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] time = 0.0487828, size = 92, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.012, size = 111, normalized size = 0.6

$$\frac{c}{bdx^4+adx^2-bcx^2-ac}\sqrt{-dx^2+c}\sqrt{-bx^2-a}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/
(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2-a}\sqrt{-dx^2+c}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(-d*x^2 + c)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.289 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2
])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]
*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.129412, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {423, 427, 426, 424, 421, 419}

$$-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```


```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2
])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]
*Sqrt[-c + d*x^2])
```


Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c+dx^2}}\,dx}{b}$$

$$= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}}$$

$$= -\frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-a-bx^2}\sqrt{-c+dx^2}}$$

$$= -\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\right.}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{-c+dx^2}}$$

Mathematica [A] time = 0.0452211, size = 93, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.012, size = 167, normalized size = 0.8

$$\frac{1}{\left(bdx^4+adx^2-bcx^2-ac\right)b}\sqrt{dx^2-c}\sqrt{-bx^2-a}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}\left(ad\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+c\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(-b*x^2-a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(-b
*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))/(b*d*x^4+a*
d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^2-a}\sqrt{dx^2-c}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 - a)*sqrt(d*x^2 - c)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{-bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.290 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{bx^2+2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(
Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.0137813, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {418}

$$\frac{\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(
Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx=\frac{\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

Mathematica [A] time = 0.0119633, size = 37, normalized size = 0.47

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)}{\sqrt{3}\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[-b]*x)/Sqrt[2]], (2*d)/(3*b)]/(Sqrt[3]*Sqrt[-b])
```

Maple [A] time = 0.023, size = 38, normalized size = 0.5

$$\frac{\sqrt{2}}{2}\text{EllipticF}\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^
(1/2))/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}\sqrt{dx^2+3}}{bdx^4+(3\,b+2\,d)x^2+6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)/(b*d*x^4 + (3*b + 2*d)*x^2 + 6), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.291 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Rubi [A] time = 0.0200224, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {421, 419}

$$\frac{\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{c+dx^2}} = \frac{\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.0379465, size = 40, normalized size = 1.03

$$\frac{\sqrt{\frac{c+dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Maple [A] time = 0.022, size = 38, normalized size = 1.

$$\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(1/2*x,2*(-d/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^2+c}\sqrt{-x^2+4}}{dx^4+(c-4\,d)x^2-4\,c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^2 + c)*sqrt(-x^2 + 4)/(d*x^4 + (c - 4*d)*x^2 - 4*c), x)
```

Sympy [A] time = 2.21516, size = 20, normalized size = 0.51

$$\begin{cases} \frac{F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle| -\frac{4d}{c}\right)}{\sqrt{c}} & \text{for}\: x > -2 \wedge x < 2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((elliptic_f(asin(x/2), -4*d/c)/sqrt(c), (x > -2) & (x < 2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.292 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt
[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi [A] time = 0.0124724, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {418}

$$\frac{\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt
[(c + d*x^2)/(c*(4 + x^2))])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

Mathematica [C] time = 0.0346564, size = 47, normalized size = 0.77

$$-\frac{i\sqrt{\frac{c+dx^2}{c}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{2}\right),\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[(c + d*x^2)/c]*EllipticF[I*ArcSinh[x/2], (4*d)/c])/Sqrt[c + d*x^
2]
```

Maple [A] time = 0.017, size = 53, normalized size = 0.9

$$\frac{1}{2}\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\frac{1}{2}\sqrt{\frac{c}{d}}\right)\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-d/c)^(1/2),1/2*(c/d)^
(1/2))/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}\sqrt{x^2+4}}{dx^4+(c+4\,d)x^2+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*sqrt(x^2 + 4)/(d*x^4 + (c + 4*d)*x^2 + 4*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c+dx^2}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.293 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=6

$$-\text{EllipticF}\left(\cos^{-1}(x),2\right)$$

[Out] -EllipticF[ArcCos[x], 2]

Rubi [A] time = 0.0060251, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {420}

$$-F\left(\cos^{-1}(x)\middle|2\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] -EllipticF[ArcCos[x], 2]

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -F\left(\cos^{-1}(x)\middle|2\right)$$

Mathematica [B] time = 0.0245561, size = 27, normalized size = 4.5

$$\frac{\sqrt{1-2x^2}\,\text{EllipticF}\left(\sin^{-1}(x),2\right)}{\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticF[ArcSin[x], 2])/Sqrt[-1 + 2*x^2]
```

Maple [A] time = 0.016, size = 25, normalized size = 4.2

$$\mathrm{EllipticF}\left(x,\sqrt{2}\right)\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}{2\,x^4-3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)/(2*x^4 - 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x+1\right)}\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.294 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\text{EllipticF}\left(\sin^{-1}(cx),-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rubi [A] time = 0.02485, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
```

```
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + \frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}\end{aligned}$$

Mathematica [A] time = 0.0089855, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] time = 0.015, size = 28, normalized size = 1.2

$$\frac{(2\,\mathrm{EllipticF}\,(x\mathrm{csgn}\,(c)\,c,i) - \mathrm{EllipticE}\,(x\mathrm{csgn}\,(c)\,c,i))\,\mathrm{csgn}\,(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.295 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rubi [A] time = 0.0752468, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx &= 2\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx \\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{\left(3+dx^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0089538, size = 37, normalized size = 0.2

$$
\frac{\sqrt{2}E\left(\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]
```

Maple [A] time = 0., size = 37, normalized size = 0.2

$$\sqrt{2}\,\mathrm{EllipticE}\left(\frac{x\sqrt{3}}{3}\sqrt{-d}, \frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/d*b)^(1/2))*2^(1/
2)/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.296 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rubi [A] time = 0.0065772, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {425}

$$-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rule 425

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> -Simp
[(Sqrt[a - (b*c)/d]*EllipticE[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)])/
(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c
, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0259826, size = 35, normalized size = 1.84

$$\frac{\sqrt{3x^2-1}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3-9x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (Sqrt[-1 + 3*x^2]*EllipticE[ArcSin[Sqrt[3/2]*x], 2])/Sqrt[3 - 9*x^2]
```

Maple [A] time = 0.014, size = 37, normalized size = 2.

$$-\frac{\sqrt{3}}{3}\text{EllipticE}\left(\frac{x\sqrt{2}\sqrt{3}}{2},\sqrt{2}\right)\sqrt{-3\,x^2+1}\frac{1}{\sqrt{3\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*EllipticE(1/2*x*2^(1/2)*3^(1/2),2^(1/2))*(-3*x^2+1)^(1/2)*3^(1/2)/(3*x
^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)/(3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

3.297 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Rubi [A] time = 0.132217, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {424}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Mathematica [A] time = 0.107819, size = 95, normalized size = 1.

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\,\frac{1}{\sqrt{1-2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

```
[Out] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(bx^2+\sqrt{b^2-4\,ac}x^2-2\,a\right)\sqrt{\frac{bx^2+\sqrt{b^2-4\,ac}x^2+2\,a}{a}}\sqrt{-\frac{bx^2-\sqrt{b^2-4\,ac}x^2-2\,a}{a}}}{4\left(cx^4-bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/4*(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)*sqrt((b*x^2 + sqrt(b^2
- 4*a*c)*x^2 + 2*a)/a)*sqrt(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)/(c*x^
4 - b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.298 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[c])
```

Rubi [A] time = 0.101995, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {424}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[c])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Mathematica [A] time = 0.0871633, size = 95, normalized size = 1.01

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/
(Sqrt[2]*Sqrt[c])
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int \sqrt{1-2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1-2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

```
[Out] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqr
t(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(bx^2+\sqrt{b^2-4\,ac}x^2-2\,a\right)\sqrt{-\frac{bx^2+\sqrt{b^2-4\,ac}x^2-2\,a}{a}}\sqrt{-\frac{bx^2-\sqrt{b^2-4\,ac}x^2-2\,a}{a}}}{4\left(cx^4-bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-1/4*(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)*sqrt(-(b*x^2 + sqrt(b^2
 - 4*a*c)*x^2 - 2*a)/a)*sqrt(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)/(c*x
^4 - b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.299 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=478

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2}{b-\sqrt{}}}}{\sqrt{2}\sqrt{c}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*
Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.36674, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {422, 418, 492, 411}

$$\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+}}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx}{b-\sqrt{b^2}}}{\frac{2cx}{\sqrt{b^2-4}}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*
Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx = \frac{(2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx}{b-\sqrt{b^2-4ac}} + \int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx$$

$$= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \int \frac{}{(1}$$

$$= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.116894, size = 102, normalized size = 0.21

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
- Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(S
qrt[2]*Sqrt[c])
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

```
[Out] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(
b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+\sqrt{b^2-4\,ac}x^2+2\,a\right)\sqrt{\frac{bx^2+\sqrt{b^2-4\,ac}x^2+2\,a}{a}}\sqrt{\frac{bx^2-\sqrt{b^2-4\,ac}x^2+2\,a}{a}}}{4\left(cx^4+bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)*sqrt((b*x^2 + sqrt(b^2 -
 4*a*c)*x^2 + 2*a)/a)*sqrt((b*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)/(c*x^4
+ b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.300 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt{2}b\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))])/(Sq
rt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) + (Sqrt[2]*b*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/
(b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.207668, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {423, 424, 419}

$$\frac{\sqrt{2}bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```


```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))])/(Sq
rt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) + (Sqrt[2]*b*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/
(b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```


Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
```

```
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{(2b)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}}$$

$$= -\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.0897935, size = 102, normalized size = 0.47

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
- Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[2]*Sqrt[c])
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \sqrt{1-2\,{\frac{c{x}^{2}}{b-\sqrt{-4\,ac+{b}^{2}}}}}{\frac{1}{\sqrt{1+2\,{\frac{c{x}^{2}}{b+\sqrt{-4\,ac+{b}^{2}}}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

```
[Out] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{2\,c x^{2}}{b-\sqrt{b^{2}-4\,a c}}+1}}{\sqrt{\frac{2\,c x^{2}}{b+\sqrt{b^{2}-4\,a c}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+\sqrt{b^2-4\,ac}x^2+2\,a\right)\sqrt{-\frac{bx^2+\sqrt{b^2-4\,ac}x^2-2\,a}{a}}\sqrt{\frac{bx^2-\sqrt{b^2-4\,ac}x^2+2\,a}{a}}}{4\left(cx^4+bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)*sqrt(-(b*x^2 + sqrt(b^2
- 4*a*c)*x^2 - 2*a)/a)*sqrt((b*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)/(c*x^4
 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.301 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2^{-m-2}\sqrt{x^2}\left(2-4x^2\right)^{m+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\left(1-2x^2\right)^2\right)}{(m+1)x}$$

```
[Out] -((2^(-2 - m)*Sqrt[x^2]*(2 - 4*x^2)^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (1 - 2*x^2)^2])/((1 + m)*x))
```

Rubi [C] time = 0.010505, antiderivative size = 23, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$xF_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] x*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(1-2x^2\right)^m}{\sqrt{1-x^2}}\,dx = xF_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)$$

Mathematica [C] time = 0.123661, size = 122, normalized size = 1.97

$$\frac{3x\left(1-2x^2\right)^m F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)}{\sqrt{1-x^2}\left(x^2\left(F_1\left(\frac{3}{2};-m,\frac{3}{2};\frac{5}{2};2x^2,x^2\right)-4mF_1\left(\frac{3}{2};1-m,\frac{1}{2};\frac{5}{2};2x^2,x^2\right)\right)+3F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] (3*x*(1 - 2*x^2)^m*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2])/(Sqrt[1 - x^2]*
(3*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2] + x^2*(-4*m*AppellF1[3/2, 1 - m,
 1/2, 5/2, 2*x^2, x^2] + AppellF1[3/2, -m, 3/2, 5/2, 2*x^2, x^2])))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \left(-2x^2+1\right)^m \frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

```
[Out] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-2x^2+1\right)^m}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\left(-2\,x^2+1\right)^m}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*(-2*x^2 + 1)^m/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(1-2x^2\right)^m}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((1 - 2*x**2)**m/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-2\,x^2+1\right)^m}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

3.302 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}(x),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*Sqrt[3]]*S
qrt[-1 + x^2])
```

Rubi [A] time = 0.0407638, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {421, 419}

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*Sqrt[3]]*S
qrt[-1 + x^2])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.0817452, size = 48, normalized size = 1.04

$$\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}(x),\frac{1}{4\sqrt{3}-7}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], (-7 + 4*Sqrt[3])^(-1)])/(Sqrt[7 - 4*Sqr
t[3]]*Sqrt[-1 + x^2])
```

Maple [B] time = 0.106, size = 117, normalized size = 2.5

$$\frac{-i\left(-2+\sqrt{3}\right)}{\left(4\sqrt{3}-7\right)\left(-x^4+4\sqrt{3}x^2-6x^2-4\sqrt{3}+7\right)}\text{EllipticF}\left(\frac{ix}{-2+\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{-\left(-x^2+4\sqrt{3}-7\right)\left(7-4\sqrt{3}\right)}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x)
```

```
[Out] -I*EllipticF(I*x/(-2+3^(1/2)),2*I-I*3^(1/2))*(-x^2+1)^(1/2)*(-(-x^2+4*3^(1/
2)-7)*(7-4*3^(1/2)))^(1/2)/(4*3^(1/2)-7)*(-2+3^(1/2))*(x^2-1)^(1/2)*(7+x^2-
4*3^(1/2))^(1/2)/(-x^4+4*3^(1/2)*x^2-6*x^2-4*3^(1/2)+7)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - 4\sqrt{3} + 7}\sqrt{x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^2 + 4\sqrt{3} + 7\right)\sqrt{x^2 - 4\sqrt{3} + 7}\sqrt{x^2 - 1}}{x^6 + 13\,x^4 - 13\,x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 4*sqrt(3) + 7)*sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)/(x^6
 + 13*x^4 - 13*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{x^2 - 4\sqrt{3} + 7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 - 4*sqrt(3) + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - 4\sqrt{3} + 7}\sqrt{x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

3.303 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

```
[Out] -(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3]
)/2])/6
```

Rubi [A] time = 0.0616624, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {420}

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x]
```

```
[Out] -(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3]
)/2])/6
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx=-\frac{1}{6}\sqrt{3+\sqrt{3}}F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Mathematica [A] time = 0.147288, size = 81, normalized size = 1.72

$$\frac{\sqrt{-2\sqrt{3}x^2+3\sqrt{3}-3}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1+\sqrt{3}}x}{\sqrt[4]{3}}\right),2-\sqrt{3}\right)}{3^{3/4}\sqrt{4\sqrt{3}x^2-6\sqrt{3}+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^
2]),x]
```

```
[Out] (Sqrt[-3 + 3*Sqrt[3] - 2*Sqrt[3]*x^2]*EllipticF[ArcSin[(Sqrt[1 + Sqrt[3]]*x
)/3^(1/4)], 2 - Sqrt[3]])/(3^(3/4)*Sqrt[6 - 6*Sqrt[3] + 4*Sqrt[3]*x^2])
```

Maple [B] time = 0.181, size = 207, normalized size = 4.4

$$\frac{\sqrt{2}\left(-3+\sqrt{3}\right)}{18\left(\sqrt{3}-1\right)^{2}\left(2x^{4}\sqrt{3}-2x^{4}-6\sqrt{3}x^{2}+6x^{2}+3\sqrt{3}-3\right)\sqrt{\left(2\sqrt{3}-3\right)\left(\sqrt{3}-1\right)}}\sqrt{\sqrt{3}x^{2}-3x^{2}+3}\sqrt{3-3\sqrt{3}+2\sqrt{3}x^{2}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/18*(3^(1/2)*x^2-3*x^2+3)^(1/2)*(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2)*2^(1/2)/
(3^(1/2)-1)^2*(-(4*3^(1/2)*x^2-6*x^2-3*3^(1/2)+3)*(3^(1/2)-1))^(1/2)*(-(3-3
*3^(1/2)+2*3^(1/2)*x^2)*(3^(1/2)-1))^(1/2)*EllipticF(1/3*x*2^(1/2)*3^(1/2)/
(3^(1/2)-1)*((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2),1/(3^(1/2)-1)*((3^(1/2)-1)*(1
+3^(1/2)))^(1/2))*(-3+3^(1/2))/(2*x^4*3^(1/2)-2*x^4-6*3^(1/2)*x^2+6*x^2+3*3
^(1/2)-3)/((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{\sqrt{3}x^2 - 3\,x^2 + 3}\sqrt{\sqrt{3}\left(2\,x^2 - 3\right) + 3}\left(\sqrt{3} + 1\right)}{6\left(2\,x^4 - 6\,x^2 + 3\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-1/6*sqrt(sqrt(3)*x^2 - 3*x^2 + 3)*sqrt(sqrt(3)*(2*x^2 - 3) + 3)*(
sqrt(3) + 1)/(2*x^4 - 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + \sqrt{3}x^2 + 3}\sqrt{2\sqrt{3}x^2 - 3\sqrt{3} + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*3**(1/2)*x**2)**(
1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*x**2 + sqrt(3)*x**2 + 3)*sqrt(2*sqrt(3)*x**2 - 3*sqrt(3
) + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\sqrt{3} - 3\right) + 3}\sqrt{2\,\sqrt{3}x^2 - 3\,\sqrt{3} + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

3.304 $\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi [A] time = 0.0191751, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```


```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```


Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+3x^2}\left(4+3x^2\right)}\,dx = -\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.106849, size = 135, normalized size = 1.05

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{\sqrt[4]{3x^2+2}\left(3x^2+4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)-4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2, (-3*x^2)/4])/((2 + 3*x^2)^(1/4
)*(4 + 3*x^2)*(-4*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2, (-3*x^2)/4] + x^2*
(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/2, (-3*x^2)/4] + AppellF1[3/2, 5/4,
1, 5/2, (-3*x^2)/2, (-3*x^2)/4])))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{3x^2+4}\frac{1}{\sqrt[4]{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x)
```

```
[Out] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2+4\right)\left(3x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

Fricas [B] time = 27.7936, size = 1520, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3
 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 + 4
)*sqrt(3*x^2 + 2) + 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2
) + 72*(3*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) + 48*sqrt(2
)*(3*x^2 + 2)^(3/4))*sqrt((3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(
1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) - 1/72*18^
(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x^3 - 54*x^4
 + 24*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 + 4)*sqrt(3*
x^2 + 2) - 72*x^2 - (18^(3/4)*sqrt(2)*(3*x^3 - 4*x)*sqrt(3*x^2 + 2) - 72*(3
*x^2 + 2)^(1/4)*x^2 + 6*18^(1/4)*sqrt(2)*(3*x^3 + 4*x) - 48*sqrt(2)*(3*x^2
+ 2)^(3/4))*sqrt((3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x +
4*sqrt(3*x^2 + 2))/(3*x^2 + 4)))/(9*x^4 - 24*x^2 - 16)) + 1/288*18^(3/4)*sq
rt(2)*log(36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sq
rt(3*x^2 + 2))/(3*x^2 + 4)) - 1/288*18^(3/4)*sqrt(2)*log(36*(3*sqrt(2)*x^2
- 2*18^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 4*sqrt(3*x^2 + 2))/(3*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{3x^2+2}\left(3x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
```

```
[Out] Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

3.305 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi [A] time = 0.0158787, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```


```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```


Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.120949, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}\,(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x^2)^(1/4)*
(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*A
ppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2
, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [B] time = 28.2156, size = 1553, normalized size = 12.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2} - 4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")

[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.306 $\int \frac{1}{\sqrt[4]{2+bx^2}(4+bx^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0228716, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1
/4))]/(2*2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+bx^2}\left(4+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.130916, size = 144, normalized size = 1.12

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{\sqrt[4]{bx^2+2}\left(bx^2+4\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)-12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2)/2, -(b*x^2)/4])/((2 + b*x^2)^(1/
4)*(4 + b*x^2)*(-12*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2)/2, -(b*x^2)/4] + b*
x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -(b*x^2)/2, -(b*x^2)/4] + AppellF1[3/2, 5
/4, 1, 5/2, -(b*x^2)/2, -(b*x^2)/4])))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+4}\frac{1}{\sqrt[4]{bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

```
[Out] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+4\right)\left(bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Fricas [B] time = 72.5485, size = 2201, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(b*x^
2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^
2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x + 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 + 4*b)*s
qrt(b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(b*x^2 + 2)^(1/4)*b*x^2 + 2*sq
rt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2
+ 2)^(3/4)*b*sqrt(b^(-2)) + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^
2 + 2)*(b^(-2))^(1/4))*sqrt((2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b
^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b*x^2 + 2))/(b*x^2
 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) - 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*
arctan((2*sqrt(2)*(1/2)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 - b^
2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*
x^2 - 4*sqrt(1/2)*(b^2*x^2 + 4*b)*sqrt(b*x^2 + 2)*sqrt(b^(-2)) + 2*sqrt(1/2
)*(4*(b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 + 4*b^2*x)*(b
^(-2))^(3/4) + 16*sqrt(1/2)*(b*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2
)^(1/4)*(b^2*x^3 - 4*b*x)*sqrt(b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*
(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4)))/(b^2*x^4 - 8*b*x^2 - 16)) + 1/1
6*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(b*x^2
+ 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(b
*x^2 + 2))/(b*x^2 + 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*
(2*sqrt(2)*(1/2)^(3/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1/2)*b
^2*sqrt(b^(-2))*x^2 - 2*sqrt(b*x^2 + 2))/(b*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx^2+2}\left(bx^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

```
[Out] Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+4\right)\left(bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

3.307 $\int \frac{1}{\sqrt[4]{2-bx^2}\left(4-bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0204322, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\,\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\,\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.131649, size = 145, normalized size = 1.17

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{\sqrt[4]{2-bx^2}\,(bx^2-4)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)+12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2, (b*x^2)/4])/((2 - b*x^2)^(1/4)
*(-4 + b*x^2)*(12*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2, (b*x^2)/4] + b*x^2*
(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/2, (b*x^2)/4] + AppellF1[3/2, 5/4, 1,
 5/2, (b*x^2)/2, (b*x^2)/4])))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2+4}\frac{1}{\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

```
[Out] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Fricas [B] time = 73.0235, size = 2229, normalized size = 17.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x
^2 + 2)^(1/4)*b^2*(b^(-2))^(1/4)*x^3 + b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*
x^2 + 2)^(3/4)*b^2*(b^(-2))^(3/4)*x - 4*b*x^2 + 4*sqrt(1/2)*(b^2*x^2 - 4*b)
*sqrt(-b*x^2 + 2)*sqrt(b^(-2)) - 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 +
2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b
*x^2 + 2)^(3/4)*b*sqrt(b^(-2)) - sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt
(-b*x^2 + 2)*(b^(-2))^(1/4))*sqrt(-(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4
)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 + 2*sqrt(-b*x^2 + 2
))/(b*x^2 - 4)))/(b^2*x^4 + 8*b*x^2 - 16)) + 1/4*sqrt(2)*(1/2)^(1/4)*(b^(-2
))^(1/4)*arctan(-(2*sqrt(2)*(1/2)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(1/
4)*x^3 - b^2*x^4 + 8*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(3/4)*b^2*(b^(-2))^(3
/4)*x + 4*b*x^2 - 4*sqrt(1/2)*(b^2*x^2 - 4*b)*sqrt(-b*x^2 + 2)*sqrt(b^(-2))
 + 2*sqrt(1/2)*(4*(-b*x^2 + 2)^(1/4)*b*x^2 - 2*sqrt(2)*(1/2)^(3/4)*(b^3*x^3
 - 4*b^2*x)*(b^(-2))^(3/4) + 16*sqrt(1/2)*(-b*x^2 + 2)^(3/4)*b*sqrt(b^(-2))
 + sqrt(2)*(1/2)^(1/4)*(b^2*x^3 + 4*b*x)*sqrt(-b*x^2 + 2)*(b^(-2))^(1/4))*s
qrt((2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x - sqrt(1
/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)))/(b^2*x^4 + 8*b
*x^2 - 16)) + 1/16*sqrt(2)*(1/2)^(1/4)*(b^(-2))^(1/4)*log(-1/2*(2*sqrt(2)*(
1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(3/4)*x + sqrt(1/2)*b^2*sqrt(b^(
-2))*x^2 + 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4)) - 1/16*sqrt(2)*(1/2)^(1/4)*(b^(
-2))^(1/4)*log(1/2*(2*sqrt(2)*(1/2)^(3/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-2))^(
3/4)*x - sqrt(1/2)*b^2*sqrt(b^(-2))*x^2 - 2*sqrt(-b*x^2 + 2))/(b*x^2 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2+2}-4\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

3.308 $\int \frac{1}{\sqrt[4]{a+3x^2}(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(2*Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rubi [A] time = 0.0198623, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(2*Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Mathematica [C] time = 0.140052, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/((a + 3*x^2
)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)
/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + A
ppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2+2\,a}\frac{1}{\sqrt[4]{3\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

```
[Out] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Fricas [B] time = 58.6065, size = 826, normalized size = 6.88

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2+a}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{-a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3
)^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(-a*sqrt(-1/a^
3)) - (1/36)^(1/4)*(3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) - 1/4*(1/36)^(1/4
)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4)
+ (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) +
(3*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2
*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (3*x^2 + a)^(3/4))/(3*x
^2 + 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+3x^2}\left(2a+3x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

3.309 $\int \frac{1}{\sqrt[4]{a-3x^2}\left(2a-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rubi [A] time = 0.0171325, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-3x^2}\left(2a-3x^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Mathematica [C] time = 0.153428, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{\sqrt[4]{a-3x^2}\left(3x^2-2a\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((a - 3*x^2)^
(1/4)*(-2*a + 3*x^2)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*
a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/a, (3*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2+2\,a}\frac{1}{\sqrt[4]{-3\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

```
[Out] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3\,x^2-2\,a\right)\left(-3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

Fricas [B] time = 62.8448, size = 836, normalized size = 6.97

$$\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{-3x^2+a}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{a\sqrt{-\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\right)}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] (1/36)^(1/4)*(-1/a^3)^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(-1/a^3
)^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 + a)*a*(-1/a^3)^(1/4))*sqrt(a*sqrt(-1/a^
3)) - (1/36)^(1/4)*(-3*x^2 + a)^(1/4)*a*(-1/a^3)^(1/4))/x) + 1/4*(1/36)^(1/
4)*(-1/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/
4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4
) - (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) - 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*lo
g((18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (-3*x^2 + a)^(1/
4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4
))/(3*x^2 - 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2-2\,a\right)\left(-3\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

3.310 $\int \frac{1}{\sqrt[4]{a+bx^2}(2a+bx^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4
))]/(2*a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0214657, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {397}

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4
))]/(2*a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.148803, size = 165, normalized size = 1.38

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{a+bx^2}\,(2a+bx^2)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(2*a)])/((a + b*x^
2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*x^
2)/(2*a)] - b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a
)] + AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+2\,a}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\left(2a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+2\,a\right)\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

3.311 $\int \frac{1}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.020752, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.153459, size = 162, normalized size = 1.31

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{\sqrt[4]{a-bx^2}\left(2a-bx^2\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((a - b*x^2)^(
1/4)*(2*a - b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)
] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/a, (b*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2+2a}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2-2a\right)\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2-2\,a\right)\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

3.312 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0088104, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.136884, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(-1 + 3*x^
2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[
3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*
x^2)/2])))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [B] time = 30.8715, size = 281, normalized size = 4.61

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-}{9x^4-12x^2+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.313 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0098623, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.124174, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)}{\sqrt[4]{-3x^2-1}\,(3x^2+2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)\right)-2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^2)/2])/((-1 - 3*x^2)^(1/4)*(2
 + 3*x^2)*(-2*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^2)/2] + x^2*(2*Appel
lF1[3/2, 1/4, 2, 5/2, -3*x^2, (-3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, -3*x
^2, (-3*x^2)/2])))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{-3x^2-2}\frac{1}{\sqrt[4]{-3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3x^2+2)(-3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Fricas [C] time = 25.7431, size = 683, normalized size = 11.2

$$-\frac{1}{24}\sqrt{6}\log\left(\frac{\sqrt{6}\sqrt{-3x^2-1}x-\sqrt{6}x+2\left(-3x^2-1\right)^{\frac{3}{4}}-2\left(-3x^2-1\right)^{\frac{1}{4}}}{3\left(3x^2+2\right)}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{\sqrt{6}\sqrt{-3x^2-1}x-\sqrt{6}x-2\left(-3x\right.}{3\left(3x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*log(1/3*(sqrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x + 2*(-3*x^2 -
 1)^(3/4) - 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + 1/24*sqrt(6)*log(-1/3*(sqr
t(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x - 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)
^(1/4))/(3*x^2 + 2)) + 1/24*I*sqrt(6)*log(1/3*(I*sqrt(6)*sqrt(-3*x^2 - 1)*x
 + I*sqrt(6)*x + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
- 1/24*I*sqrt(6)*log(1/3*(-I*sqrt(6)*sqrt(-3*x^2 - 1)*x - I*sqrt(6)*x + 2*(
-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3x^2+2)(-3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

3.314 $\int \frac{1}{\left(-2+bx^2\right)\sqrt[4]{-1+bx^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rubi [A] time = 0.013133, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2+bx^2\right)\sqrt[4]{-1+bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [C] time = 0.157584, size = 132, normalized size = 1.71

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)}{\left(bx^2-2\right)\sqrt[4]{bx^2-1}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2])/((-2 + b*x^2)*(-1 + b*x^
2)^(1/4)*(6*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2] + b*x^2*(2*AppellF
1[3/2, 1/4, 2, 5/2, b*x^2, (b*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, b*x^2, (
b*x^2)/2])))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2-2}\frac{1}{\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-1\right)^{\frac{1}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Fricas [B] time = 77.9659, size = 701, normalized size = 9.1

$$\left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}(bx^2-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{bx^2-1}bx^2+4bx^2-4\sqrt{2}(bx^2-1)^{\frac{3}{4}}\sqrt{b}x-4}{b^2x^4-4bx^2+4}\right)}{8b}, \frac{2\sqrt{2}\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sqr
t(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*sq
rt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)/
(b^2*x^4 - 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x^2
- 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 + 2*sqrt(2)*(b*
x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(
2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.315 $\int \frac{1}{\left(-2-bx^2\right)\sqrt[4]{-1-bx^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rubi [A] time = 0.0137537, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b]) - Arc
Tanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2-bx^2\right)\sqrt[4]{-1-bx^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [C] time = 0.142623, size = 137, normalized size = 1.73

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)}{\sqrt[4]{-bx^2-1}\left(bx^2+2\right)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -(b*x^2)/2])/((-1 - b*x^2)^(1/4)*
(2 + b*x^2)*(-6*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -(b*x^2)/2] + b*x^2*(2
*AppellF1[3/2, 1/4, 2, 5/2, -(b*x^2), -(b*x^2)/2] + AppellF1[3/2, 5/4, 1, 5
/2, -(b*x^2), -(b*x^2)/2])))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-2}\frac{1}{\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Fricas [B] time = 85.1679, size = 675, normalized size = 8.54

$$\left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4bx^2-2\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3+2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4bx^2+4}\right)}{8b}, \frac{2\sqrt{2}\sqrt{-b}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sq
rt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(2
)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^4
 + 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1
/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1)*b
*x^2 - 4*b*x^2 + 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3/4)
*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

3.316 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rubi [A] time = 0.0178728, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a+3x^2\right)\sqrt[4]{-a+3x^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [C] time = 0.160353, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{\left(3x^2-2a\right)\sqrt[4]{3x^2-a}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((-2*a + 3*x^2
)*(-a + 3*x^2)^(1/4)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*
a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/a, (3*x^2)/(2*a)] + AppellF
1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2*a)])))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2-2\,a}\frac{1}{\sqrt[4]{3\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-a\right)^{\frac{1}{4}}\left(3\,x^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Fricas [B] time = 64.0755, size = 826, normalized size = 9.72

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\frac{1}{a^3}^{\frac{3}{4}}+\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{3x^2-a}\,a\frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{a\sqrt{\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) + (1/36)^(1/4)*sqrt(3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(a*sqrt(a^(-3
))) - (1/36)^(1/4)*(3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) - 1/4*(1/36)^(1/4
)*(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x
+ (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x +
(3*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log(-(1
8*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2
*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x - (3*x^2 - a)^(3/4))/(3*x
^2 - 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2a+3x^2\right)\sqrt[4]{-a+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^2-a\right)^{\frac{1}{4}}\left(3\,x^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

3.317 $\int \frac{1}{\left(-2a-3x^2\right)\sqrt[4]{-a-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rubi [A] time = 0.0139391, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a-3x^2\right)\sqrt[4]{-a-3x^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [C] time = 0.146947, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{-a-3x^2}\left(2a+3x^2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/((-a - 3*x^2
)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)
/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)] + A
ppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{-3x^2-2a}\frac{1}{\sqrt[4]{-3x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(3x^2+2a\right)\left(-3x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

Fricas [B] time = 54.896, size = 838, normalized size = 9.86

$$-\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{2\left(\sqrt{\frac{1}{2}}\left(6\left(\frac{1}{36}\right)^{\frac{3}{4}}a^3\frac{1}{a^3}^{\frac{3}{4}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\sqrt{-3x^2-a}a\frac{1}{a^3}^{\frac{1}{4}}\right)\sqrt{-a\sqrt{\frac{1}{a^3}}}-\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-3x^2-a\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}\right)}{x}\right)+\frac{1}{4}\left(\frac{1}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/36)^(1/4)*(a^(-3))^(1/4)*arctan(2*(sqrt(1/2)*(6*(1/36)^(3/4)*a^3*(a^(-3
))^(3/4) - (1/36)^(1/4)*sqrt(-3*x^2 - a)*a*(a^(-3))^(1/4))*sqrt(-a*sqrt(a^(
-3))) - (1/36)^(1/4)*(-3*x^2 - a)^(1/4)*a*(a^(-3))^(1/4))/x) + 1/4*(1/36)^(
1/4)*(a^(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/
4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4
)*x - (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) - 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*
log((18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (-3*x^2 - a)^(
1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3
/4))/(3*x^2 + 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(3\,x^2+2\,a\right)\left(-3\,x^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

3.318 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0193968, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2a+bx^2\right)\sqrt[4]{-a+bx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.156617, size = 163, normalized size = 1.61

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{\left(2a-bx^2\right)\sqrt[4]{bx^2-a}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((2*a - b*x^2
)*(-a + b*x^2)^(1/4)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*
a)] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/a, (b*x^2)/(2*a)] + Appel
lF1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2*a)])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2-2\,a}\frac{1}{\sqrt[4]{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^{\frac{1}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-2a + bx^2\right)\sqrt[4]{-a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 - a\right)^{\frac{1}{4}}\left(bx^2 - 2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

3.319 $\int \frac{1}{\left(-2a-bx^2\right)\sqrt[4]{-a-bx^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0212074, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4
)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*S
qrt[2]*a^(3/4)*Sqrt[b])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.155954, size = 168, normalized size = 1.63

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{-a-bx^2}\,(2a+bx^2)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(2*a)])/((-a - b*
x^2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*
x^2)/(2*a)] - b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -(b*x^2)/(2
*a)] + AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-2\,a}\frac{1}{\sqrt[4]{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2+2\,a)(-bx^2-a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{2a\sqrt[4]{-a-bx^2}+bx^2\sqrt[4]{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

3.320 $\int \frac{1}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1 +
 x^2)^(1/4))]/(2*Sqrt[2])
```

Rubi [A] time = 0.0081616, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {398}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1 +
 x^2)^(1/4))]/(2*Sqrt[2])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(2-x^2\right)\sqrt[4]{-1+x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.137361, size = 115, normalized size = 2.17

$$-\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\left(x^2-2\right)\sqrt[4]{x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (-6*x*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/((-2 + x^2)*(-1 + x^2)^(1/4)*
(6*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/
2, x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+2}\frac{1}{\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] int(1/(-x^2+2)/(x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(x^2-1\right)^{\frac{1}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [B] time = 23.3346, size = 254, normalized size = 4.79

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{x^{4}+2\sqrt{2}\left(x^{2}-1\right)^{\frac{1}{4}}x^{3}+4\sqrt{x^{2}-1}x^{2}+4\sqrt{2}\left(x^{2}-1\right)^{\frac{3}{4}}x+4x^{2}-4}{x^{4}-4x^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/8*sqrt(2)*log(-(x^4 + 2*
sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 + 4*sqrt(2)*(x^2 - 1)^(3/
4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x^{2}\sqrt[4]{x^{2}-1}-2\sqrt[4]{x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{\left(x^{2}-1\right)^{\frac{1}{4}}\left(x^{2}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.321 $\int \frac{\left(a+bx^2\right)^{7/4}}{c+dx^2}\,dx$

Optimal. Leaf size=362

$$-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}-\frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad}{}$$

```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a + b*x^2)^(1
/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)
^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*d*(a + b*x^2)^(1/4))
 + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)
^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]
), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b*c) +
 a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x)
```

Rubi [A] time = 0.265366, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {402, 195, 229, 227, 196, 399, 490, 1218}

$$-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}-\frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```



```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a + b*x^2)^(1
/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)
^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*d*(a + b*x^2)^(1/4))
 + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)
^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]
), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b*c) +
 a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x)
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
```

```
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{7/4}}{c+dx^2}\,dx = \frac{b\int \left(a+bx^2\right)^{3/4}\,dx}{d} - \frac{(bc-ad)\int \frac{\left(a+bx^2\right)^{3/4}}{c+dx^2}\,dx}{d}$$

$$= \frac{2bx\left(a+bx^2\right)^{3/4}}{5d} + \frac{(3ab)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5d} - \frac{(b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{d^2}$$

$$= \frac{2bx\left(a+bx^2\right)^{3/4}}{5d} + \frac{\left(2(bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^2x} + \frac{\left(3ab\sqrt[4]{1+\frac{bx^2}{a}}\right.}{5d\sqrt[4]{a}}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx\left(a+bx^2\right)^{3/4}}{5d} - \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\right.}{d^{5/2}x}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx\left(a+bx^2\right)^{3/4}}{5d} - \frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}} + \frac{2\sqrt{a}\sqrt{b}(}{}$$

Mathematica [C] time = 0.465512, size = 346, normalized size = 0.96

$$\frac{x\left(\frac{6\left(bx^2\left(a+bx^2\right)\left(c+dx^2\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3ac\left(5a^2d+2abdx^2+2b^2x^2\left(c+dx^2\right)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}+\frac{bx^2\sqrt[4]{\frac{bx^2}{a}+}}{}\right.}{15d\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (x*((b*(-5*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(5*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/
c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
 + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(15*d*(a
 + b*x^2)^(1/4))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{7}{4}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.322 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2} dx$

Optimal. Leaf size=302

$$\frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d^2\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\sqrt{-}\cdots}{\cdots}$$

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a + b*x^2)^(3/4)) - (2*S
qrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b
*x^2)^(1/4)/a^(1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/(d^2*x)
```

Rubi [A] time = 0.206693, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {402, 195, 233, 231, 401, 108, 409, 1218}

$$\frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a + b*x^2)^(3/4)) - (2*S
qrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b
*x^2)^(1/4)/a^(1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/(d^2*x)
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/4}}{c+dx^2}\,dx = \frac{b\int \sqrt[4]{a+bx^2}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx}{d}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{(ab)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3d} - \frac{(b(bc-ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{d^2}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{2d^2x} + \frac{\left(ab\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{}{\left(1+\right.}}{3d\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}} - \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan\right.}{d^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}} - \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan\right.}{d^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}} - \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan\right.}{d^2\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.480034, size = 348, normalized size = 1.15

$$\frac{x\left(\frac{6\left(bx^2\left(a+bx^2\right)\left(c+dx^2\right)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3ac\left(3a^2d+2abdx^2+2b^2x^2\left(c+dx^2\right)\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)} + \frac{bx^2\left(\frac{bx^2}{a}\right.}{}\right.}{9d\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (x*((b*(-3*b*c + 4*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(3*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2
)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(9*d
*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2 + c} \left(bx^2 + a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{4}}}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{4}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

3.323 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2} dx$

Optimal. Leaf size=244

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}+\frac{2bx}{d\sqrt[4]{a+bx^2}}$$

```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*El
lipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(1/4)) + (a^(1/4)
*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1
/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```

Rubi [A] time = 0.150341, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {402, 229, 227, 196, 399, 490, 1218}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}+\frac{2bx}{d\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```



```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*El
lipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(1/4)) + (a^(1/4)
*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1
/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```


Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{d}$$

$$= -\frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx} + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{d\sqrt[4]{a+bx^2}}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} + \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^{3/2}x} - \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right.}{}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+}}{\sqrt[4]{}}\right.\right.}{d^{3/2}x}$$

Mathematica [C] time = 0.158206, size = 161, normalized size = 0.66

$$\frac{6acx\left(a+bx^2\right)^{3/4}F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(x^2\left(3bcF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-4adF_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+6acF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(3/4)*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + 3*b*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{4}}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.324 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$

Optimal. Leaf size=199

$$\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\right.}{dx}$$

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rubi [A] time = 0.147482, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {402, 233, 231, 401, 108, 409, 1218}

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\right.}{d\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{d}$$

$$= -\frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2dx} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{d\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}} + \frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx, x, \sqrt[4]{a}\right.}{dx}$$

$$= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx} - \frac{\sqrt{-}}{}$$

$$= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}}{}$$

Mathematica [C] time = 0.154318, size = 160, normalized size = 0.8

$$\frac{6acx\sqrt[4]{a+bx^2}F_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(x^2\left(bcF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-4adF_1\left(\frac{3}{2};-\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+6acF_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(1/4)*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + b*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{d x^{2}+c}\sqrt[4]{b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{\frac{1}{4}}}{d x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+b x^{2}}}{c+d x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{dx^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

3.325 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
 - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rubi [A] time = 0.108276, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {399, 490, 1218}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
 - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx = \frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x}$$

$$= -\frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt{d}x} + \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\cdots}\right.}{\sqrt{d}x}$$

$$= \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\sqrt{-bc+ad}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\sqrt{-bc+ad}x}$$

Mathematica [C] time = 0.0527963, size = 160, normalized size = 0.96

$$-\frac{6acxF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((a + b*x
^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

3.326 $\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}$$

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x) + (a^(1/4)*Sqr
t[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x)
```

Rubi [A] time = 0.117927, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {401, 108, 409, 1218}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x) + (a^(1/4)*Sqr
t[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x)
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
```

```
x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx = \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2x}$$

$$= -\frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x}$$

$$= \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\right.}{(bc-ad)x}$$

$$= \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-\right.}{(bc-ad)x}$$

Mathematica [A] time = 0.0374438, size = 123, normalized size = 0.81

$$\frac{bx\left(\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};-\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)+\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};-\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{a^{3/4}\sqrt{-\frac{bx^2}{a}}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (b*x*(EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), -ArcSin[(a + b*x^
2)^(1/4)/a^(1/4)], -1] + EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], -
ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]))/(a^(3/4)*(b*c - a*d)*Sqrt[-((b*x^2
)/a)])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{4}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

3.327 $\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=233

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\sqrt[4]{b}\cdots\right.\right.}{x(ad-bc)^{3/2}}$$

```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((
b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rubi [A] time = 0.160015, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {403, 197, 196, 399, 490, 1218}

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\sqrt[4]{b}\cdots\right.\right.}{x(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((
b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx = \frac{b\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{bc-ad}$$

$$= -\frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{a(bc-ad)\sqrt[4]{a+bx^2}}$$

$$= \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\left(\sqrt{d}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a}\right.}{(bc-ad)x}$$

$$= \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(-bc+ad)^{3/2}x} - \frac{\sqrt[4]{}}{}$$

Mathematica [C] time = 0.268941, size = 327, normalized size = 1.4

$$\frac{x\left(\frac{bdx^2\sqrt[4]{\frac{bx^2}{a}+1}F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} + \frac{6\left(bx^2\left(c+dx^2\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3ac\left(ad-b\left(c+2dx^2\right)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{d}{}\right.\right.}{\left(c+dx^2\right)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right.}{3a\sqrt[4]{a+bx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c + (6*(3*a*c*(a*d - b*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*AppellF1[3/2,
1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*
x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*
x^2)/c)])))))/(3*a*(-(b*c) + a*d)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{d x^{2}+c}\left(b x^{2}+a\right)^{-\frac{5}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+a\right)^{\frac{5}{4}}\left(d x^{2}+c\right)}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b x^{2}\right)^{\frac{5}{4}}\left(c+d x^{2}\right)}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

3.328 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{2bx}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}$$

```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(b*c - a*d)*(a
+ b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^2*x) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```

Rubi [A] time = 0.15701, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {403, 199, 233, 231, 401, 108, 409, 1218}

$$\frac{2bx}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}-\frac{\sqrt[4]{a}d}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```



```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(b*c - a*d)*(a
+ b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^2*x) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```


Rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

```
&& LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx = \frac{b\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{bc-ad}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3a(bc-ad)} - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + -$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}(bc-ad)\left(a+bx^2\right)^{3/4}} - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2bx}{3a(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}(bc-ad)\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}}{\sqrt{-bc}}\right.}{(bc}$$

Mathematica [C] time = 0.274444, size = 331, normalized size = 1.3

$$\frac{x\left(\frac{6\left(bx^2\left(c+dx^2\right)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3ac\left(3ad-3bc-2bdx^2\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{\left(c+dx^2\right)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)} - \frac{bdx^2\left(\frac{bx^2}{a}+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\right.}{c}\right.}{9a\left(a+bx^2\right)^{3/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (x*(-((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])/c) + (6*(3*a*c*(-3*b*c + 3*a*d - 2*b*d*x^2)*AppellF1[1/2,
 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*Appell
F1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 3
/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)])))))/(9*a*(-(b*c) + a*d)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{d x^2 + c} \left(b x^2 + a\right)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^2 + a\right)^{\frac{7}{4}} \left(d x^2 + c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{7}{4}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

3.329 $\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=274

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}(bc-ad)^2}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{ad-}}\right.}{x(ad-}$$

```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c - 8*a*d)*(1
 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)
*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```

Rubi [A] time = 0.381708, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 229, 227, 196, 399, 490, 1218}

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}(bc-ad)^2}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{ad-}}\right.}{x(ad-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```


```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c - 8*a*d)*(1
 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)
*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}dx = \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} - \frac{2\int \frac{\frac{1}{2}(-3bc+5ad)-\frac{3}{2}bdx^2}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}dx}{5a(bc-ad)}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{4\int \frac{\frac{1}{4}\left(-3b^2c^2+8abcd+5a^2d^2\right)-\frac{1}{4}bd(3bc-8ad)x}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}}{5a^2(bc-ad)^2}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}dx}{(bc-ad)^2} - \frac{(b(3bc-8a}{5a^2(b}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\left(2d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+\right.}\right.}{(bc-ad)^2x}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} - \frac{\left(d^{3/2}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2\sqrt{b}(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx}{a}}}{}$$

Mathematica [C] time = 0.703583, size = 419, normalized size = 1.53

$$\frac{x\left(\frac{bdx^2\sqrt[4]{\frac{bx^2}{a}+1}(8ad-3bc)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} - \frac{6\left(3ac\left(-a^2bd\left(10c+13dx^2\right)+5a^3d^2+ab^2\left(5c^2-16d^2x^4\right)+3b^3cx^2\left(c+2dx^2\right)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2(c\right.}{\left(a+bx^2\right)\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\right.\right.\right.}\right.}{15a^2\sqrt[4]{a+bx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*(-3*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5
/2, -((b*x^2)/a), -((d*x^2)/c)])/c - (6*(3*a*c*(5*a^3*d^2 + 3*b^3*c*x^2*(c
+ 2*d*x^2) - a^2*b*d*(10*c + 13*d*x^2) + a*b^2*(5*c^2 - 16*d^2*x^4))*Appell
F1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(9*a^2
```

```
*d - 3*b^2*c*x^2 - 4*a*b*(c - 2*d*x^2))*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x
^2)/c)])))))/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{9}{4}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

3.330 $\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=304

$$\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{2bx(5bc-12ad)}{21a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-}\right.}{x(bc-ad)^3}$$

```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*d)*x)/(21*
a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*c - 12*a*d)*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*(b*c
 - a*d)^2*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/((b*c - a*d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a
]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*
c - a*d)^3*x)
```

Rubi [A] time = 0.342644, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 233, 231, 401, 108, 409, 1218}

$$\frac{2bx(5bc-12ad)}{21a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+}}{\sqrt[4]{a}}\right.\right.}{x(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```


```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*d)*x)/(21*
a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*c - 12*a*d)*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*(b*c
 - a*d)^2*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/((b*c - a*d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a
]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*
c - a*d)^3*x)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx = \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} - \frac{2\int \frac{\frac{1}{2}(-5bc+7ad)-\frac{5}{2}bdx^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{7a(bc-ad)}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{4\int \frac{\frac{1}{4}\left(5b^2c^2-12abcd+21a^2d^2\right)+\frac{1}{4}bd(5b}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}}{21a^2(bc-ad)^2}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{d^2\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{(bc-ad)^2} + \frac{(b(5b}{}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{\left(d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+b}\right.}{2(bc-ad)^2x}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F}{21a^{3/2}(bc-ad)^2\left(a\right.}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F}{21a^{3/2}(bc-ad)^2\left(a\right.}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F}{21a^{3/2}(bc-ad)^2\left(a\right.}$$

Mathematica [C] time = 0.809633, size = 431, normalized size = 1.42

$$-\frac{x\left(\frac{6\left(3ac\left(-3a^2bd\left(14c+3dx^2\right)+21a^3d^2+ab^2\left(21c^2-20cdx^2-24d^2x^4\right)+5b^3cx^2\left(3c+2dx^2\right)\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bx^2\left(c+dx^2\right)\left(15a^2d+ab\left(12dx^2-8c\right)-5b^2cx^2\right)\left(\right.}{\left(a+bx^2\right)\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a}\right.\right.}\right.}{63a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] -(x*((b*d*(-5*b*c + 12*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1,
 5/2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(3*a*c*(21*a^3*d^2 + 5*b^3*c*x^2*
(3*c + 2*d*x^2) - 3*a^2*b*d*(14*c + 3*d*x^2) + a*b^2*(21*c^2 - 20*c*d*x^2 -
 24*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^
2*(c + d*x^2)*(15*a^2*d - 5*b^2*c*x^2 + a*b*(-8*c + 12*d*x^2))*(4*a*d*Appel
lF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4
, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*a*c*A
ppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1
[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1
, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(63*a^2*(b*c - a*d)^2*(a + b*x^2)^(
3/4))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{11}{4}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

3.331 $\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2} dx$

Optimal. Leaf size=340

$$\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{}}\right.\right.}{4cd^{5/2}x}$$

```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*(a + b*x^2
)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c - a*d)*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^
(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)
*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin
[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rubi [A] time = 0.294429, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {413, 530, 229, 227, 196, 399, 490, 1218}

$$\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{}}\right.\right.}{4cd^{5/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*(a + b*x^2
)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c - a*d)*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^
(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)
*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin
[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{a(bc+ad)+\frac{1}{2}b(5bc-ad)x^2}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} + \frac{(b(5bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd^2} - \frac{((bc-ad)(5bc+2ad))\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} - \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2cd^2x}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} + \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-}}\right.}{4cd^{5/2}x}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x\left(a+bx^2\right)^{3/4}}{2cd\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-b}}{}$$

Mathematica [C] time = 0.335801, size = 340, normalized size = 1.

$$\frac{x\left(\frac{6c\left(x^2\left(a+bx^2\right)(ad-bc)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6ac\left(2a^2d+abdx^2-b^2cx^2\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)} - bx^2\sqrt[4]{\frac{bx^2}{a}+1}(ad\right.}{12c^2d\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(-(b*(-5*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)]) + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x
^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)
*x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/
c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d
*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*
(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF
1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a + b*x^2)^
(1/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^{2}+c\right)^{2}}\left(bx^{2}+a\right)^{\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{\frac{7}{4}}}{\left(dx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{7}{4}}}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.332 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2} dx$

Optimal. Leaf size=279

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(1/4))/(2*c*d*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(
3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2,
 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(4*c*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/
4)/a^(1/4)], -1])/(4*c*d^2*x)
```

Rubi [A] time = 0.270767, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {413, 530, 233, 231, 401, 108, 409, 1218}

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(1/4))/(2*c*d*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(
3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2,
 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(4*c*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/
4)/a^(1/4)], -1])/(4*c*d^2*x)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/4}}{\left(c+dx^2\right)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} + \frac{\int \frac{a(bc+ad)+\frac{1}{2}b(3bc+ad)x^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} + \frac{(b(3bc+ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{4cd^2} - \frac{((bc-ad)(3bc+2ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} - \frac{\left((bc-ad)(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{8cd^2x} + \frac{\left(b(3\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}} + \frac{\left((bc-ad)(3bc+2ad\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}} - \frac{\left((3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\cdots}{\cdots}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\cdots}{\cdots}$$

Mathematica [C] time = 0.330475, size = 341, normalized size = 1.22

$$\frac{x\left(\frac{6c\left(x^2\left(a+bx^2\right)(ad-bc)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6ac\left(2a^2d+abdx^2-b^2cx^2\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}+bx^2\left(\frac{bx^2}{a}+1\right)^{3/4}}{12c^2d\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(b*(3*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -(
(b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x^2)*
AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)*x^2
*(a + b*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
+ 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x
^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF
1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a + b*x^2)^
(3/4))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{4}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{4}}}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{4}}}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

3.333 $\int \frac{\left(a+bx^2\right)^{3/4}}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{bx}{2cd\sqrt[4]{a+}}$$

```
[Out] -(b*x)/(2*c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c + d*x^2))
+ (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/4)*(b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c +
 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d],
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x)
```

Rubi [A] time = 0.222382, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {412, 530, 229, 227, 196, 399, 490, 1218}

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{bx}{2cd\sqrt[4]{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```



```
[Out] -(b*x)/(2*c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c + d*x^2))
+ (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/4)*(b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c +
 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d],
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x)
```


Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
```

```
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} - \frac{\int \frac{-a+\frac{bx^2}{2}}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{2c}$$

$$= \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} - \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd} + \frac{(bc+2ad)\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{4cd}$$

$$= \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} + \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2cdx} - \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{}{\sqrt[4]{}}}{4cd\sqrt[4]{a+b}}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} - \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right.}{4cd^{3/2}x}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\right.}{4cd^{3/2}\sqrt{}}$$

Mathematica [C] time = 0.202731, size = 232, normalized size = 0.75

$$\frac{x\left(\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}\right)}{c+dx^2}-\frac{bx^2\sqrt[4]{\frac{bx^2}{a}+1}F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}\right)}{12\sqrt[4]{a+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(-((b*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c^2) + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
)/(c + d*x^2)))/(12*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{4}}}{\left(c + dx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.334 $\int \frac{\sqrt[4]{a+bx^2}}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd\left(a+bx^2\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}$$

```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^
(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(3/4)
) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*S
qrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*
(b*c - a*d)*x)
```

Rubi [A] time = 0.205519, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {412, 530, 233, 231, 401, 108, 409, 1218}

$$\frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```


```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^
(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(3/4)
) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*S
qrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*
(b*c - a*d)*x)
```


Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
```

```
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{\left(c+dx^2\right)^2}\,dx = \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} - \frac{\int \frac{-a-\frac{bx^2}{2}}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2c}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{4cd} - \frac{\left(\frac{bc}{2}-ad\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2cd}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{4cdx} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3}}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} + \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad\right.}\right.}{cdx}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1}}\right.}{2cd(bc-ad)x}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\right.\right.}{4cd(bc-ad)x}$$

Mathematica [C] time = 0.192904, size = 232, normalized size = 0.83

$$\frac{x\left(\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}\right)}{c+dx^2}+\frac{bx^2\left(\frac{bx^2}{a}+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}\right)}{12\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((b*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/c^2 + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 3/4, 1, 3/2, -
((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a)
, -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*
x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))
/(c + d*x^2)))/(12*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a + bx^2}}{\left(c + dx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{\left(dx^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

3.335 $\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=336

$$\frac{bx}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\right.}{4c\sqrt{d}x(ad-bc)}$$

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4))/(2*c*(b
*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (a^
(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqr
t[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b
*c) + a*d)^(3/2)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticP
i[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rubi [A] time = 0.254961, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {414, 530, 229, 227, 196, 399, 490, 1218}

$$\frac{bx}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\right.}{4c\sqrt{d}x(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4))/(2*c*(b
*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (a^
(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqr
t[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b
*c) + a*d)^(3/2)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticP
i[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
```

```
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2}\,dx = -\frac{dx\left(a+bx^2\right)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad+\frac{1}{2}bdx^2}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4c(bc-ad)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^2\right)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2c(bc-ad)x} +$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx\left(a+bx^2\right)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right.}\right.}{4c\sqrt{d}(bc-ad)x}$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx\left(a+bx^2\right)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{a}(}{}$$

Mathematica [C] time = 0.242853, size = 392, normalized size = 1.17

$$\frac{-dx^3\left(6c\left(a+bx^2\right)-bx^2\sqrt[4]{\frac{bx^2}{a}+1}\left(c+dx^2\right)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\right.\right.}{12c^2\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]*(-6*c*(-2*
b*c + 2*a*d + b*d*x^2) + b*d*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1
[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]) - d*x^3*(6*c*(a + b*x^2) -
b*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2
)/a), -((d*x^2)/c)])*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{1}{4}} \left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

3.336 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2} dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2c\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\right.\right.}{4cx(bc-ad)^2}$$

```
[Out] -(d*x*(a + b*x^2)^(1/4))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c
 - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*El
lipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/
a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x
^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)
^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rubi [A] time = 0.23078, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {414, 530, 233, 231, 401, 108, 409, 1218}

$$-\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-\right.}{4cx(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x*(a + b*x^2)^(1/4))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c
 - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*El
lipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/
a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x
^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)
^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{1}{2}bdx^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{4c(bc-ad)} + \frac{(5bc-2ad)\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{8c(bc-ad)x} - \cdots$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} - \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\cdots\right.}{\cdots}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\cdots\right.}{\cdots}$$

$$= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} + \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{b\cdots}{}}}{\cdots}$$

Mathematica [C] time = 0.345522, size = 336, normalized size = 1.15

$$\frac{x\left(\frac{bdx^2\left(\frac{bx^2}{a}+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{ad-bc} + \frac{c\left(36ac\left(2ad-2bc+bdx^2\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6dx^2\left(a+bx^2\right)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\cdots\right.\right.\right.}{\left(c+dx^2\right)(bc-ad)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\cdots\right.\right.}\right)}{12c^2\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/(-(b*c) + a*d) + (c*(36*a*c*(-2*b*c + 2*a*d + b*d*x^2)*Appe
llF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 6*d*x^2*(a + b*x^2)*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF
1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((b*c - a*d)*(c + d*x^2)
*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*
d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3
/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*(a + b*x^2)^(3/4)
)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

3.337 $\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=314

$$-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c\sqrt[4]{a+bx^2}(bc-ad)^2}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{si}\right.}{4cx(ad-bc)^{5/2}}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c +
a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*
Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) - (a^(1/4)*Sqrt[d]*(7*b*c - 2*a*
d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^(1
/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])
/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) +
 a*d)^(5/2)*x)
```

Rubi [A] time = 0.395199, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 229, 227, 196, 399, 490, 1218}

$$-\frac{dx}{2c\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c\sqrt[4]{a+bx^2}(bc-ad)^2}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\text{si}\right.}{4cx(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```


```
[Out] -(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c +
a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*
Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) - (a^(1/4)*Sqrt[d]*(7*b*c - 2*a*
d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^(1
/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])
/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) +
 a*d)^(5/2)*x)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{3}{2}bdx^2}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(2b^2c^2+4abcd-a^2d^2\right)+\frac{1}{4}bd(4bc+}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}}{ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} - \frac{(d(7bc-2ad))\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx}}{4c(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} - \frac{\left(d(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}}{2c}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} + \frac{\left(\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-}}\right.}{4c(bc-ad)^2x}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\left(c+dx^2\right)} + \frac{\sqrt{b}(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{a}\sqrt{a}}{}$$

Mathematica [C] time = 0.498465, size = 380, normalized size = 1.21

$$\frac{x\left(\frac{c\left(36ac\left(2a^2d^2+abd\left(dx^2-4c\right)+2b^2c\left(c+2dx^2\right)\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6x^2\left(a^2d^2+abd^2x^2+4b^2c\left(c+dx^2\right)\right)\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\right.\right.\right.}{\left(c+dx^2\right)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}}{12ac^2\sqrt[4]{a+bx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(-(b*d*(4*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)]) + (c*(36*a*c*(2*a^2*d^2 + a*b*d*(-4*c + d*x^
2) + 2*b^2*c*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] - 6*x^2*(a^2*d^2 + a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d*Appell
F1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1
```

```
, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4
, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])))))/(12*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

3.338 $\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}c\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{bx(3ad+4bc)}{6ac\left(a+bx^2\right)^{3/4}(bc-ad)^2}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}$$

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (d*x)/(2*c*
(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c + 3*a*d)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*
(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*S
qrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rubi [A] time = 0.382358, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 233, 231, 401, 108, 409, 1218}

$$\frac{bx(3ad+4bc)}{6ac\left(a+bx^2\right)^{3/4}(bc-ad)^2}-\frac{dx}{2c\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c\left(a+bx^2\right)^{3/4}(bc-ad)^2}-\sqrt[4]{a}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (d*x)/(2*c*
(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c + 3*a*d)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*
(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*S
qrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{5}{2}bdx^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-2b^2c^2+12abcd-3a^2d^2\right)}{\left(a+bx^2\right)^{3/4}\left(c\cdots\right.}}{3ac(bc-\cdots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{(d(9bc-2ad))\int \frac{\cdots}{\left(a+bx^{2}\cdots\right.}}{4c(bc-ad)\cdots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} - \frac{\left(d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\cdots}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx\cdots}{a}\right.}{6\sqrt{a}c(bc-a\cdots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx\cdots}{a}\right.}{6\sqrt{a}c(bc-a\cdots}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx\cdots}{a}\right.}{6\sqrt{a}c(bc-a\cdots}$$

Mathematica [C] time = 0.538072, size = 387, normalized size = 1.12

$$\frac{x\left(\frac{c\left(36ac\left(6a^2d^2+3abd\left(dx^2-4c\right)+2b^2c\left(3c+2dx^2\right)\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6x^2\left(3a^2d^2+3abd^2x^2+4b^2c\left(c+dx^2\right)\right)\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\cdots\right.\right.\right.}{\left(c+dx^2\right)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\cdots}{36ac^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(b*d*(4*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)] + (c*(36*a*c*(6*a^2*d^2 + 3*a*b*d*(-4*c + d*x
^2) + 2*b^2*c*(3*c + 2*d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] - 6*x^2*(3*a^2*d^2 + 3*a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d
*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/
2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1
[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3
/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)])))))/(36*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^{2}+c\right)^{2}}\left(bx^{2}+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{7}{4}}\left(dx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

3.339 $\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=371

$$\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\right|}{4cx(ad-bc)^{7/2}}-$$

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) - (d*x)/(2*c
*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*(12*b^2*c^2 - 52*a*b
*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)
*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(
7/2)*x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(
Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rubi [A] time = 0.551699, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 229, 227, 196, 399, 490, 1218}

$$\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\right|}{4cx(ad-bc)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) - (d*x)/(2*c
*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*(12*b^2*c^2 - 52*a*b
*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)
*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(
7/2)*x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(
Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{7}{2}bdx^2}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-6b^2c^2+20abcd-5a^2d^2 \ldots\right.}{\left(a+bx^2\right)^{5/4}\left(\ldots\right.}}{5ac(bc-\ldots}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} + \frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} + \frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} + \frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} - \frac{\left(d^{3/2}(11bc-2ad)\sqrt{-\ldots}\right.}{\ldots}$$

$$= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} + \frac{\sqrt{b}\left(12b^2c^2-52abcd \ldots\right.}{10a\ldots}$$

Mathematica [C] time = 1.09193, size = 536, normalized size = 1.44

$$\frac{bdx^3\sqrt[4]{\frac{bx^2}{a}+1}\left(5a^2d^2+52abcd-12b^2c^2\right)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{6c\left(x^3\left(a^2b^2d\left(56c^2+56cdx^2+5d^2x^4\right)+10a^3bd^3x^2+5a^4d^3+4ab^3c\left(-4c\ldots\right.\right.\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*d*(-12*b^2*c^2 + 52*a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(1/4)*Appel
lF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*x*(10*a^4*
d^3 + 15*a^3*b*d^2*(-2*c + d*x^2) - 6*b^4*c^2*x^2*(c + 2*d*x^2) + a^2*b^2*d
*(30*c^2 + 26*c*d*x^2 + 5*d^2*x^4) + 2*a*b^3*c*(-5*c^2 + 5*c*d*x^2 + 26*d^2
*x^4))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^3*(5*a^4*
d^3 + 10*a^3*b*d^3*x^2 - 12*b^4*c^2*x^2*(c + d*x^2) + a^2*b^2*d*(56*c^2 + 5
6*c*d*x^2 + 5*d^2*x^4) + 4*a*b^3*c*(-4*c^2 + 9*c*d*x^2 + 13*d^2*x^4))*(4*a*
d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2
, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(6*a
*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Appe
llF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(60*a^2*c^2*(b*c - a*d)^3*(a + b*x
^2)^(1/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

3.340 $\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=419

$$\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{42a^{3/2}c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{42a^2c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{}}{}$$

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) + (b*(20*b^2
*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d)^3*(a + b*x^2)^(3/4
)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)*(c + d*x^2)) + (Sqrt[b]*(20*b
^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) +
(a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt
[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
 - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1
])/(4*c*(b*c - a*d)^4*x)
```

Rubi [A] time = 0.484125, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 527, 530, 233, 231, 401, 108, 409, 1218}

$$\frac{bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{42a^2c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{42a^{3/2}c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13b}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) + (b*(20*b^2
*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d)^3*(a + b*x^2)^(3/4
)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)*(c + d*x^2)) + (Sqrt[b]*(20*b
^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) +
(a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt
[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
 - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
```

```
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1
])/(4*c*(b*c - a*d)^4*x)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
```

```
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)^2}\,dx = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)} + \frac{\int \frac{2bc-ad-\frac{9}{2}bdx^2}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)} - \frac{\int \frac{\frac{1}{2}\left(-10b^2c^2+28abcd-7a^2\right.}{\left(a+bx^2\right)^{7/4}}}{7ac(bc}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7}}$$

Mathematica [C] time = 0.975074, size = 550, normalized size = 1.31

$$\frac{bdx^3\left(\frac{bx^2}{a}+1\right)^{3/4}\left(21a^2d^2+76abcd-20b^2c^2\right)F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ad-bc)^3} + \frac{6c\left(x^3\left(a^2b^2d\left(88c^2+88cdx^2+21d^2x^4\right)+42a^3bd^3x^2+21a^4d^3+4ab^3c\left(-8c^2+11cdx^2+19d^2x^4\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] ((b*d*(-20*b^2*c^2 + 76*a*b*c*d + 21*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(3/4)*App
ellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-(b*c) + a*d)^3 + (6*
c*(-6*a*c*x*(42*a^4*d^3 + 63*a^3*b*d^2*(-2*c + d*x^2) - 10*b^4*c^2*x^2*(3*c
 + 2*d*x^2) + a^2*b^2*d*(126*c^2 - 38*c*d*x^2 + 21*d^2*x^4) + 2*a*b^3*c*(-2
1*c^2 + 41*c*d*x^2 + 38*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^3*(21*a^4*d^3 + 42*a^3*b*d^3*x^2 - 20*b^4*c^2*x^2*(c + d*
x^2) + 4*a*b^3*c*(-8*c^2 + 11*c*d*x^2 + 19*d^2*x^4) + a^2*b^2*d*(88*c^2 + 8
8*c*d*x^2 + 21*d^2*x^4))*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])
))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)*(6*a*c*AppellF1[1/2, 3/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d
*x^2)/c)]))))/(252*a^2*c^2*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{11}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}\left(dx^2+c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

3.341 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.0429968, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx \\ &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q \, dx \\ &= x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\end{aligned}$$

Mathematica [B] time = 0.220684, size = 172, normalized size = 2.18

$$\frac{3acx\left(a+bx^2\right)^p \left(c+dx^2\right)^q F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a
), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)
/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p\left(dx^2+c\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.342 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=296

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-9a^2bcd^2(2p+7)+15a^3d^3+3ab^2c^2d\left(4p^2+24p+35\right)-b^3c^3\left(8p^3+60p^2+142p+105\right)\right)\,{}_2}{b^3(2p+3)(2p+5)(2p+7)}$$

```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))*x*(a + b*
x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*(5*a*d - b*c*(11 + 2
*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*(5 + 2*p)*(7 + 2*p)) + (d*x*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d
^2*(7 + 2*p) + 3*a*b^2*c^2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 6
0*p^2 + 8*p^3))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.27632, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {416, 528, 388, 246, 245}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-9a^2bcd^2(2p+7)+15a^3d^3+3ab^2c^2d\left(4p^2+24p+35\right)-b^3c^3\left(8p^3+60p^2+142p+105\right)\right)\,{}_2}{b^3(2p+3)(2p+5)(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^3,x]
```


```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))*x*(a + b*
x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*(5*a*d - b*c*(11 + 2
*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*(5 + 2*p)*(7 + 2*p)) + (d*x*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d
^2*(7 + 2*p) + 3*a*b^2*c^2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 6
0*p^2 + 8*p^3))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx = \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^2}{b(7+2p)} + \frac{\int \left(a+bx^2\right)^p \left(c+dx^2\right)\left(-c(ad-bc(7+2p))-d(5ad-bc(11\right.}{b(7+2p)}$$

$$= -\frac{d(5ad-bc(11+2p))x\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)}{b^2(5+2p)(7+2p)} + \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^2}{b(7+2p)} + \frac{\int \left(a+bx^2\right.}{}$$

$$= \frac{d\left(15a^2d^2-8abcd(6+p)+b^2c^2\left(57+28p+4p^2\right)\right)x\left(a+bx^2\right)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))}{b^2(5+}$$

$$= \frac{d\left(15a^2d^2-8abcd(6+p)+b^2c^2\left(57+28p+4p^2\right)\right)x\left(a+bx^2\right)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))}{b^2(5+}$$

$$= \frac{d\left(15a^2d^2-8abcd(6+p)+b^2c^2\left(57+28p+4p^2\right)\right)x\left(a+bx^2\right)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))}{b^2(5+}$$

Mathematica [A] time = 5.06293, size = 136, normalized size = 0.46

$$\frac{1}{35}x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(dx^2\left(35c^2\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+dx^2\left(21c\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+5dx^2\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*(35*c^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(35*c^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + d*x^2*(21*c*Hyp
ergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)] + 5*d*x^2*Hypergeometric2F1[7/2,
 -p, 9/2, -((b*x^2)/a)]))))/(35*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)^3 \left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 52.5896, size = 121, normalized size = 0.41

$$a^pc^3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+a^pc^2dx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{3a^pcd^2x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^pd^3x^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**3,x)
```

```
[Out] a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*c**2*
d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*a**p*c*d**2*x
**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*d**3*x**7*h
```

```
yper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)^3 \left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

3.343 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 dx$

Optimal. Leaf size=176

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2d^2-2abcd(2p+5)+b^2c^2\left(4p^2+16p+15\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)}-\frac{dx\left(a+bx^2\right)^{p+1}(3ad-bc}{b^2(2p+3)(2p+5}$$

```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p)*(5 + 2*p
))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) + ((3*a^2*d^2 - 2
*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p^2))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Rubi [A] time = 0.120852, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 388, 246, 245}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2d^2-2abcd(2p+5)+b^2c^2\left(4p^2+16p+15\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)}-\frac{dx\left(a+bx^2\right)^{p+1}(3ad-bc}{b^2(2p+3)(2p+5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^2,x]
```



```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p)*(5 + 2*p
))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) + ((3*a^2*d^2 - 2
*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p^2))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)
/a)^p)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 dx = \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)}{b(5+2p)} + \frac{\int \left(a+bx^2\right)^p \left(-c(ad-bc(5+2p))-d(3ad-bc(7+2p))x^2\right) \, d}{b(5+2p)}$$

$$= -\frac{d(3ad-bc(7+2p))x\left(a+bx^2\right)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)}{b(5+2p)} + \frac{\left(3a^2d^2-2abcd(5+2\right.}{}$$

$$= -\frac{d(3ad-bc(7+2p))x\left(a+bx^2\right)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)}{b(5+2p)} + \frac{\left(\left(3a^2d^2-2abcd(5+2\right.\right.}{}$$

$$= -\frac{d(3ad-bc(7+2p))x\left(a+bx^2\right)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)}{b(5+2p)} + \frac{\left(3a^2d^2-2abcd(5+2\right.}{}$$

Mathematica [A] time = 5.04245, size = 106, normalized size = 0.6

$$\frac{1}{15}x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\left(10c\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+3dx^2\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*(15*c^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(10*c*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 3*d*x^2*Hypergeom
etric2F1[5/2, -p, 7/2, -((b*x^2)/a)])))/(15*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)^2 \left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^4+2\,cdx^2+c^2\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 22.8613, size = 88, normalized size = 0.5

$$a^p c^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{2a^p c d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p d^2 x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**2,x)
```

```
[Out] a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + 2*a**p*c*d
*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*d**2*x**5
*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)^2 \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

3.344 $\int \left(a+bx^2\right)^p \left(c+dx^2\right) dx$

Optimal. Leaf size=93

$$\frac{dx\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ad-bc(2p+3))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}$$

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - ((a*d - b*c*(3 + 2*p))*x*(a + b*x
^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x
^2)/a)^p)
```

Rubi [A] time = 0.0390779, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{dx\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((c - (a*d)/(3*b + 2*b*p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^p \left(c+dx^2\right) dx &= \frac{dx\left(a+bx^2\right)^{1+p}}{b(3+2p)} - \left(-c+\frac{ad}{3b+2bp}\right)\int \left(a+bx^2\right)^p dx \\ &= \frac{dx\left(a+bx^2\right)^{1+p}}{b(3+2p)} - \left(\left(-c+\frac{ad}{3b+2bp}\right)\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx \\ &= \frac{dx\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(c-\frac{ad}{3b+2bp}\right)x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.029336, size = 90, normalized size = 0.97

$$\frac{x\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left((bc(2p+3)-ad)\, {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+d\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\right)}{b(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*(d*(a + b*x^2)*(1 + (b*x^2)/a)^p + (-(a*d) + b*c*(3 + 2*p)
)*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(b*(3 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)\left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^2 + c\right)\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 10.7052, size = 53, normalized size = 0.57

$$a^p c x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c),x)
```

```
[Out] a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*x**3*h
yper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)\left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

3.345 $\int \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=44

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rubi [A] time = 0.0092847, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^p\,dx = \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0034102, size = 44, normalized size = 1.

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Maple [F] time = 0.002, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p,x)
```

```
[Out] int((b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [C] time = 2.45905, size = 22, normalized size = 0.5

$$a^p x_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

3.346 $\int \frac{(a+bx^2)^p}{c+dx^2} \, dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*
(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.026394, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*
(1 + (b*x^2)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{c+dx^2}\,dx$$
$$= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

Mathematica [B] time = 0.178261, size = 162, normalized size = 2.84

$$-\frac{3acx(a+bx^2)^p F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(2x^2\left(adF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-bcpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)*(-3*a*c*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/
c)] + 2*x^2*(-(b*c*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c
)]) + a*d*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^p}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**p/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

3.347 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
2*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0251872, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
2*(1 + (b*x^2)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(c+dx^2)^2}\,dx$$
$$= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

Mathematica [B] time = 0.178998, size = 162, normalized size = 2.84

$$-\frac{3acx(a+bx^2)^p F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^2\left(-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2adF_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^2*(-3*a*c*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 2*a*d*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{d^2x^4+2\,cdx^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

3.348 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
3*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0251613, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
3*(1 + (b*x^2)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{\left(c+dx^2\right)^3}\,dx = \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\left(c+dx^2\right)^3}\,dx$$
$$= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

Mathematica [B] time = 0.234635, size = 162, normalized size = 2.84

$$-\frac{3acx\left(a+bx^2\right)^p F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)^3\left(-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-3adF_1\left(\frac{3}{2};-p,4;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^3*(-3*a*c*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 3*a*d*AppellF1[3/2, -p, 4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

3.349 $\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2*
a*d)))
```

Rubi [A] time = 0.0200724, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {381}

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/(2*b*c
 - 2*a*d)),x]
```

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2*
a*d)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx=\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Mathematica [A] time = 0.0311385, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^2\right)^{\frac{bc}{2ad-2bc}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/
(2*b*c - 2*a*d)),x]
```

```
[Out] (x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d))
)/(a*c)
```

Maple [A] time = 0.004, size = 71, normalized size = 1.3

$$\frac{x}{ac}\left(bx^2+a\right)^{1-\frac{2\,ad-3\,bc}{2\,ad-2\,bc}}\left(dx^2+c\right)^{1-\frac{3\,ad-2\,bc}{2\,ad-2\,bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x)
```

```
[Out] (b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d-2*b*c)/(a
*d-b*c))/a/c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^{-\frac{bc}{2\,(bc-ad)}-1}\left(dx^2+c\right)^{\frac{ad}{2\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Fricas [A] time = 1.67663, size = 182, normalized size = 3.43

$$\frac{bdx^5 + (bc + ad)x^3 + acx}{\left(bx^2 + a\right)^{\frac{3\,bc-2\,ad}{2\,(bc-ad)}}\left(dx^2 + c\right)^{\frac{2\,bc-3\,ad}{2\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*a*d)/(b*c
- a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-2*a*d+2
*b*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{-\frac{bc}{2\,(bc-ad)}-1}\left(dx^2 + c\right)^{\frac{ad}{2\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```